

Introduction

Stellar Magnitudes:

Greek astronomers were thought to be the first people who classified stars by their brightness. Around 129 B. C. Hipparchus divided the stars into six brightness classes and he called the stars that appeared brightest (by using naked eye, without telescopes) “first magnitude” stars, and the faintest visible stars the “sixth magnitude” stars. He ranked the 1,080 stars in his catalogue in this simple way. Our knowledge of Hipparchus is via Claudius Ptolemy, who, in the 2nd century A. D., utilized a similar stellar brightness scheme in his Almagest. Of the 1,028 stars in his catalogue, 156 were given the descriptions of a little more or a little less than the integer values, but his precision was off by ± 0.6 magnitudes.

1. Apparent magnitude (m): is the apparent brightness of stars as seen by the human eyes on earth. Astronomers found that Hipparchus magnitude scale was roughly Logarithmic. That is, each magnitude step corresponded to a fixed brightness ratio or factor. The first magnitude stars are roughly 2.5 times as bright as the second magnitude stars; the second magnitude stars are roughly 2.5 times as bright as the third magnitude stars etc.

Old astronomers used magnitude scale as follow:

Star magnitude	1	2	3	4	5	6
Brightness Ratio	1	2.512	6.310	15.85	39.81	100

Based on the Hipparchus magnitude system, but using modern brightness measurements. Norman Pogson (1829-1891) decided to define a magnitude system where 5 magnitudes correspond to exactly a factor 100 in brightness or flux. Thus, each magnitude is exactly $100^{1/5}$ or about 2.512 times as bright as the next.

- * The brightest stars has magnitude = 1
- * The faintest stars that could be observed by the naked eye = 6.
- * stars of magnitudes larger than 6 needs telescopes
- * The larger the number the fainter the star. Therefore we can say that the magnitude scale is a measure of how the star is faint.

- * Arabs used this magnitude scale to measure and test the eye efficiency. Who cannot observe star of magnitude 6 his eye measure is 5/6 or lower depending on the stellar magnitudes that he can see. Who can see the stars with $m = 6$ is the best eye visibility.
- * We can reorganize the magnitude scale as shown in Table-1:

Table-1: Shows apparent magnitudes for some of stars and limits.

Approximate V Apparent Magnitudes for some bright stars and limits		
Object	V	Note
Sun	-26.7	
Full Moon	-12.0	
Venus	-4.7	at brightest
Sirius	-1.4	α Canis Major
Vega	0.0	α Lyra
Castor	0.0	α Gemini
Deneb	0.1	α Cygnus
Altair	0.2	α Aquila
Polaris	0.6	α Ursa Minor
Pollux	1.0	β Gemini
Betelgeuse	1.5	α Orion
Aldebaran	1.5	α Taurus
Antares	1.9	α Scorpius
Naked eye limit	~6.5	dark sky
Visual limit - 6 inch telescope	~13	dark sky
CCD 5 minutes - 6 inch telescope	~16	dark sky (S/N=5)
HST	~30	deep field

The introduction of the telescope allowed astronomers to see fainter stars. Galileo (1564-1642) estimated he could see magnitude +12 stars. However, it wasn't until William Herschel (1738-1822) that magnitude accuracy reached ± 0.2 magnitudes. He felt that brightness was just based on stellar distance, and (unsuccessfully) attempted to delineate the Milky Way based on the distribution of stars of different magnitudes. Today, the eyeball has been supplanted by CCD photometry for very accurate magnitude estimation.

2. Brightness (b): is the flux of radiation received (arrived at Earth) from the star.

The relation between the magnitudes and the brightness are established in the late 18th and 19th centuries, several astronomers performed experiments to see how the magnitude scale was related to the amount of energy received. It appeared that a given difference in magnitude, at any point in the magnitude scale, corresponded to a ratio of the brightness's. In 1856 Pogson proposed that the value of the ratio, corresponding to a magnitude difference of **five**, should be

100. Thus, the ratio of two stellar brightness's, B1 and B2, can be related to their magnitudes, m1 and m2, by the equation

$$\frac{B_1}{B_2} = 2.512^{-(m_1 - m_2)} \dots\dots\dots 1$$

Since (2.512)⁵ equals 100. This is known as **Pogson's equation**. The negative sign before the bracketed exponent reflects the fact that magnitude values increase as the brightness falls. By taking logarithms of equation above, we obtain

$$\log_{10} \left(\frac{B_1}{B_2} \right) = -(m_1 - m_2) \log_{10}(2.512) = -0.4(m_1 - m_2)$$

$$m_1 - m_2 = -2.5 \log_{10} \left(\frac{B_1}{B_2} \right) \dots\dots\dots 2$$

More generally, Pogson's equation in the style of equation (1) can be presented in a simplified form as

$$m = k - 2.5 \log_{10} B \dots\dots\dots 3$$

Where *m* is the magnitude of the star, *B* its apparent brightness and *k* some constant. The value of *k* is chosen conveniently by assigning a magnitude to one particular star such as α Lyr, or set of stars, thus fixing the zero point to that magnitude scale. It should also be noted that the numerical coefficient of 2.5 in equation (2) is exact and is not a rounded value of 2.512 from equation (1).

3. Luminosity (L): is the energy (J) radiated by the star per time unit (s) over all wavelengths and over 4π stellar radians. Unit is Watt.

We define apparent magnitude as a logarithmic luminosity ratio of a body to some standard:

$$m = -2.5 \log \left(\frac{L}{L_0} \right) \dots\dots\dots 4$$

Where *m* is the apparent magnitude, *L* is the luminosity of a body as determined through a V

Filter and L_0 is the standard's luminosity, the luminosity of Vega. Vega has been assigned an $m = 0$ although its actual magnitude is 0.03. It follows that

$$m_1 - m_2 = -2.5 \log\left(\frac{L_1}{L_2}\right) \dots\dots\dots 5$$

Equation 4 can be rewritten as:

$$L = L_0 \times 10^{-m/2.5} \dots\dots\dots 6$$

EXAMPLE 1. The Sun is about 480,000 times more luminous than the full Moon. What is the difference in their apparent magnitude?

EXAMPLE 2. The individual apparent magnitudes of two binary stars are +2 and +4. What is the combined apparent magnitude of the binary system?

EXAMPLE 3. Sirius is 8.6 light years distant, with an apparent magnitude of -1.5. What would the apparent magnitude of the Sun be at Sirius' distance?

4. The inverse square law for light:

The brightness (B) of an object decays with distance (d) as follows:

$$B \propto \frac{1}{d^2} \dots\dots\dots 7$$

As the star distance increases its brightness decreases as the square of the star distance from us. This is a general physical law.

5. Absolute Magnitude (M):

Since stellar brightness depends on distance, astronomers utilize magnitudes at a standard distance 10 parsecs (32.62 ly), to compare stellar intrinsic luminosity. We already determined in Example 3 that if the Sun was at Sirius's distance, it would appear 28.7 magnitudes fainter.

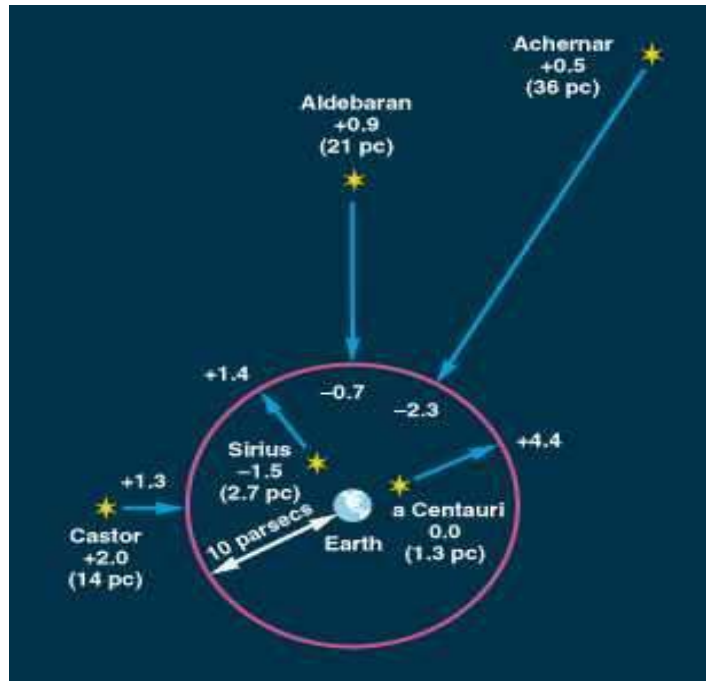
We'll relate three quantities: apparent magnitude, m ; absolute magnitude, M ; and stellar distance in parsecs, d . (1 parsec = 3.26 light years.)

$$m - M = 5 \log (d) - 5 \dots\dots\dots 8$$

Equation 4 can be rewritten as follows:

$$d = 10^{\frac{m-M+5}{5}} \dots\dots\dots 8$$

The expression $(m - M)$ is called the distance modulus. The distance modulus is negative if a star is closer than 10 parsecs.



EXAMPLE 4. Find the absolute magnitude of the Sun with $m = -26.8$.

We need the Sun's distance to be in parsecs. There are 206,265 AU in a parsec, so $d = 1/206,265$. Using Equation 7,

$$M = m - 5 \log (d) + 5 = -26.8 - 5 \log (1/206,265) + 5 = 4.77$$

EXAMPLE 5. A star's $m = 8$ and $M = -2$. Find the star's distance.

Notice that the distance modulus is $8 - (-2) = 10$. This indicates the star is more distant than 10 parsecs. Using Equation 5,

$$d = 10^{\frac{m-M+5}{5}} = 10^{\frac{8-(-2)+5}{5}} = 10^3 = 1,000 \text{ parsecs.}$$

Luminosity, Radius, and Temperature

If a star is considered simply as a spherical source radiating as a black body, according to its surface temperature and its surface area. This total output is referred to as the stellar luminosity, L , and may be expressed as.

$$L = 4\pi R^2 \sigma T^4 \quad \text{W}$$

This seems complicated, but if we express Luminosity, Radius, and Temperature in terms of the Sun, we get a much simpler form:

$$\frac{L_{star}}{L_{sun}} = \left(\frac{R_{star}}{R_{sun}}\right)^2 \left(\frac{T_{star}}{T_{sun}}\right)^4 \dots\dots\dots 9$$

Example 6. Suppose we want to find the luminosity of a star 10 times the Sun's radius but only half as hot. How luminous is it?

$$\frac{L_{star}}{L_{sun}} = \left(\frac{10}{1}\right)^2 \left(\frac{1}{2}\right)^4 = 6.25 \quad \text{the star has 6.25 times the Sun's luminosity}$$

Bolometric magnitude (M_{bol}):

If we were able to measure the radiation at all wavelength, we would get the Bolometric magnitude M_{bol} of star. This magnitude is given by

$$M_{bol} = m_v - BC$$

Where m_v is the visual magnitude (the magnitude corresponding to the sensitivity of the eye which occurred at wavelength of 550 nm. BC is the bolometric correction

Example of Magnitudes

Example 1

A star is 100 parsecs away with $m = +7.3$, What absolute magnitude is M ?

Example 2

A star is 2.5 parsecs away with $m = +1.3$, What is absolute magnitude M ?

Table 3.4 The difference in magnitude related to the *ratio* of brightness^a

$\Delta m = m_2 - m_1$	Approximation: $2.5^{\Delta m}$	f_1/f_2 (Actual)
0	1	1.000
1	2.5	2.512
2	$2.5^2 = 2.5 \times 2.5$	6.310
3	$2.5^3 = 2.5 \times 2.5 \times 2.5$	15.85
4	$2.5^4 = 2.5 \times 2.5 \times 2.5 \times 2.5$	39.81
5	100 (by definition)	100
6	$2.5^6 = 2.5^{5+1} = 100 \times 2.5$	251.2
7	$2.5^7 = 2.5^{5+2} = 100 \times 6.31$	631.0
10	$2.5^{10} = (2.5^5)^2 = 100^2$	10 000
15	$2.5^{15} = (2.5^5)^3 = 100^3$	1 000 000

^a See Appendix A, p. 230, if you do not follow all the arithmetic here.

UBV photometric system:

The optical region is that region limited by the atmosphere on the short wavelength. Within the optical region, we usually further limit the wavelengths observed by use of Filters. Filters are optical components that only allow certain wavelengths to pass through them.

One widely used filter system in the optical region of the spectrum is called the UBV system. The UBV photometric system (Ultraviolet, Blue, Visual), also called the Johnson system (or Johnson-Morgan system (1953)). The letters correspond to different filters: U for Ultraviolet, B for Blue and V for Visual. The central wavelengths of the filters are roughly U – 3600 Å, B – 4400 Å, V – 5500 Å. The passband, or wavelength range passed by each filter, is roughly 1000 Å for each filter in the broadband UBV system. For example the B filter passes only light from about 3900 Å - 4900 Å. The color of an object related to the variation of flux with wavelength. ($1\text{Å} = 10^{-10}\text{ m} = 0.1\text{nm}$).

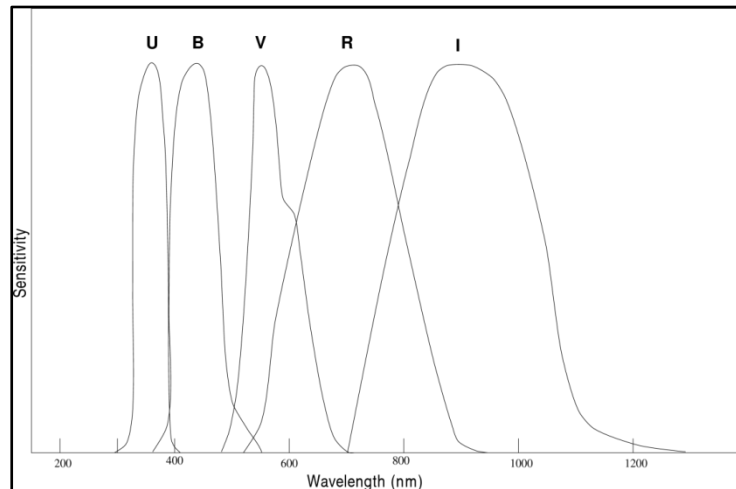


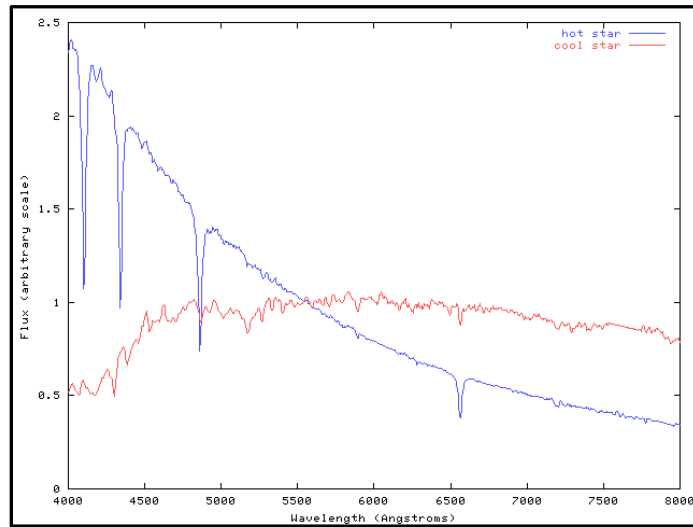
Figure 2: Relative transmission profiles of the UBVRI filters.

The difference between two color magnitudes, e.g. $B-V = m_B - m_V$ is independent of the stellar distance, but provides a direct diagnostic of the stellar temperature, sometimes called “the color temperature”.

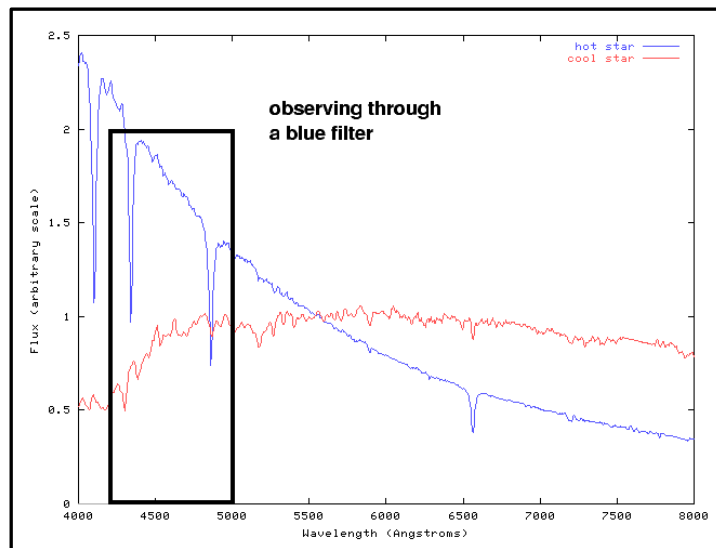
Because a larger magnitude corresponds to a lower brightness, stars with a positive B-V actually are less bright in the blue than in the visible, implying a relatively low temperature.

On the other hand, a negative B-V means blue is brighter, implying a high temperature. Figure 2 shows how the temperature of a Black-Body varies with the B-V color of the emitted Black-Body spectrum.

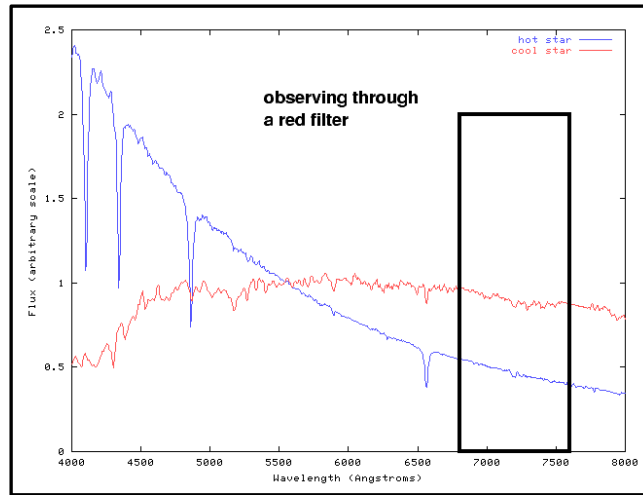
Most stars emit radiation like a classical blackbody, with a spectrum which depends mostly on their temperature:



Now, if one were to attach a blue filter to one's telescope, one would measure only the blue light emitted by stars. In that case, the hot star would appear brighter than the cool one:



On the other hand, if one were to attach a red filter to one's telescope, the cool star would appear brighter than the hot one:



So, the ratio of apparent brightness -- and, hence, the magnitude difference -- between two stars depends on the bandpass through which one observes them. A "bandpass" is the overall sensitivity of an instrument as a function of wavelength: it includes the effects of filters, plus characteristics of the detector, and telescope mirrors.

Astronomers have settled on a number of different photometric systems, each one based on a particular passband (i.e. a particular combination of filter and detector and telescope). One should always remember to specify the system when quoting the magnitude of a star.

Most astronomers working in the optical use the UBVRI photometric systems. These are five different passbands which stretch from the blue end of the visible spectrum to beyond the red end.

They were set up many years ago by several astronomers:

- **Johnson and Morgan, 1953** defines the UBV system with stars visible in the northern hemisphere.
- **Cousins, 1974** extends the UBV system to the southern sky and defines the redder R and I passbands.
- **Bessell, 1979** is a good secondary reference for UBVRI band.

When writing the magnitude of a star, astronomers use an abbreviation to denote the photometric system of the measurement:

V = 1.03 means "magnitude of this star in the V system is 1.03"
 B = 0.46 means "magnitude of this star in the B system is 0.46"

There is also a convention to use lower-case letters for raw measurements and upper-case letters for fully reduced values:

b = 1.18 means "a measurement made through a B filter"
 B = 1.22 means "the same measurement after a full reduction"

Zero point

So, once one has settled on the equipment one will use -- which sets the photometric system -- one still faces the question of the magnitude zero-point. The choice is arbitrary. Astronomers have chosen to use the bright star Vega as their starting point.



In the UBVRI systems, the star Vega is defined to have a magnitude of zero. That is,

Vega's magnitude in U-band:	U = 0.0
Vega's magnitude in B-band:	B = 0.0
Vega's magnitude in V-band:	R = 0.0
Vega's magnitude in R-band:	V = 0.0
Vega's magnitude in I-band:	I = 0.0

Stellar "colors"

Astronomers use the word "color" to measure of the magnitude difference of a star in two passbands relative to the magnitude difference of Vega in the same passbands. Let me illustrate with an example or two.

Consider the stars Vega (a hot star), Antares (a very cool star), and Alnitak (a very hot star in the constellation Orion).

We can measure their magnitudes in the B and V passbands.

	B	V	(B-V)
Vega	0.00	0.00	0.00
Antares	2.96	1.09	+1.87
Alnitak	1.59	1.79	-0.20

Plank's Law (Black Body Radiation):

It is the relation between the energies that emitted from different surface (with different temperature) and the wavelengths of the photons it emits. At higher temperature the peak of the spectrum shifts toward shorter wavelengths and the amount of energy radiated per second for each square meter of the source (Blackbody) increases. The equation that predicts the radiation of a blackbody at different temperatures is known as Planck's Law.

$$B_v = \frac{2hv^3}{c^2} \frac{1}{\exp(hv/kT)-1} \quad \text{W/m}^2/\text{Hz/sr}$$

Where h is the Planck constant (6.626×10^{-27} erg.s), k is the Boltzmann constant ($1.38064852 \times 10^{-23}$ J.K⁻¹), c is the speed of light (3×10^8 m/sec), ν is the frequency (Hz).

Wien's Law:

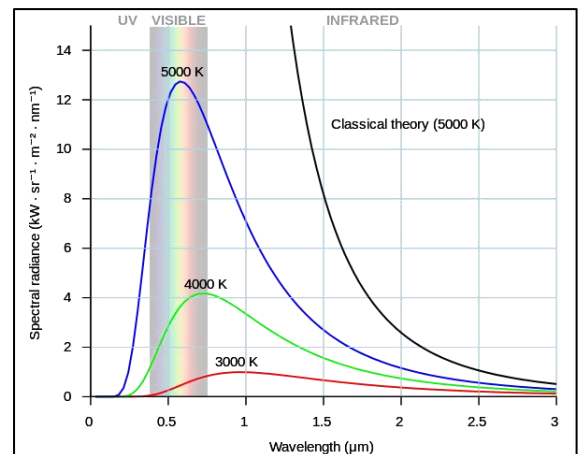
A law of physics describing the relation between the wavelength of the peak emission of the blackbody and the temperature. The peak wavelength is inversely proportional to the temperature (the higher the temperature the shorter the wavelength at which maximum spectral radiance occurs).

$$\lambda_{\max} = \frac{2900 \mu\text{m.K}}{T}$$

Where λ_{\max} is the wavelength of the maximum intensity and T is the temperature of the star in kelvin.

Wien's law relates the temperature (T) of the astronomical to the wavelength (λ_{\max}) at which it emits the most radiation simply Wien's law tell us that the hotter the object the bluer its radiation. For example an astronomical object with a temperature of 6000° K emits most of its energy in visible part of the spectrum, with a peak wavelength of 480 nm.

At 600° K the object's emission would peak at a wavelength of 4800 nm, well into the infrared portion of the spectrum. At a temperature $60,000^\circ$ K the peak would move all the way through the visible spectrum to a wavelength of 48 nm, in the Ultra-violet range. [The hot body radiates more total energy and that the wavelength of maximum intensity is shorter for hotter objects]. The hotter object will look blue to our eyes, whereas the cooler objects will look red.



Example 1: Find the distance of the star like the Sun has an observed flux of $2.4 \times 10^{-10} \text{ W/m}^2$. If the flux of the sun at the Earth is 1 kW/m^2 ?

A star like the sun has an observed flux of $2.4 \times 10^{-10} \text{ W/m}^2$. If the flux of the sun at the Earth is 1 kW/m^2 , how far away is the star?

$$L_{sun} = 4\pi d^2 f \Rightarrow L_{sun} = 4 \times 3.14 \times (1.5 \times 10^{11} \text{ m})^2 1000 \text{ W/m}^2$$

$$\Rightarrow L_{sun} = 3 \times 10^{26} \text{ W}$$

Now

$$d = \sqrt{L_{sun} / 4\pi f} \Rightarrow d = \sqrt{3 \times 10^{26} \text{ W} / (4 \times 3.14 \times 2.4 \times 10^{-10} \text{ W/m}^2)}$$

$$\Rightarrow d = 3 \times 10^{17} \text{ m} = 10 \text{ pc}$$

Introduction to CCDs

A photographic emulsion was first technological advance in astronomical detectors. It is a light-sensitive colloid used in film-based photography. The earliest known photograph of an astronomical object, the Moon, was made by J. W. Draper in 1840.

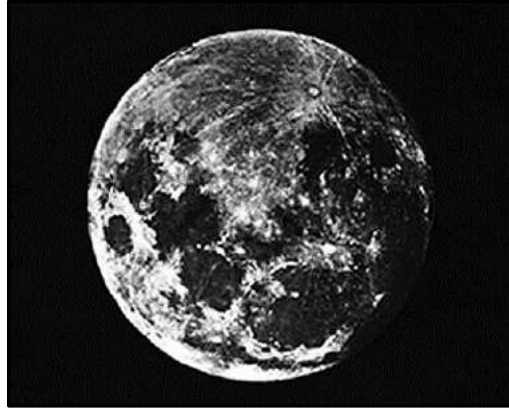


Figure 9: Shows the first image taken by photographic emulsion for moon.

Photographic emulsions can make a permanent record of astronomical objects imaged by telescopes. Photographic emulsions are not very good photometric devices for astronomy because of several drawbacks.

1. Photographic emulsions only record a small fraction (around 1%) of the photons that hit the emulsion.
2. The analog (rather than digital) nature of the image record on an emulsion, it is difficult to make quantitative measurements of star brightness's.
3. Photographic emulsions are also nonlinear with input light- if one star is twice as bright as another; it does not produce twice the output on the film.

Photographic emulsions have served as detectors in astronomy for more than a hundred years. Another detector after the Photographic emulsions is a CCD detector.

Charge Coupled Devices (CCDs) is a device used in digital photography that converts an optical image into electrical signal. The CCDs were invented in the 1970s by Boyle and Smith, originally found application as memory devices. The CCDs are involved in many aspects of everyday life such as video cameras for home use, automatically trap speeders on highways, hospital x-ray imagers, digital cameras used as quality control monitors. There are four main

methods of employing CCD imager in astronomical work: imaging, astrometry, photometry, and spectroscopy.

The CCDs are mostly useful at optical wavelengths (about 3,000 to 10,000Å). Their light sensitive properties were quickly exploited for imaging applications and they produced a major revolution in Astronomy. They improved the light gathering power of telescopes by almost two orders of magnitude. Nowadays an amateur astronomer with a CCD camera and a 15 cm telescope can collect as much light as an astronomer of the 1960s equipped with a photographic plate and a 1m telescope.

The first CCD images reported from a professional telescope were of the planets Jupiter, Saturn, and Uranus, taken in 1976 by Bradford Smith and James Janesick from Jet Propulsion Laboratory (JPL) imaged the planet Uranus at a wavelength of (8900 Å, or 0.89 μm) with the 61-inch telescope outside Tucson, Arizona.

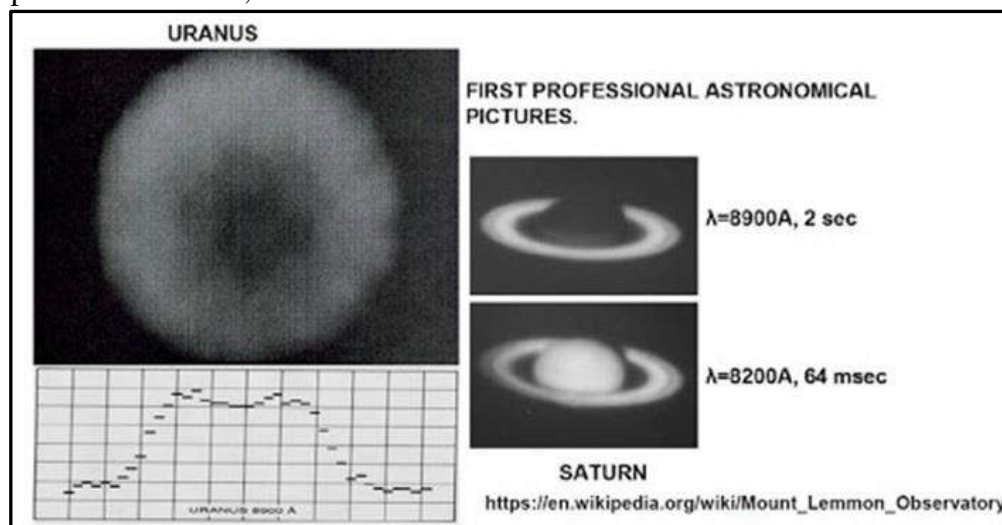
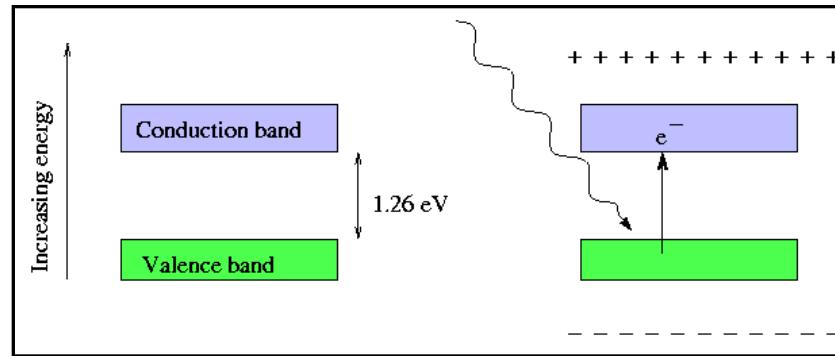


Figure 10: Shows the first image taken by CCD camera for two planets Uranus and Saturn.

CCDs work by converting light into a pattern of electronic charge in a silicon chip. This pattern of charge is converted into a video waveform, digitized and stored as an image file on a computer. There are four main methods of employing CCD imagers in astronomy work: Imaging, Astrometry, Photometry, and Spectroscopy.

The **photoelectric effect** is fundamental to the operation of a CCD. Atoms in a silicon crystal have electrons arranged in discrete energy bands. The lower energy band is called the **Valence Band**, the upper band is the **Conduction Band**. Most of the electrons occupy the **Valence Band** but can be excited into the **Conduction Band** by heating or by the absorption of a photon. The

energy required for this transition is 1.26 electron volts. Once in this **Conduction Band** the electron is free to move about in the lattice of the silicon crystal. It leaves behind a "hole" in the **Valence Band** which acts like a positively charged carrier. In the absence of an external electric field the hole and electron will quickly re-combine and be lost. In a CCD an electric field is introduced to sweep these charge carriers apart and prevent recombination.



Thermally generated electrons are indistinguishable from photo-generated electrons. They constitute a noise source known as "Dark Current" and it is important that CCDs are kept cold to reduce their number. 1.26eV corresponds to the energy of light with a wavelength of 1 micron. Beyond this wavelength silicon becomes transparent and CCDs constructed from silicon become insensitive. The CCD has many advantages- it is a linear, photon counting device which records a large fraction of the photons that fall on it. It can record a two dimensional image- i. e. there is positional information.

Gain – number of electrons that get converted to 1 count

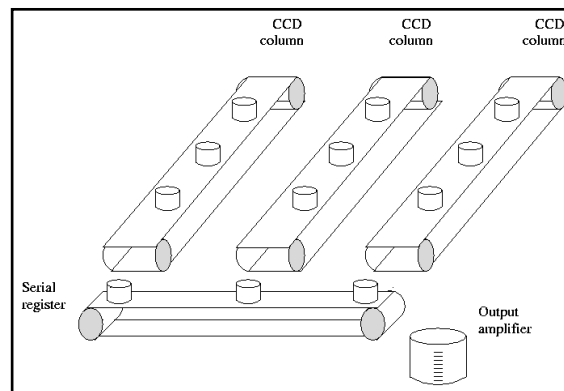
Basic Theory of a CCD

The CCD is a special integrated circuit consisting of a flat, two dimensional array of small light detectors referred to as pixels. The CCD chip is an array of Metal-Oxide-Semiconductor capacitors (MOS capacitors), each capacitor represents a pixel. Each pixel acts like a bucket for electrons. A CCD chip acquires data as light or electrical charge. During an exposure, each pixel fills up with electrons in proportion to the amount of light that enters it. The CCD takes this optical or electronic input and converts it into an electronic signal. The electronic signal is then processed by some other equipment and/or software to either produce an image or to give the user valuable information.

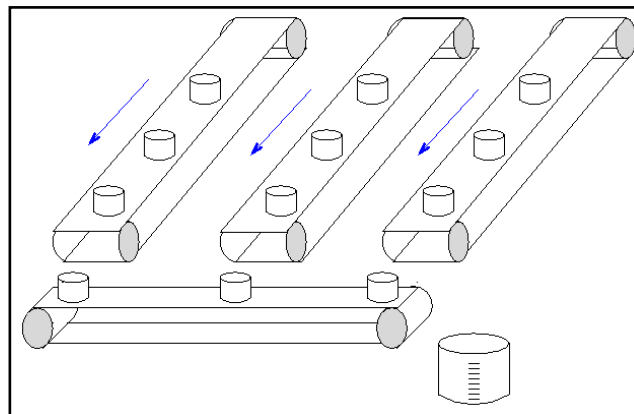
The Conveyor Belt Analogy

A common analogy for the operation of a CCD is as follows:

A number of buckets (Pixels) are distributed across a field (Focal Plane of a telescope) in a square array. The buckets are placed on top of a series of parallel conveyor belts and collect rain fall (Photons) across the field. The conveyor belts are initially stationary, while the rain slowly fills the buckets (During the course of the exposure). Once the rain stops (The camera shutter closes) the conveyor belts start turning and transfer the buckets of rain, one by one, to a measuring cylinder (Electronic Amplifier) at the corner of the field (at the corner of the CCD).



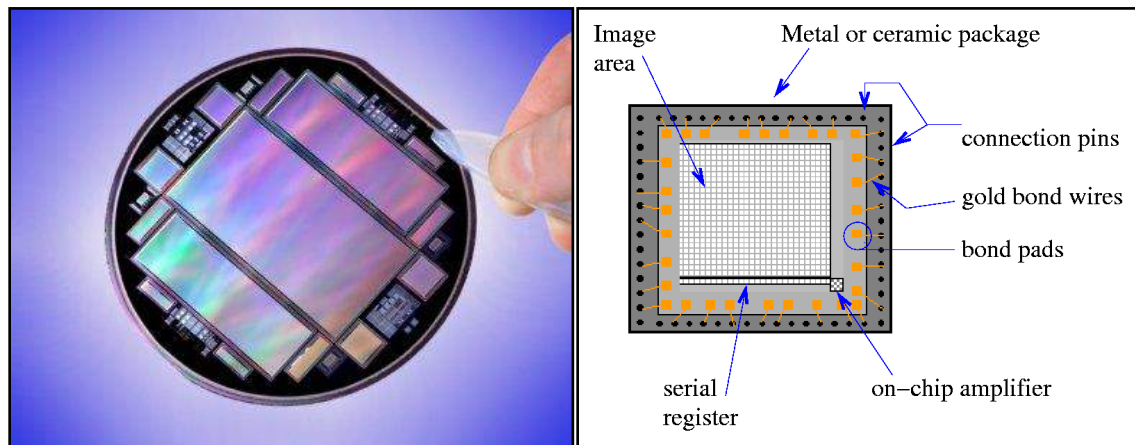
First, we open the shutter and let rain (light) fall on the array, filling the buckets (pixels). At the end of the exposure, we close the shutter. Now, shift the buckets along the conveyor belts.



Structure of a CCD

The image area of the CCD is positioned at the focal plane of the telescope. An image then builds up that consists of a pattern of electric charge. At the end of the exposure this pattern is then transferred, one pixel at a time, by way of the serial register to the on-chip amplifier.

Electrical connections are made to the outside world via a series of bond pads and thin gold wires positioned around the chip periphery.



CCDs are manufactured on silicon wafers using the same photo-lithographic techniques used to manufacture computer chips. Scientific CCDs are very big, so only a few can be fitted onto a wafer. This is one reason that they are so costly. The photo above shows a silicon wafer with three large CCDs and assorted smaller devices.

Types of CCD Camera

1- Front-Side Illuminated CCD (Thick Chip).

- 1) Silicon layers on bottom.
- 2) Insulating layer.
- 3) Electrodes on top.

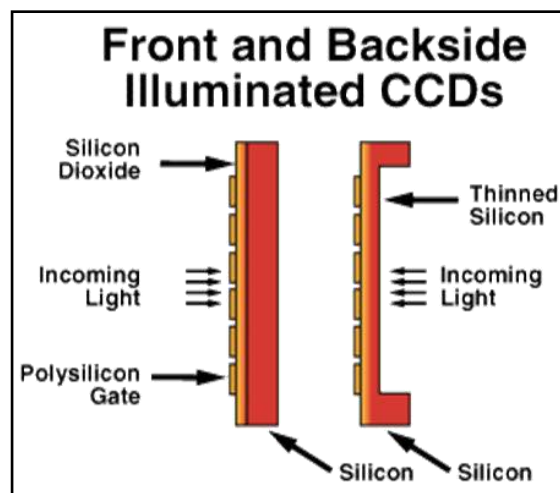
How it works: Incoming photons strike the silicon. If the photons have the right energy, they liberate electrons from the silicon atoms. The electrodes (capacitors), called “gates”, are given a positive charge and attract the freed electrons. At the end of each exposure, each gate has a remaining net charge that indicates how many photons struck that region of the silicon.

Front Illuminated CCD – disadvantage is a lot of reflection of photons by the gates (capacitors) –
QE ~ 40-60%.

2- Back-Side Illuminated CCD (Thin Chip) Increased blue sensitivity.

The silicon is chemically etched and polished down to a thickness of about 15 microns. Light enters from the rear and so the electrodes do not obstruct the photons. The QE can approach 100%. These are very expensive to produce since the thinning is a non-standard process that reduces the chip yield. These thinned CCDs become transparent to near infra-red light and the red response is poor. Response can be boosted by the application of an anti-reflective coating on the thinned rear-side. These coatings do not work so well for thick CCDs due to the surface bumps created by the surface electrodes.

Almost all Astronomical CCDs are Thinned and Backside Illuminated.



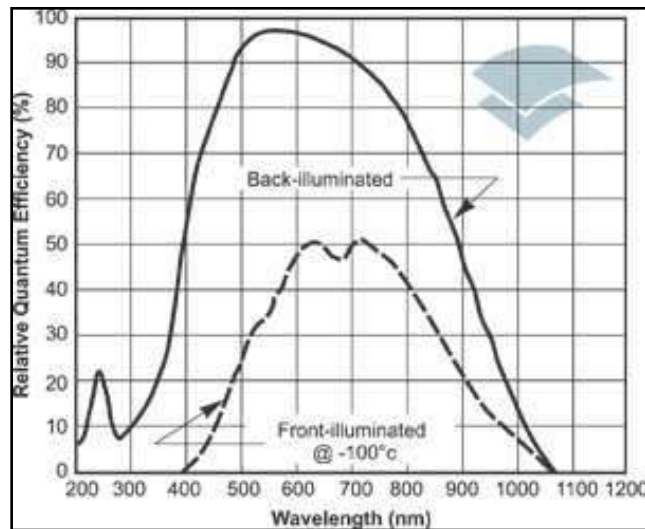
Quantum Efficiency

A CCD detects individual photons, but even the best CCD does not detect every single photon that falls on it. The fraction of photons falling on a CCD that are actually detected by the CCD is called the quantum efficiency (QE), usually expressed as a percentage. (Note: There is another way of defining QE that involves the signal to noise ratio of the input and detected signal. For CCDs in which photon noise is the dominant noise source, the fraction of photons detected and the signal to noise definitions of QE are equivalent.). Silicon can see wavelengths in the range of 100 nm to 1200 nm (UV-IR) while the human eye sensitive to wavelength in the range of 350-650 nm (Visible)!

The percentage of photons that are actually detected is known as the Quantum Efficiency (QE). For example, the human eye only has a QE of about 20%, photographic film has a QE of

around 10%, and the best CCDs can achieve a QE of over 80%. Quantum efficiency will vary with wavelength.

Thick chips have low QE in the blue, because the electronic layers absorb much of the blue light. Thin chips have better blue QE. Thin chips and thick chips have similar red QE, but the thin chips have higher QE at all wavelengths than the thick chip.



Types of noise in a CCD

When a CCD image is taken, noise will appear as well as the main CCD image. Noise can be thought of as unwanted signal which doesn't improve the quality of the image. In fact, it will degrade it. The main problem with noise is that most noise is essentially random, and so cannot be completely removed from the image. For example, if we know that a noise source contributes 10 units on each image we can subtract those 10 units from the image. If we only know that the noise is 'around' 10 units, then we can't completely remove all of this noise (as we don't know its exact value).

The main contributions to CCD noise are:

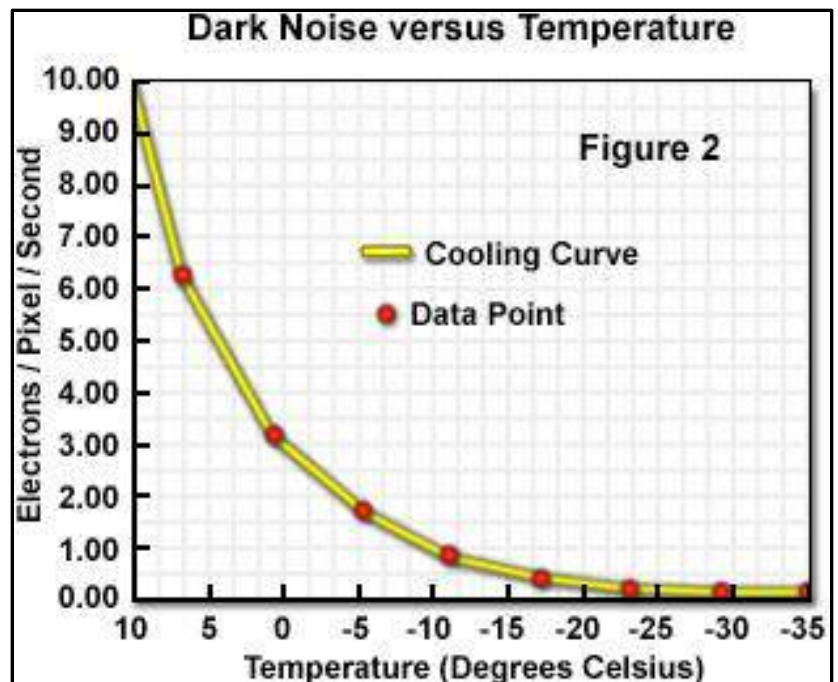
1- Dark Current (Thermally Generated Noise)

Additional electrons will be generated within the CCD not by the absorption of photons (i.e the signal) but by physical processes within the CCD itself. The number of electrons generated in a second will be dependent on the operating temperature of the CCD and hence this noise is known as thermal noise (sometimes also known as dark noise). As with the detection of the signal, the same number of electrons will not be generated in equivalent periods of time as the thermal noise will also have a Poisson distribution.

Caused by thermally generated electrons, temperature dependence.

Solution:

-Cooling



2- Pixel Non-Uniformity

Each pixel has a slightly different sensitivity to light, typically within 1% to 2% of the average signal.

Solution:

-Flat-field image

There are two types of the flat field:

1. Twilight Flats:

After the Sun sets, we take images of the twilight sky, which should be a reasonable uniform light source. One common problem with doing twilight flats is the fact that soon after sunset the sky is much brighter in the west near the horizon than elsewhere in the sky. If your telescope has any scattered light problems, then the bright western horizon could easily mess up the twilight flats. One way to eliminate the problem is to observe twilight flats east of the meridian after sunset (or west of the meridian before sunrise).

2. Dome Flats:

Another way to get flat fields is to use a screen in the dome, and illuminate the screen with artificial lights. With effort, this also works well, although there are a number of potential concerns. One is that the electric lights used to illuminate the spot are quite a bit redder than the sky. This concern can be partially addressed by using very hot lamps, or using filters that decrease the red light from the lamps.

3- Noise on the image itself ("shot noise")

The detection of photons by the CCD is a statistical process. If images are taken over several (equal) time periods, then the intensity (the number of photons recorded) will not be the same for each image but will vary slightly. If enough images are taken, it will be seen that the deviation in intensity found for each image follows the well-known Poisson distribution. In effect, we cannot be sure that the intensity we have measured in a particular image represents the "true" intensity as we know that this value will deviate

from the average. It is this deviation which is considered to be the noise associated with the image. As the deviation is known to follow a Poisson distribution, we know that the likely deviation will be plus or minus the square root of the signal intensity measured. Thus, if we measure a signal intensity of one hundred photons, then the noise on this signal will be ten photons. If we measure a signal intensity of one thousand photons in the image, then the noise on this signal will be about thirty one photons.

Solution:

-Longer exposure or combining multiple frames.

CCD Read Noise (On-chip)

There are several on-chip sources of noise that can affect a CCD. CCD manufacturers typically combine all of the on-chip noise sources and express this noise as a number of electrons RMS (e.g. $15e^-$ RMS).

Signal-to-Noise Ratio

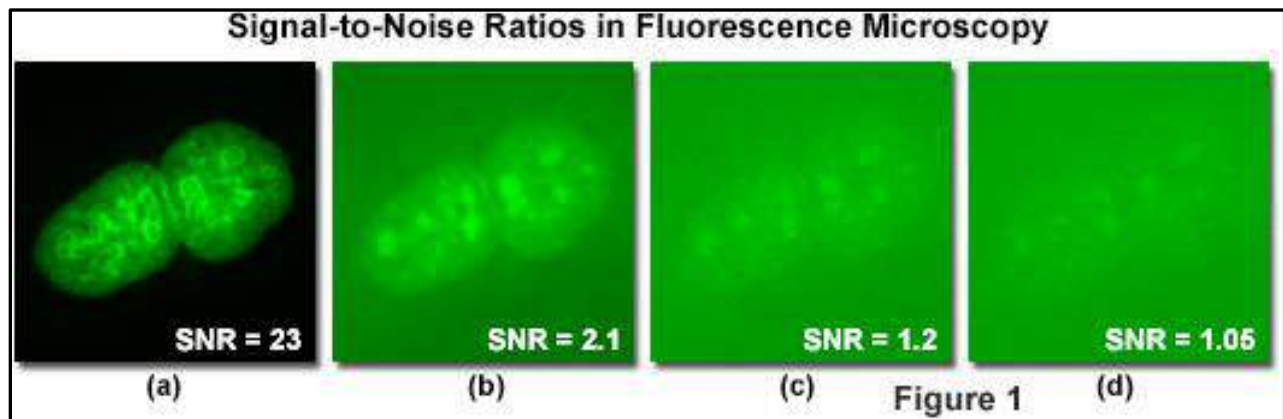
In all measurements of a target the total signal includes:

- S_t , the signal of the target,
- S_b , the “background” signal of the sky and near-by sources (objects in the side-lobes of radio telescopes),
- S_i , the “instrumental” signal from the telescope and instrument, including electronic noise from the detector.

The goal of a measurement is a reliable evaluation of the target signal S_t where the measuring errors are negligible (for a given science case) or as small as possible. The quality of a measurement is often expressed as a signal to noise ratio:

$$\frac{\text{Target Signal}}{\text{noise}} = \frac{S}{N}$$

In a first step one has to distinguish the target signal S_t from the background signal S_b and the instrumental signal S_i . Then one has to evaluate the different contributions to the noise N , the noise in the target signal N_t , the noise in the background signal N_b , and the noise due to the instrument N_i . It is important to understand these noise sources in order to find strategies to reduce them.



The following equation is commonly used to calculate CCD camera system signal-to-noise ratio:

$$\text{SNR} = PQ_e t / \sqrt{PQ_e t + Dt + N_r^2}$$

where **P** is the incident photon flux (photons/pixel/second), **Q(e)** represents the CCD quantum efficiency, **t** is the integration time (seconds), **D** is the dark current value (electrons/pixel/second), and **N(r)** represents read noise (electrons rms/pixel).

$$\text{SNR} = PQ_e t / \sqrt{(P + B)Q_e t + Dt + N_r^2}$$

B background photon flux

Image Format

CCD cameras are described according to the number of pixels they contain in multiples of one million pixels (1 megapixel). They are also described by their format in rows (M) and columns (N) as containing MxN pixels. The total number of pixels is usually rounded to the nearest power of 2 as is the row and column format. For example, a CCD with a format of 1024x1024 pixels has a total of 1,048,576 pixels. In the digital camera industry, this is called 1 megapixel. A 4 megapixel CCD has a format of 2048x2048 pixels or 4,194,304 pixels.

The word pixel can refer to an area of silicon on a CCD or to one tiny piece of the picture. A CCD is a collection of pixels arranged in rows and columns, and a picture the CCD produces is an array of pixels arranged in rows and columns. Each pixel in the image is represented by a number that is related to the amount of light that fell on each pixel on the CCD. Pixels on the CCD have a finite size. The area of sky imaged on each pixel also has a finite angular size, set by the (linear) size of the pixel on the CCD and the plate scale of the telescope. Plate scale of the image is determined by the focal length $S = 206265/F$ arcsec/mm (with F in mm).

A typical CCD has square pixels with sides of length 24 microns ($s = 2.4E-5$ m). On a 4 meter, f/2.7 telescope, with a focal length of $f = 10.8$ meters, each pixel subtends an angle of $\Theta = s/f$ or $2.22E-6$ radians, or about 0.46 arcsec.

Image Format – FITS

There are a number of different image processing programs used by astronomers, and astronomers use many types of computers that store files in different ways. To allow images to be easily transferred between computers, astronomers have developed something called the **flexible image transport system (FITS)**. FITS is an image interchange format. Each image processing program has a task to read FITS images, converting from FITS to the internal format required by the program and computer, and each program has a task to write images into FITS files. So if you want to send an image to someone, you don't need to know what kind of computer or program she is using- simply write a FITS file, send it, then the person at the other end will read the FITS file, converting it to the internal format required by her program and computer.

Quantum efficiency (QE): A CCD detects individual photons, but even the best CCD does not detect every single photon that falls on it. The fraction of photons falling on a CCD that are actually detected by the CCD is called the quantum efficiency (QE), usually expressed as a percentage. (Note:

There is another way of defining QE that involves the signal to noise ratio of the input and detected signal. For CCDs in which photon noise is the dominant noise source, the fraction of photons detected and the signal to noise definitions of QE are equivalent).

Counts: So, are those numbers that we read out of the CCD the actual number of photons that fell on each pixel? Well, no. First off, part of the number is an electrical offset called the bias (see below) and part may be due to dark current (see below). After we subtract these components, the signal is related to the number of electrons liberated by photons in each pixel. Only a fraction QE of photons generate electrons, so that the number of electrons is: (number of photons) \times QE. For several technical reasons, the numbers the CCD gives out are related to the number of electrons by a divisive number called the gain. The gain value is set by the electronics that read out the CCD chip. Gain is expressed in units of electrons per count. So, basically, the number of photons that fell on a pixel is related to the output number (sometimes called “DN” for data number) as follows:

$$\text{Photons} = \frac{\text{number of electrons}}{\text{QE}} = \frac{\text{gain} \times \text{DN}}{\text{QE}}$$

here the number of electrons is only those that came from photons- i.e. the bias and dark contribution have been subtracted off.

Integration time: is the exposure time of the detector. The dark adapted eye integrates photons for $\sim 1/8$ to $1/4$ second. CCDs can integrate for hours. The long exposure time means many photons can be collected from the source. The integration time (or exposure time) is controlled by a mechanical shutter (like in a camera) or electrically (changing voltages in CCD).

Read noise: After an integration (exposure), the CCD must be “read out” to find the signal value at each pixel - because the signal may be as low as a few electrons per pixel, this step involves some very sophisticated amplifiers that are part of the CCD itself (“on chip” amps). Unfortunately, but inevitably, the read out process itself generates some electronic noise. The average noise per pixel is called the read noise. Modern CCDs typically have a read noise of 5 to 20 electrons per pixel per read out (read noise is the same whether exposure is 0.1 sec or 3 hours).

For averaged bias frames the random read-out noise should decrease like the square-root of the number of the frames averaged. This simple test is useful as check whether really random read-out noise is measured. The bias level and read-out noise can evolve and it should be monitored during the

night (e.g. for CCDs a few frames at the beginning of the night, in the middle of the night, and at the end of the night).

Bias frame: If we simply read out the CCD, without making an integration, (or think of a zero second integration), there will be a signal called the bias signal. This bias signal must be measured (it changes somewhat with things like CCD temperature) and subtracted from the images we take. Bias frames have read noise associated with them. To minimize noise introduced when we subtract the bias, we take many bias frames and then combine them to “beat down the noise”.

Dark frame: If we allow the CCD to integrate for some amount of time, WITHOUT any light falling on it, there will be a signal (and more importantly noise associated with that signal) caused by thermal excitation of electrons in the CCD. This is called the dark signal. The dark signal is very sensitive to temperature (lower temperature = lower dark signal), and that is why CCDs used in astronomy are cooled (often to liquid nitrogen temperature). Even with cooling, some CCDs have a non-negligible dark current. This must be measured and subtracted from the image. As for the bias, we want to take many dark frames and combine them to beat down the noise. (The dark frame and bias frames are *NOT* the same thing!).

Flat frame: All CCDs have non-uniformities. That is, uniformly illuminating the CCD will NOT generate an equal signal in each pixel (even ignoring noise for the moment). Small scale (pixel to pixel) non-uniformities (typically a few percent from one pixel to next) are caused by slight differences in pixel sizes. Larger scale (over large fraction of chip) non uniformities are caused by small variations in the silicon thickness across the chip, non-uniform illumination caused by telescope optics. These can be up to may be 10% variations over the chip. To correct for these, we want to shine a uniform light on the entire CCD and see what the signal (image) looks like. This frame (called a flat) can then be used to correct for the non-uniformities.

Data (object) frame: To take an image of an astronomical object, we point the telescope at the right place in the sky, and open a shutter to allow light to fall on the CCD. We allow the signal to build up (integrate) on the CCD for some length of time (anywhere from 1 second to 1 hour) and then read it out. The exposure time used depends on many things. The basic goal is to get an image of the source

with the best signal to noise ratio (S/N) possible in a given amount of available telescope time. Now, since the signal is composed of photons, there is an unavoidable noise associated with photon counting statistics. There is NO WAY to get rid of this noise. HOWEVER, by collecting more photons, we can improve the S/N (the signal goes linearly with time, while the noise goes as the square root of time). During the integration, the dark signal is also building up.

We also have to worry about other sources of noise- readout, dark, and also the effects of cosmic ray particles, which give a spurious signal. The first thing to insure is that these other sources of noise are much less than the photon noise, so that we are not limiting ourselves unnecessarily.

This argues for a long exposure. However, cosmic rays argues for several shorter exposures which can then be combined (as you might imagine, the CCDs aboard the HST have real problems with cosmic rays!). *Getting the optimum exposure time is very complicated!*

The basic steps in observing and reducing a CCD image

The basic steps in observing and reducing a CCD image taken with a telescope are as follows:

- 1- Collect a number of bias frames - median combine them to a single low noise bias frame.
- 2- Collect a number of dark frames (no light, finite integration time equal to the data frame integration). If dark current is non- negligible, combine dark frames into a single low-noise frame (after subtracting bias frame, of course).
- 3- Collect a flat frame in each filter- flat frames can be made by pointing the telescope at the twilight sky, or by pointing at the inside of the dome. Bias frames (and dark frames, if the dark current is non- negligible over the time interval covered by the exposure) must be subtracted from the flat frame. The signal level in the flat is arbitrary- it is related to how bright the twilight sky was etc- all we need is the information on the differences of the signal across the chip. Thus, we normalize the at so that the average signal in each pixel is 1.00 (we do this simply by dividing by the average signal).
- 4- Subtract low - noise bias frame and low noise dark frame from object frame. Then divide this by the normalized at frame.

Symbolically:

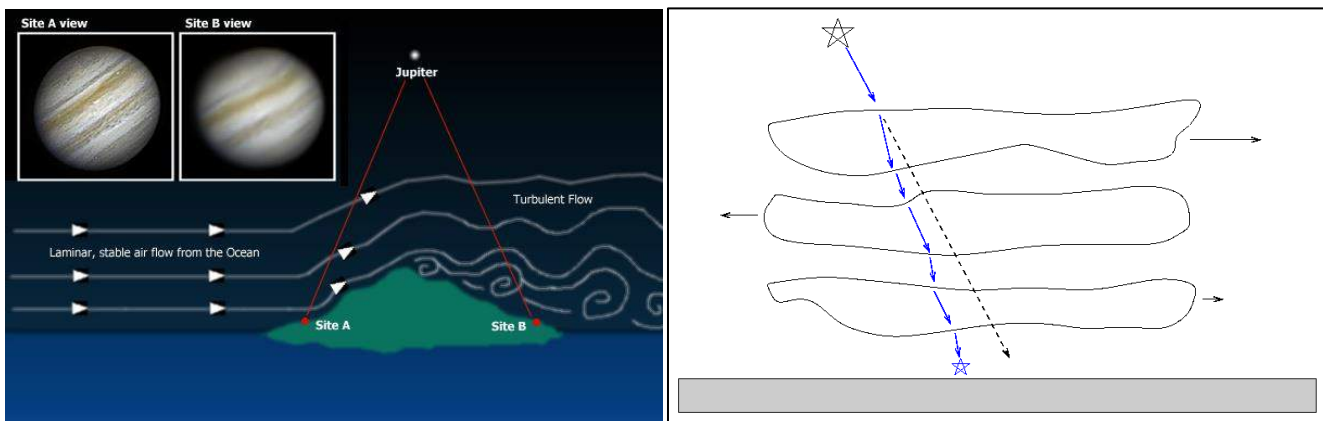
$$\text{Reducedframe} = \frac{(\text{Rawobjectframe}) - (\text{lownoisebiasframe}) - (\text{lownoisedarkframe})}{(\text{normalizedflatframe})}$$

Astronomical Seeing:

Seeing is the term astronomer's use for the blurring and twinkling of light from celestial objects due to its passage through the Earth's atmosphere. Seeing makes the images of stars appear much larger than the limit set by diffraction, and makes the images of extended objects.

Astronomers characterize the seeing by the angular FWHM (full width at half maximum), which is the angular size of the star image at a level of half the peak level.

As light makes its way through the Earth's atmosphere, it passes through many different layers of air. Each layer has a slightly different temperature, pressure and density; there may also be slight differences in chemical composition, dust and water content. That means that the index of refraction of each layer is a little different. As ray of light travels from layer to layer, it is bent by slightly different angles. By the time it reaches the ground, the ray has probably shifted to a slightly different position than it would have had in the absence of an atmosphere.



At left is a diagram showing how mountains break up stable airflow into turbulence. Note the difference in the probable views from site A (facing into the prevailing winds off the ocean) and site B (Located on the downwind side of the mountain peaks.).

Seeing does not refer to the loss of light, but only to lose the detail cause by distortion of light ray. The atmosphere also does cause light to be dimmed somewhat, a process called atmospheric extinction.

The higher altitude the telescope, the better the seeing tends to be, because there is less air to look through the higher one goes.

Over the past decade or so, astronomers have begun to realize that, at least at the best astronomical sites, a significant fraction of the smearing of astronomical images occurs during the last few meters the light travels before detection. There are two reasons for turbulence near the dome slit can be caused if:

1. The mirror is warmer than the air in the dome, there will be turbulence and temperature inhomogeneities as the hot air rises above the mirror.
2. The dome air is warmer than the air surrounding the dome.
To reduce this turbulence, astronomers are trying various ideas to minimize temperature inhomogeneities in the air in the dome, such as:
 1. Removing sources of heat in the dome environment, and keeping the mirror cool during the day with refrigeration units.
 2. Fans and large vents are used to try to keep the temperature inside the dome as close as possible to the outside temperature.

At first, you might think that fans would result in more turbulence and hence worse seeing, but it seems that seeing is caused more by passage of light through parcels of air with different temperature and densities, rather than simply through moving air. Thus, fans can help by homogenizing the temperature of the air within the dome and between dome and outside. This is because the index of refraction of air changes with temperature and changes of index of refraction is what bends light ray.

Eliminations of Seeing:

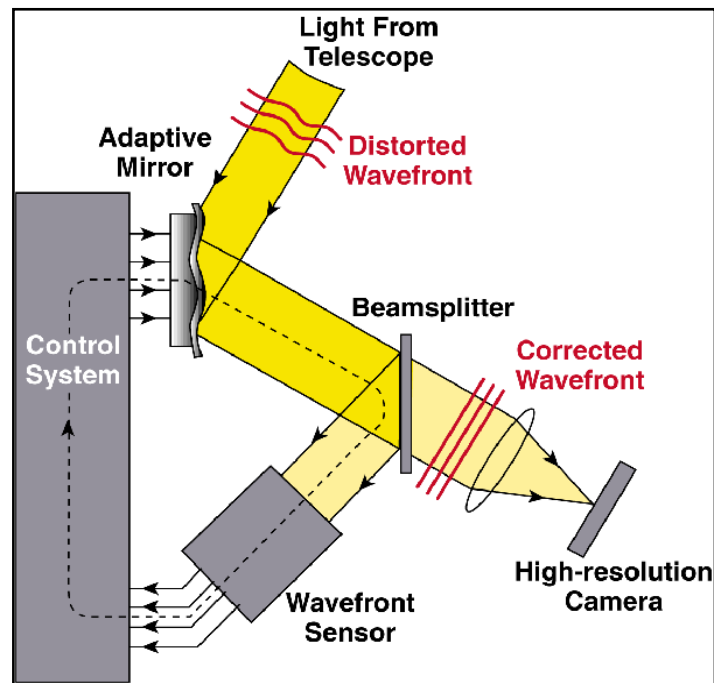
1. Lucky Imaging (LI).

The origin of lucky imaging is credited to Fried (1978). Lucky imaging is an elimination method which is used for partial removal of seeing effects in astrophotography. Its principle is based on taken of the large number of images with very short exposure time (fractions of seconds).

A Lucky Imaging system takes very short exposures, on the order of the atmospheric coherence time. The rapidly changing turbulence leads to a very variable PSF; the variability of the PSF leads to some frames acquired by the Lucky Imaging system being better quality than the rest. Only the best frames are selected, aligned and co-added to give a final image with much improved angular resolution.

2. Adaptive Optics (AO):

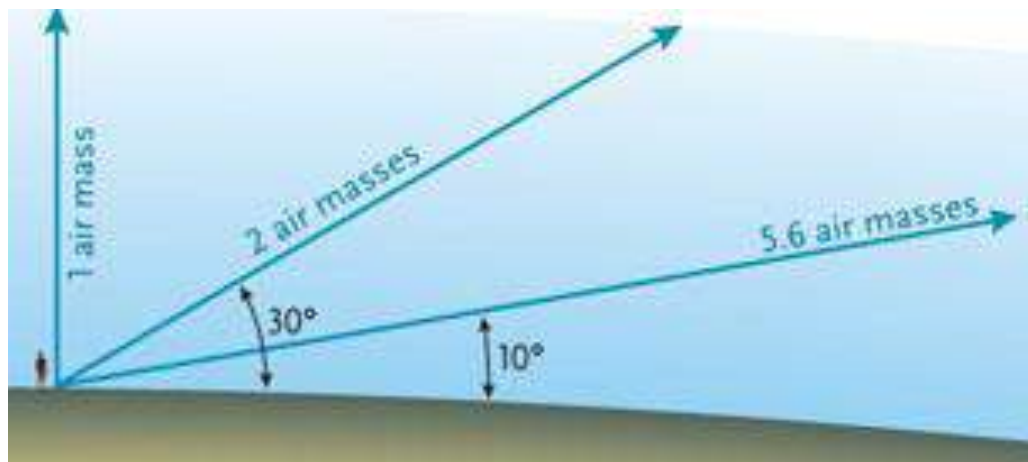
Adaptive optics (AO) aims at removing wavefront distortions by inserting one or more adjustable optical elements into the path between source and detector. In practice, the adjustable elements are usually reflective, and usually located near the telescope focal plane. The shape of the reflecting surface is adjusted to exactly cancel distortions generated by atmospheric turbulence. In the figure, a partially reflecting mirror splits the distorted wavefront in the telescope into two fronts, w and w_2 . These fronts have identical distortions. Front w proceeds to a sensor, S , which detects the magnitude of its distortion at some number of locations on the front. The other front, w_2 , is reflected from an adjustable mirror, A , onto the detector. Meanwhile, the computer has read the distortions sensed by S , and commands A to adjust the shape of its surface so as to exactly cancel them. If all goes perfectly well, the compensated image formed at the detector will be diffraction-limited, with all effects due to the atmosphere removed.



Atmosphere Extinction

is the absorption and scattering of electromagnetic radiation by dust and gas between an emitting astronomical object and the observer.

An effect which must be corrected when calibrating instrumental magnitudes is the atmospheric extinction or the dimming of starlight by the terrestrial atmosphere. The longer the path length the starlight traverses through the atmosphere the more it is dimmed. Thus, a star close to the horizon will be dimmed more than one close to the zenith, and the observed brightness of a given star will change throughout a night, as its zenith distance varies.



Consider light rays from a particular star as they head towards a particular observer on the Earth's surface. As light makes its way through the Earth's atmosphere, some photons collide with atoms, molecules, water droplets, grains of dust, and other objects. These photons may be absorbed by the objects (in which case they cease to exist), or they may be scattered into a different direction. Either way, they no longer reach the observer on the ground. As a result, the observer detects fewer light rays from the star, we call this dimming of stellar light **extinction**.

The path length through the atmosphere is known as the air mass. Consider an observation through the blanket of the atmosphere around the curved surface of the Earth. At any particular wavelength, λ , we can relate $m_0(\lambda)$, the magnitude of the observed object outside the atmosphere, to $m(\lambda)$, the magnitude of the observed object at the surface of the earth, by:

$$m(\lambda) = m_0(\lambda) + \kappa(\lambda)X(z) \text{-----}(1)$$

where $X(z)$ is the air mass, $\kappa(\lambda)$ is the extinction coefficient at wavelength λ and z is the zenith distance (the angular distance of the object from the zenith at the time of observation). X is defined as the number of times the quantity of air seen along the line of sight is greater than the quantity of air in the direction of the zenith and will vary as the observed line of sight moves away from the zenith, that is, as z increases. Note that the air mass is a normalized quantity and the air mass at the zenith is one.

For small zenith angles $X = \sec z$ is a reasonable approximation, but as z increases, refraction effects, curvature of the atmosphere and variations of air density with height can become important, gives a more refined relationship:

$$X = \sec z(1 - 0.0012(\sec^2 z - 1)) \text{-----}(2)$$

The atmospheric extinction coefficient, $\kappa(\lambda)$, can be determined by observing the same object (through an appropriate filter) at several times during the night at varying zenith angles. The extinction due to dust scattering varies from night to night, but is usually less than a few tenths of a magnitude per unit airmass. The total vertical extinction in V band.

Point Spread Function (PSF):

Is the intensity distribution of a point source as seen through the aperture of a telescope. Or the Point Spread Function (PSF) represents the distribution of light from a point source at the detector.

When a plane wave front from a distant point source passes through a circular aperture (telescope), the corresponding image of the source observed through the aperture will have a Bessel function. This is known as the PSF of the telescope.

The PSF for a perfect optical system, based on circular elements, would be an “Airy Pattern,” which is derived from Fraunhofer diffraction theory. The intensity distribution of the airy disk is given by:

$$I(\vartheta) = I_0 \left(\frac{2J_1(kD\sin(\vartheta))}{kD\sin(\vartheta)} \right)^2 \quad \text{-----(3)}$$

with J_1 , the Bessel function of the first kind and order, the aperture of the telescope D and the wavenumber k . With the substitution:

$$m = kD\sin(\vartheta) \quad \text{-----(4)}$$

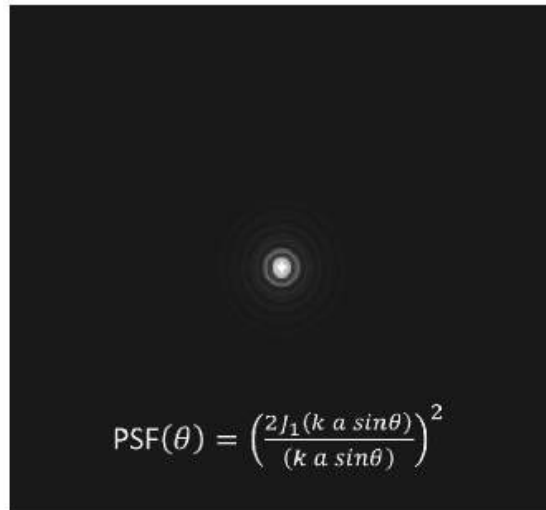
It follows:

$$I(m) = I_0 \left(\frac{2J_1(m)}{m} \right)^2 \quad \text{-----(5)}$$

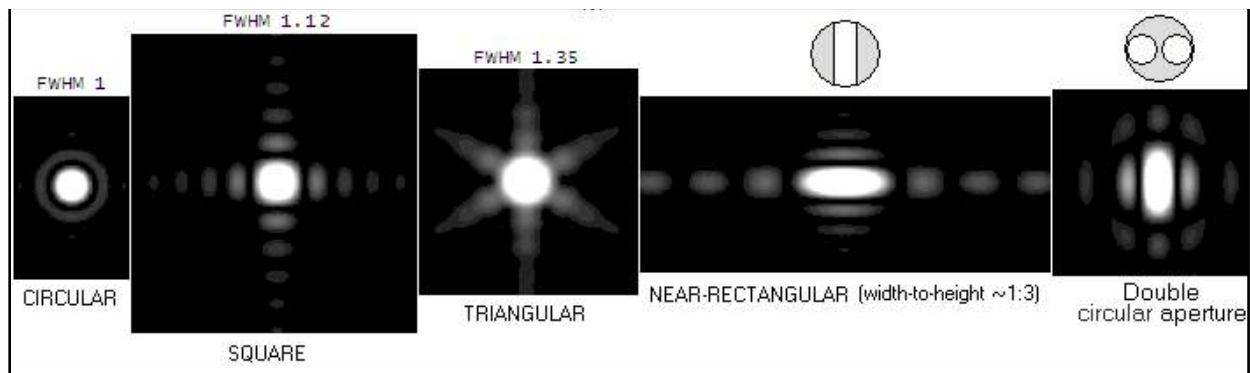
This expression is the result of a Fourier transform of a circular aperture. By considering the Rayleigh criterion, you get the minimum of the intensity function I_0 by calculating the first order m_0 of the Bessel function, which is $m_0 = 3.8317$ or $m_0 = 1.22\pi$. By entering m_0 into the expression for J_1 , the expression for the diffraction limited angular resolution is given by:

$$\sin(\vartheta) \approx \vartheta = 1.22 \frac{\lambda}{D} \quad \text{-----(6)}$$

The figure below shows the diffraction-limited PSF in two dimensions. Diffraction pattern of a point source for a circular aperture.



The shape of the PSF is depended on the shape of the aperture that the wave front is going to pass through, as shown in the figure.



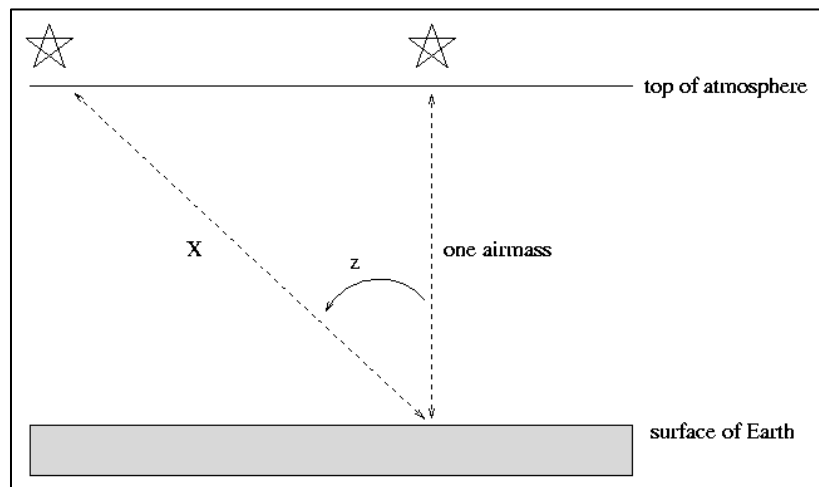
Airmass

is the "amount of air that one is looking through" when seeing a star or other celestial source from below Earth's atmosphere.

We know, previously that the airmass is essentially secant θ_z . The real atmosphere is not plane-parallel, due to the curvature of the Earth. This results in a correction to the secant θ_z formula that is very small except near the horizon. So, how do we calculate secant θ_z ? Without going into all the trig, we can express θ_z in terms of the observables as follows (for a plane-parallel atmosphere approximation):

$$\sec \theta_z = \frac{1}{[\sin \lambda \sin \delta + \cos \lambda \cos \delta \cos h]}$$

where λ is the latitude of the observatory, δ is the declination of the star, and h is the hour angle of the object at the time of the observation. The hour angle h is usually expressed in hours, minutes, and seconds of time, but must be converted into angle units for this equation. For example, an h of 1 hour 30 minutes would be an angle of 22.5 degrees.



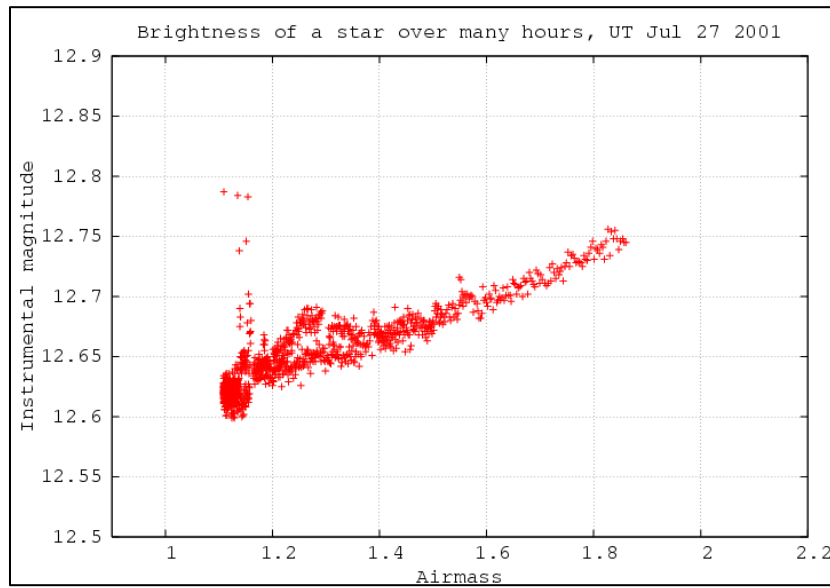
The fact that the real atmosphere is not plane-parallel, but is curved due to the curvature of the Earth, can be taken into account with a correction (ΔX), which is small, except near the horizon:

$$\Delta X = 0.00186(\sec \theta_z - 1) + 0.002875(\sec \theta_z - 1)^2 + 0.0008083(\sec \theta_z - 1)^3$$

So, the final equation for airmass, X , can be written as:

$$X = \sec(\theta_z) - \Delta X$$

Nowadays, most computer control systems at research telescopes automatically calculate the zenith angle and airmass and write that information in the headers of the images as they are taken.



Limiting Magnitude of Telescope:

The limiting magnitude is the apparent magnitude of the faintest object that is visible with the naked-eye or a telescope. The limiting magnitude of a telescope depends on the size of the aperture and the duration of the exposure. Generally, the longer the exposure, the fainter the limiting magnitude.

How faint an object can your telescope see:

$$M = 2.7 + 5 \log D$$

D = objective telescope diameter in millimeter.

Where **m** is the limiting magnitude of the telescope, for example telescope diameter is (240 mm).

The limiting magnitude of the telescope will be:

$$M = 2.7 + 5 \log (240)$$

$$M = 14.7$$

Atmospheric Extinction

Clearly, if the light has to pass through a larger path in the Earth's atmosphere, more light will be scattered/absorbed; hence one expects the least amount of absorption directly overhead (zenith), increasing as one looks down towards the horizon.

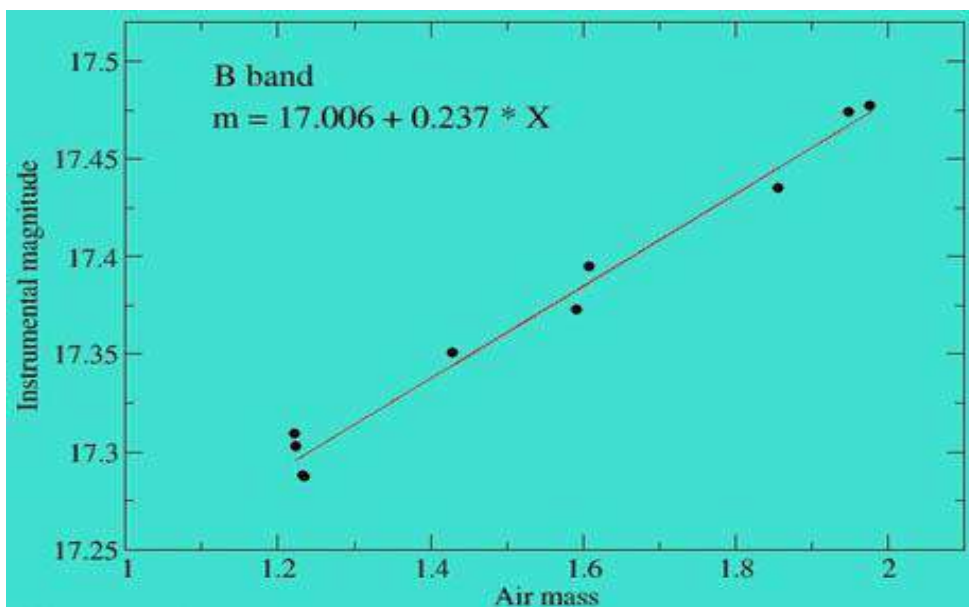
The airmass **X** is defined as the path length that the light from a celestial source must travel through the Earth's atmosphere to get to the observatory, relative to that for a source at the zenith ($X=1$ at $Z=0$), where **Z** is the zenith angle.

The magnitude outside the atmosphere, m_0 , is related to the observed magnitude, m , by

$$m_0 = m + k_\lambda X$$

where K_λ is the extinction coefficient. It can vary from night to night, so if you are interested in accurate photometry, you need to measure it on your night. Also remember that the extinction coefficient is wavelength dependent, so you need a separate number for each filter.

Air mass	Inst. mag.
1.224	17.303
1.977	17.477
1.949	17.474
1.235	17.287
1.233	17.288
1.223	17.309
1.430	17.351
1.591	17.373
1.609	17.372



The extinction coefficient can be determined by making multiple observations of a star at different airmasses. Then you can obtain the values of K_λ and m_0 by fitting a straight line to the data (observed magnitudes versus airmass).

Note that you need to sample a good range of airmasses to get good accuracy on the fit, and you must bracket the airmasses of all of your program objects. Thus, in the example above the extinction coefficients are $K_B = 0.237$.

How do we correct the equation for distance when accounting for extinction? Without extinction,

$$d = 10^{0.2(m - M + 5)}$$

If you want to account for extinction just remember that lower magnitudes are brighter, so you want to subtract AV from the apparent magnitude. The revised equation is thus:

$$d = 10^{0.2(m - M + 5 - A_V)}$$

where d is the distance in parsecs. AV can be determined from the observed and expected color index $B-V$ as

$$\begin{aligned} AV &= 3 \times E(B-V) \\ &= 3 \times [(B-V)_{\text{obs}} - (B-V)_0] \end{aligned}$$

where $(B-V)_{\text{obs}}$ is the observed color index and $(B-V)_0$ is the expected color index for the particular object that we are observing. The values of $(B-V)_0$ can be found in books on Astronomy. We give below a table relating the spectral type of the star and the $(B-V)_0$.

This exercise can only be performed if the spectral type (or intrinsic colour) of your target object is known]

1. We can find the intrinsic color of the star B0V in the Simbad Data archive.
2. The intrinsic colour for such a star is $(B-V)_0 = -0.27$.
3. Obtain the colour excess $E(B-V) = (B-V) - (B-V)_0$, where $(B-V)$ is the value obtain in the previous exercise and derive the extinction in the V band,

$$A_V = 3.1 * E(B-V).$$

$$E(B-V) = (B-V) - (B-V)_0 = 1.044 - (-0.27) = 1.314$$

$$A_V = 3.1 * E(B-V) = 3.1 * 1.314 = 4.07.$$

4. Find out the absolute magnitude, M_V , of your object (see e.g. Wegner 2000, 319, 771, Greiner et al. 1985, 145, 331; Mikami & Heck, 1982, PASJ, 32, 529)
The absolute magnitude of a B0V star is $M_V = -4.2$.
5. Calculate the distance (in parsecs) of your object using the distance-modulus equation

$$d = 10^{0.2(V - M_V + 5 - A_V)}$$

$$d = 10^{0.2(V - M_V + 5 - A_V)} = 10^{0.2(14.31 + 4.2 + 5 - 4.07)} \sim 7700 \text{ pc} \sim 7.7 \text{ kpc}$$