

جامعة بغداد

University of Baghdad

كلية العلوم

College of science

قسم التحسس النائي

Remote sensing Dept.

التعليم الالكتروني

E-learning



# Aerial Photogrammetry

المساحة/القياس التصويري الجوي

ا. د. فيصل غازي محمد

2020-2021

Video  
LECTURES

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

# *Aerial Photography & Photogrammetry*

## Lecture 1

### Introduction:

### Elements of photographic systems

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# Grading

1. Homework's : **5%**
  2. Mid Exam 1,2 : **25%**
  3. Quiz : **5%**
  4. Attendance : **5%**
- End over (sum of above)= **40%**
  - Final – (Electronic Exam = **42%** + Electronic Report = **42%**)
  - Total: End over + Final = **100%**
  - **Project**: Extra Credits you can earn up to **50%** end over.
  - Class participation will help your grade!

# Student Civility

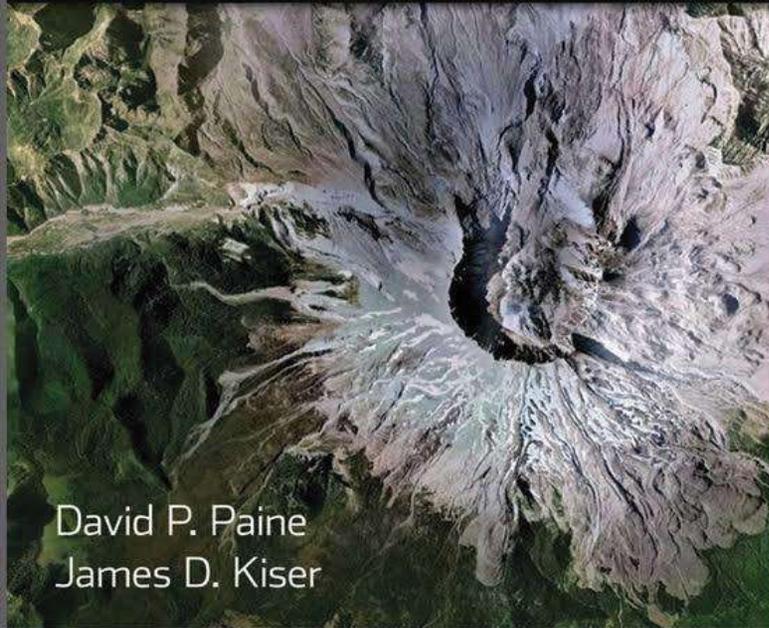
- In an effort to make this class enjoyable for everybody...
  - Please **be on time** to class!
  - Please **do not talk to your friends** and neighbors in class! It disturbs everyone, and makes it hard to concentrate. If you have a question, just ask me!
  - Please **turn** your pagers and cell-phones off!



# Course Text Book

## AERIAL PHOTOGRAPHY AND IMAGE INTERPRETATION

Third Edition



David P. Paine  
James D. Kiser

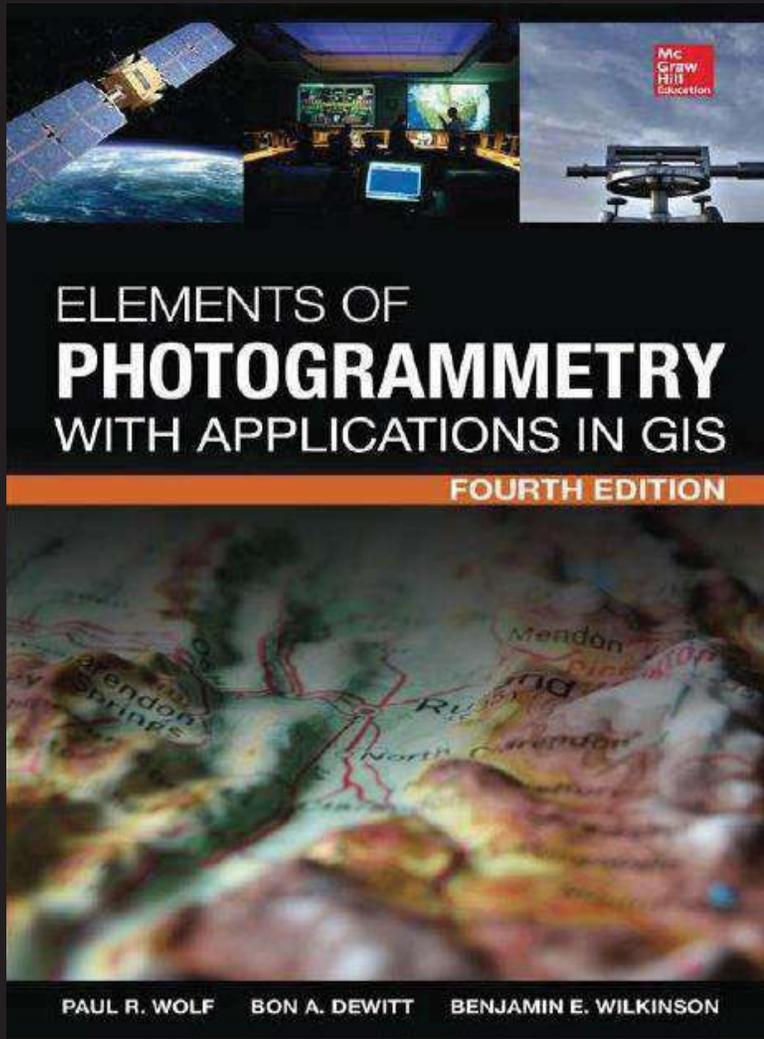
## AERIAL PHOTOGRAPHY AND IMAGE INTERPRETATION

by

David P. Paine and James  
D. Kiser ,  
3rd ed.

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Wiley & Sons, Inc

# Course Reference Book



## Elements of Photogrammetry with Applications in GIS

by

*Paul R. Wolf, Bon A. Dewitt,  
Benjamin E. Wilkinson,*

**4th Edition**

© McGraw-Hill  
Education

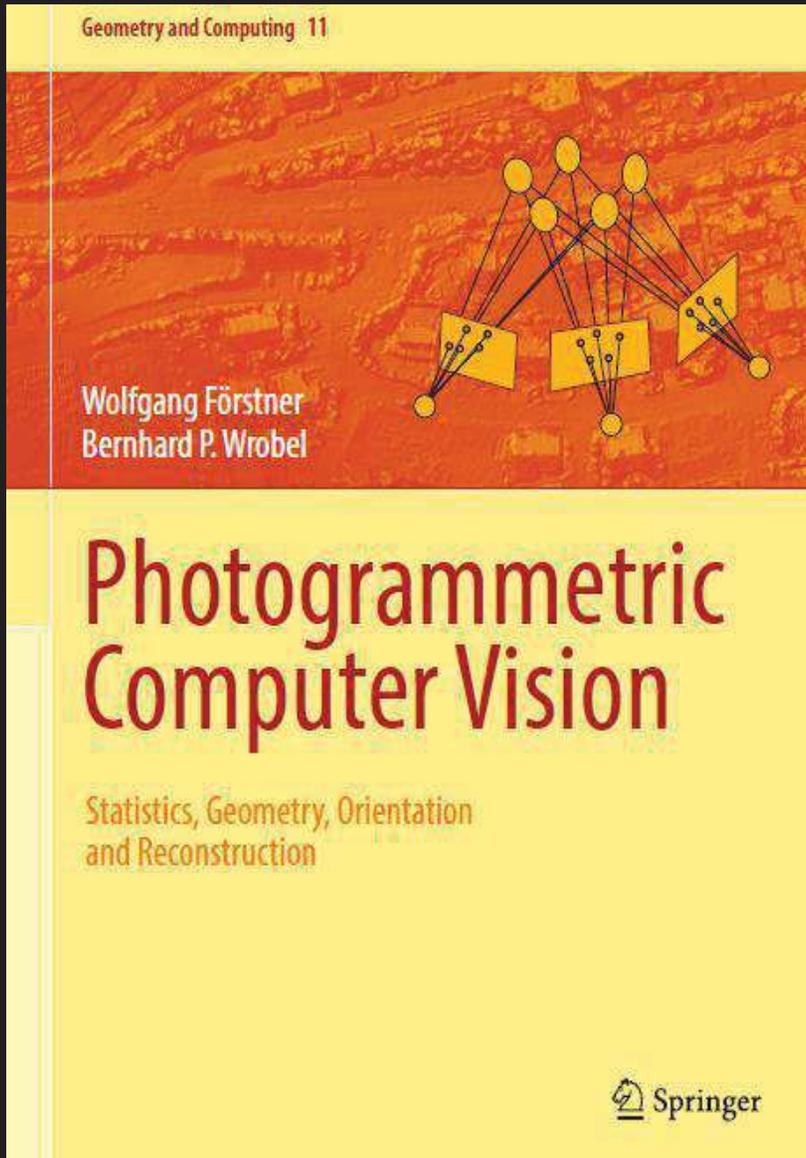
@2014

Elements of Photogrammetry with Applications in GIS

Paul R. Wolf, Ph.D

2014

# Course Reference Book



## Photogrammetric Computer Vision

Statistics, Geometry, Orientation  
and Reconstruction

by

*Wolfgang Förstner , Bernhard P.  
Wrobel*

© Springer International  
Publishing Switzerland  
2016

# Reading Chapters

*“Aerial Photography and Image Interpretation”, by David ..., 2012.*

	<b>Material</b>	<b>Chapter Sections</b>	<b>Page</b>	<b>Exercises page</b>	<b>LABORATORY EXERCISE page</b>
<u>1</u>	<b>Introduction</b>	all	1	23	83
<u>2</u>	Geometry of a Vertical Aerial Photograph		27		83
<u>3</u>	Principles of Stereoscopic Vision		44		83
<u>4</u>	Scale of a Vertical Aerial Photograph		68		101
<u>5</u>	Horizontal Measurements - Distance, Bearings and Areas		86		127
<u>6</u>	Vertical Measurements		105		127
<u>7</u>	Acquisition of Aerial Photography		131		154

# OBJECTIVES

After a thorough understanding of this chapter, you will be able to:

- Write precise definitions to differentiate clearly among the following terms: remote sensing, photogrammetry, and photo interpretation.
- Fully define the following terms: electromagnetic spectrum, atmospheric window, f-stop, film exposure, depth of field, and fiducial marks.
- Draw a diagram and write a paragraph to explain fully reflectance, transmittance, absorption, and refraction of light.
- List the wavelengths (bands) that can be detected by the human eye, film, and terrestrial digital cameras (both visible and photographic infrared bands).
- Draw complete diagrams of the energy-flow profile (a) from the sun to the sensor located in an aircraft or spacecraft and (b) within the camera.

# OBJECTIVES

- Draw a diagram of a simple frame camera (film or digital), showing the lens shutter, aperture, focal length, and the image captured.
- Given the first and subsequent photographs taken by a typical, large-format, aerial film camera in the United States, thoroughly explain the meaning of the information printed on the top of most photographs.
- Given a list of characteristics (or abilities) of various types of cameras discussed in this chapter, state whether each characteristic applies to film cameras only, digital cameras only, or both types of cameras.
- In a paragraph, briefly discuss the concept of pixel size and the number of pixels associated with digital cameras as related to resolution.

# What is Photogrammetry ?

- “photo” = light
- “gram” = film
- “metry” = metrics

Photogrammetry

= measuring with photographs

# What is Photogrammetry?

Estimation of the **geometric** and **semantic** properties of objects based on images or observations from similar sensors.

**What are “similar sensors” ?**

# What is Photogrammetry?

**Photogrammetry** is the science of making measurements from photographs, especially for recovering the exact positions of surface points.

**Photogrammetry** is as old as modern photography, dating to the mid-19th century and in the simplest example, the distance between two points that lie on a plane parallel to the photographic image plane, can be determined by measuring their distance on the image, if the scale ( $s$ ) of the image is known.

# What Do We Measure?

- Camera orientation
- Location of objects/scenes
- 3D reconstruction of objects/scenes
- Object recognition
- Interpretation of images
- Semantic interpretation
- ...

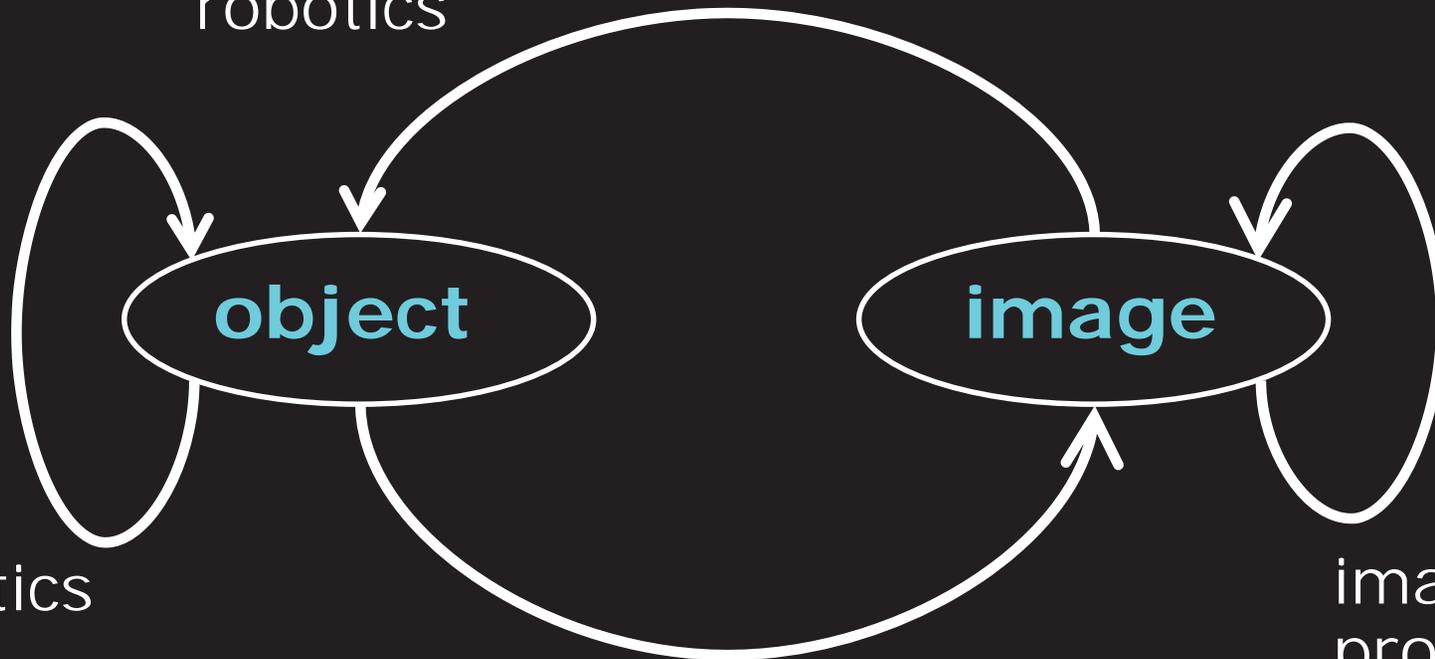
# Connections

## Remote sensing

Photogrammetry

computer vision

robotics



robotics

image  
processing

computer graphics

# Connections

- Developed for surveying purposes and is a part of the **geodetic sciences**
- Photogrammetry and image interpretation can be seen as a form of optical **remote sensing**
- Digital photogrammetry has strong connections to digital image processing and **computer vision**
- Strong links between photogrammetry and **state estimation** in robotics

## Advantages (1)

- Contact-free sensing

**Why is contact-free  
sensing relevant ?**

## Advantages (1)

- Contact-free sensing is important for
  - inaccessible (but visible) areas
  - sensitive material
  - hot/cold material
  - toxic material

# *Homework's*

(from 1 to 6 only)

Questions and Problem

in Page 23

are required

## QUESTIONS AND PROBLEMS (Page 23)

1. Fully define these terms—remote sensing, photogrammetry, and photo interpretation—in such a manner that clearly illustrates the differences among them.
2. Fully define these terms: electromagnetic spectrum, atmospheric windows, f-stop, exposure, depth of field, fiducial marks, pixels, silver halides, hard and soft copy display, photograph versus an image, focal length, and aperture.
3. Draw a diagram and write a paragraph to explain reflectance, transmittance, absorptance, and refraction.
4. Draw a diagram illustrating a typical energy-flow profile from the sun, or other source of energy, to a sensor located in an aircraft or spacecraft.
5. Draw a diagram of the electromagnetic spectrum showing the human visible and film- visible portions labeling the wavelengths
6. Draw a diagram of a simple film or digital frame camera showing the lens, shutter, aperture, focal length, and relative position of the image-capturing device.

*Thank you*

Any Questions ?



**END**

**of Lecture**

# Lecture 1-b

## Digital Image Processing:

### Introduction

*Lecturer:*

*Dr. Faisal Ghazi Mohammed*

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# *Course Contents*



1. Segmentation

2. Mathematical Morphology

3. Object Description and Representation

4. Color Image Processing

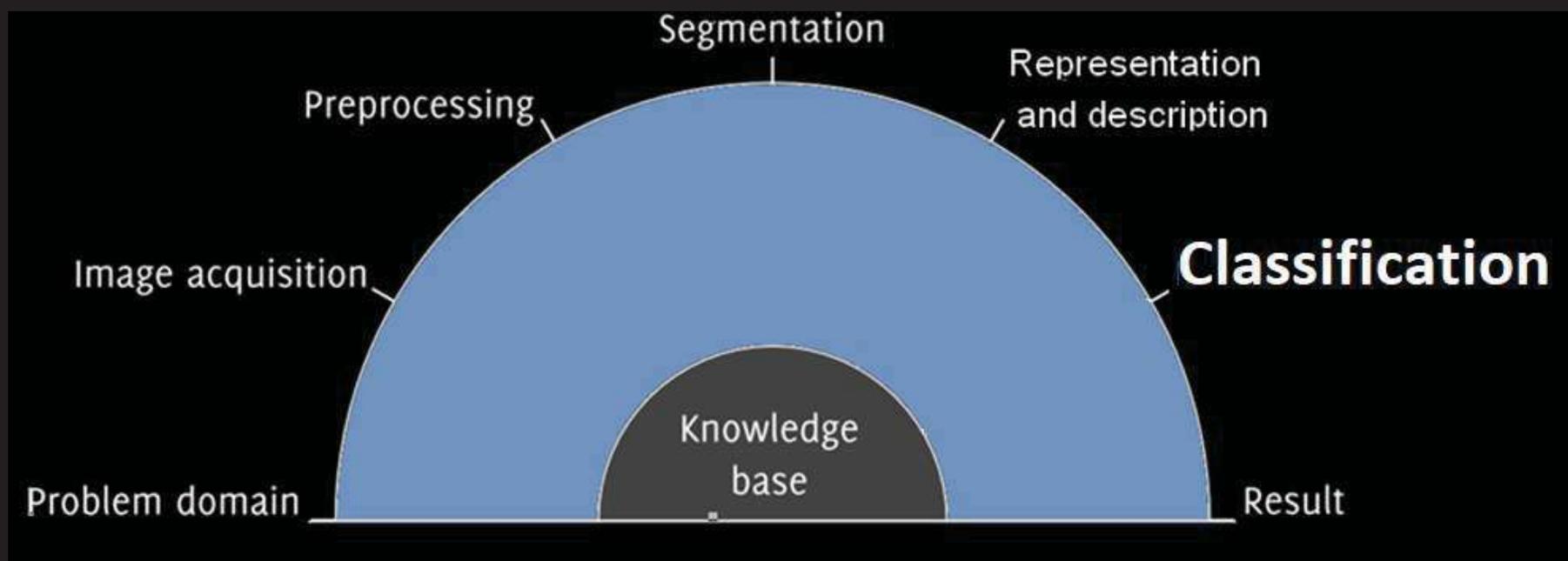
5. Classification

6. Image coding and compression: intro.

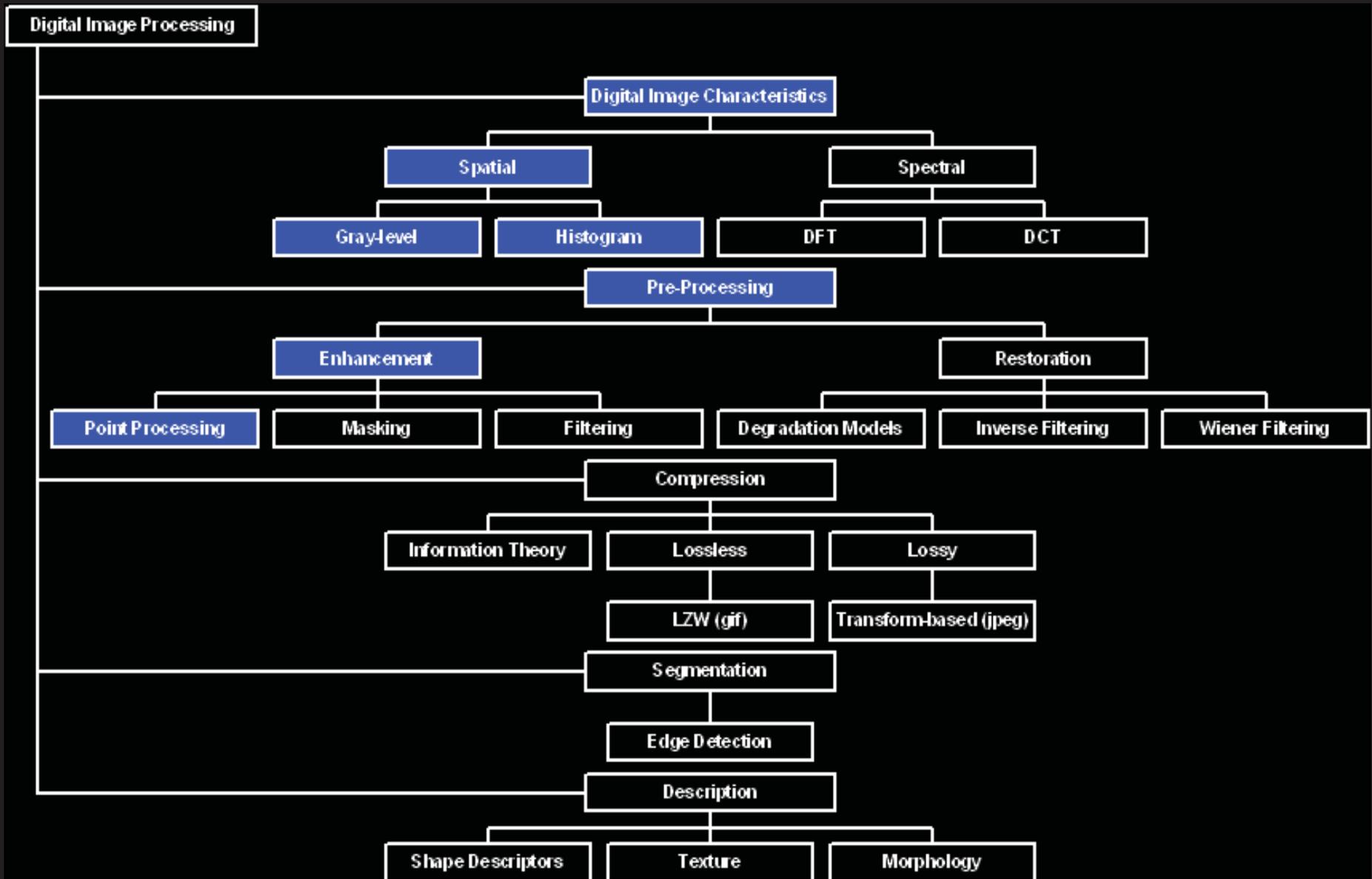
# *1. An overview on classification*



## Our progress in the analysis process



# DIP: Topics



# *Contents*

This lecture will cover:

1. What is a digital image?
2. What is digital image processing?
3. History of digital image processing
4. State of the art examples of digital image processing
5. Key stages in digital image processing

# Image Resolution

1. Intensity resolution
  - Each pixel has only “Depth” bits for colors/intensities
2. Spatial resolution
  - Image has only “Width” x “Height” pixels
3. Temporal resolution
  - Monitor refreshes images at only “Rate” Hz

Typical  
Resolutions

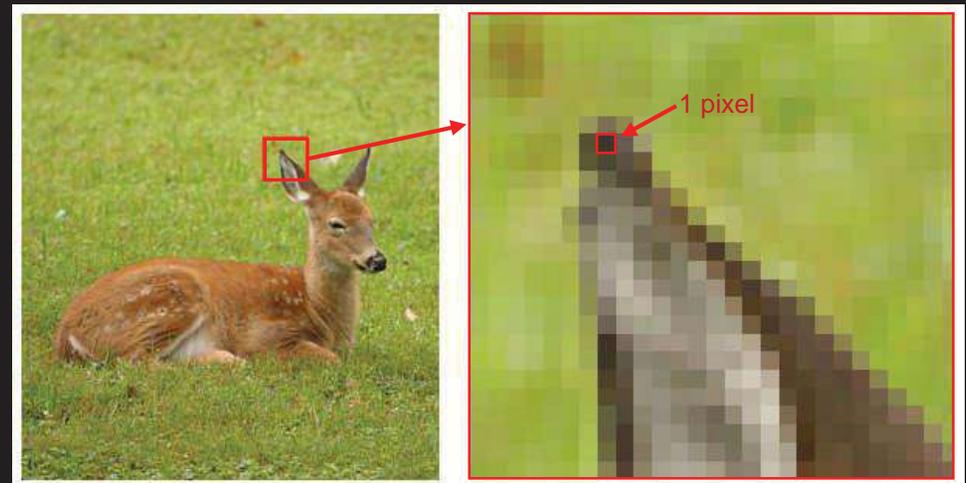
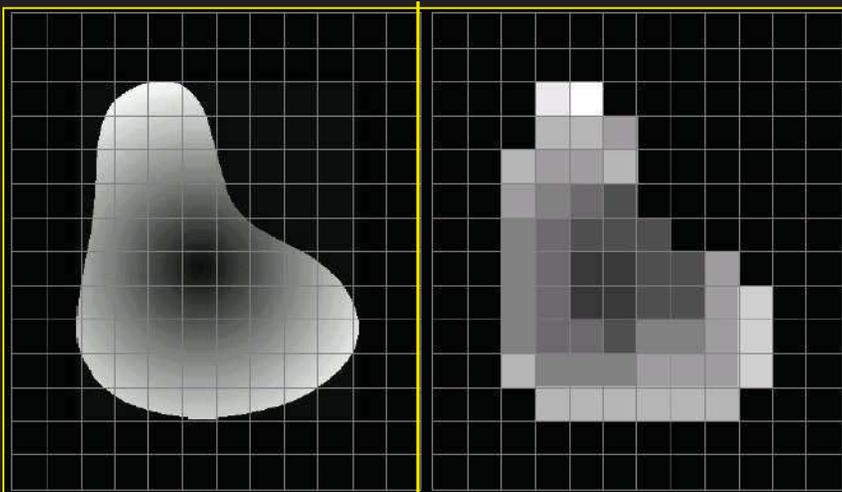
	Width x Height	Depth	Rate
NTSC	640 x 480	8	30
Workstation	1280 x 1024	24	75
Film	3000 x 2000	12	24
Laser Printer	6600 x 5100	1	-

# 1-What is a Digital Image?

**Pixel** : The elements of a digital image

Pixel values typically represent gray levels, colours, heights, opacities etc

**Remember** *digitization* implies that a digital image is an *approximation* of a real scene



# *1-What is a Digital Image?*

Common image formats include:

- 1 sample per point (B&W or Grayscale)
- 3 samples per point (Red, Green, and Blue)
- 4 samples per point (Red, Green, Blue, and “Alpha”,

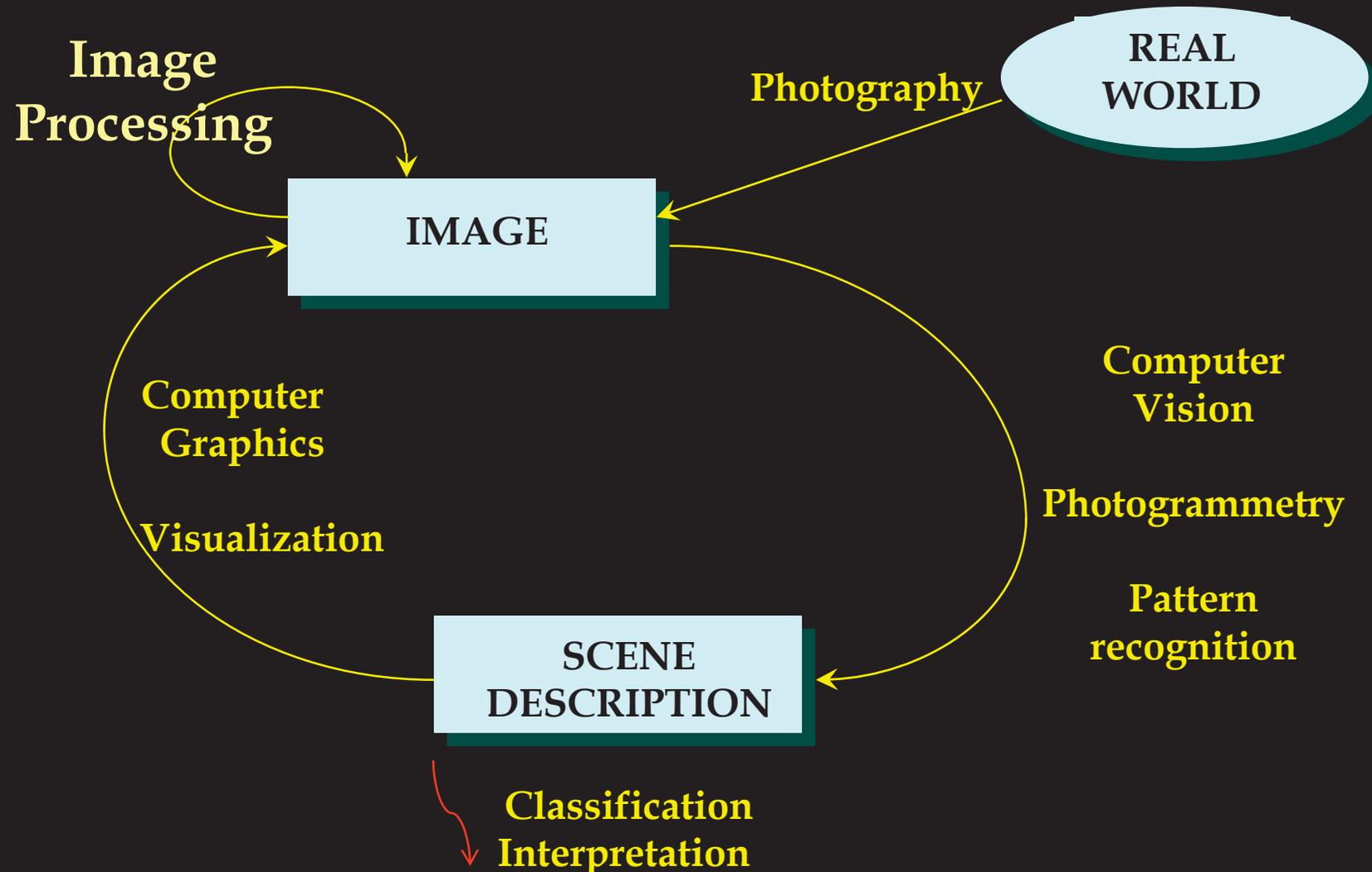
For most of this course we will **focus** on grey-scale images



# *1-What is a Digital Image?*

- Image stored in memory as 2D pixel array
- Value of each pixel controls color
- Depth of image is information per pixel
  - **1 bit**: black and white display
  - **8 bit**: 256 colors at any given time via colormap
  - **16 bit**: 5, 6, 5 bits (R,G,B),  $2^{16} = 65,536$  colors
  - **24 bit**: 8, 8, 8 bits (R,G,B),  $2^{24} = 16,777,216$  colors

# Vision, Image Processing and Visualization



## *2-What is a Digital Image Processing?*

Digital image processing focuses on two major tasks:

- **Improvement** of pictorial information for human interpretation
- **Processing** of image data for storage, transmission and representation for autonomous machine perception

Some argument about where image processing ends and fields such as image analysis and computer vision start

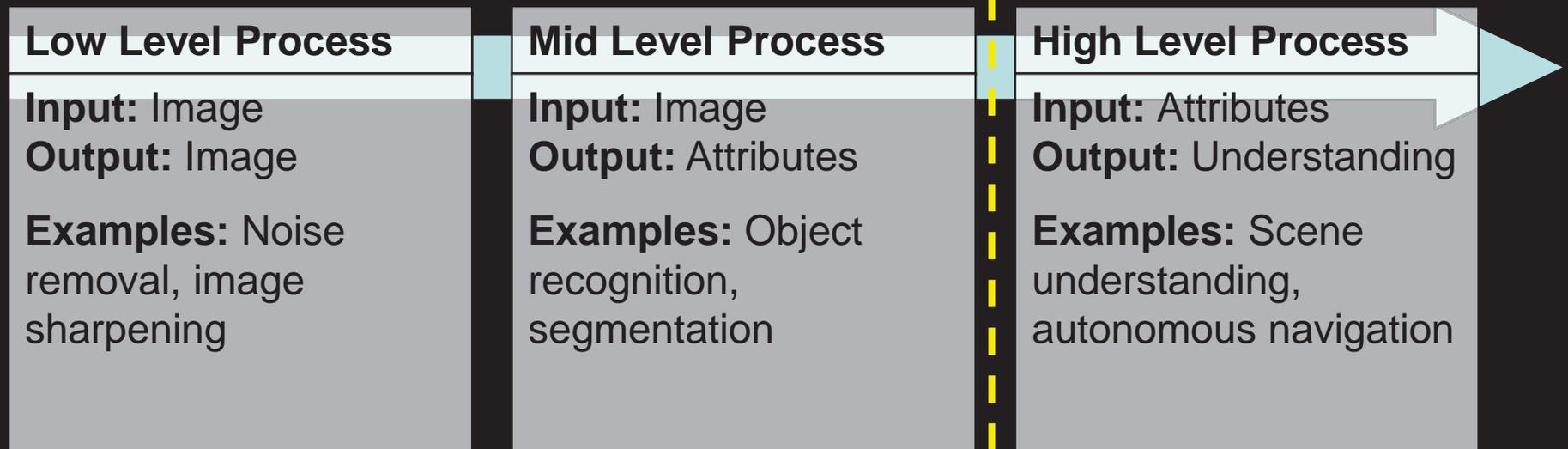
## 2-What is a Digital Image Processing?

process digital images by means of computer, it covers **low-, mid-, and high-level processes**

**low-level:** inputs and outputs are images

**mid-level:** outputs are attributes extracted from input images

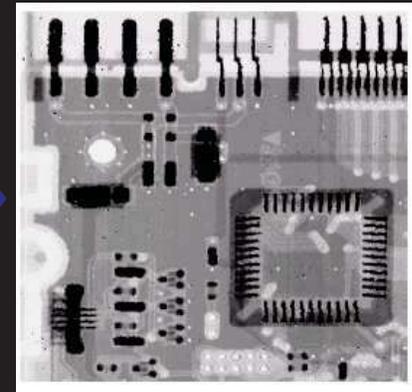
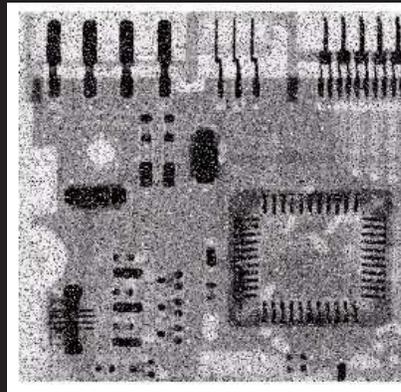
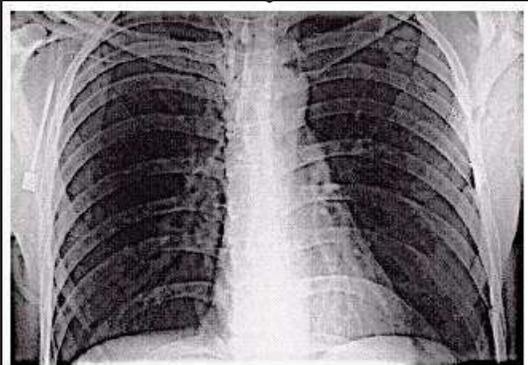
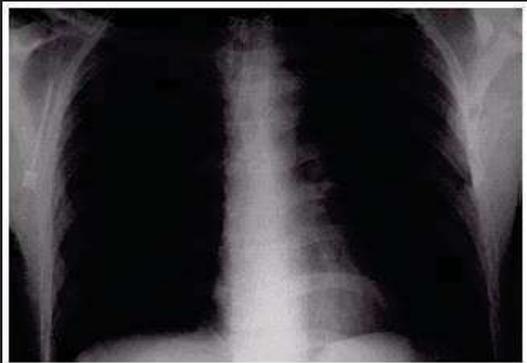
**high-level:** an ensemble of recognition of individual objects



# 4-Examples:

## Image Enhancement

One of the most common uses of DIP techniques: improve quality, remove noise etc



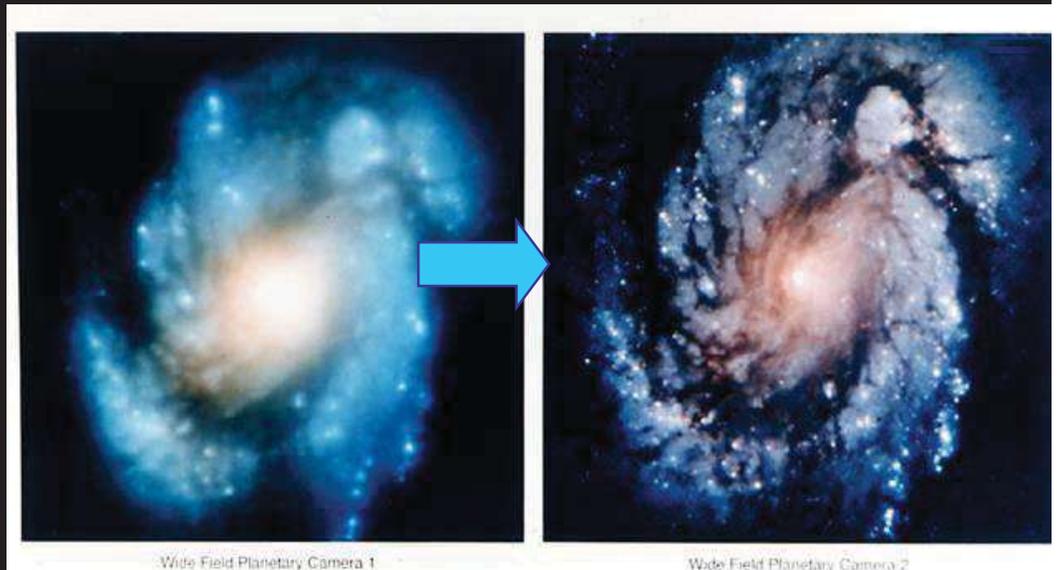
# 4-Examples:

## The Hubble Telescope

Launched in 1990 the Hubble telescope can take images of very distant objects

However, an incorrect mirror made many of Hubble's images useless

- Image processing techniques were used to fix this



Wide Field Planetary Camera 1

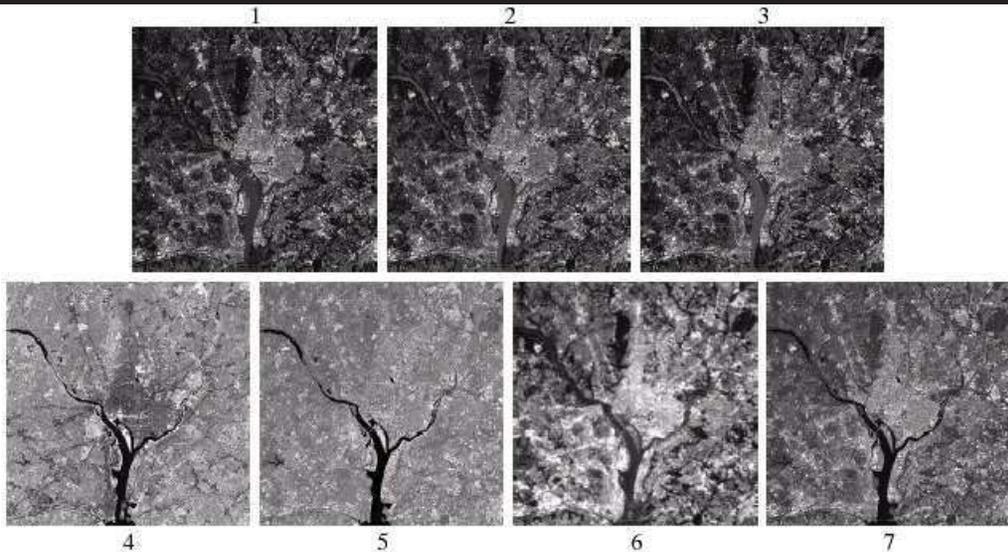
Wide Field Planetary Camera 2

# 4-Examples:

## GIS

### Geographic Information Systems

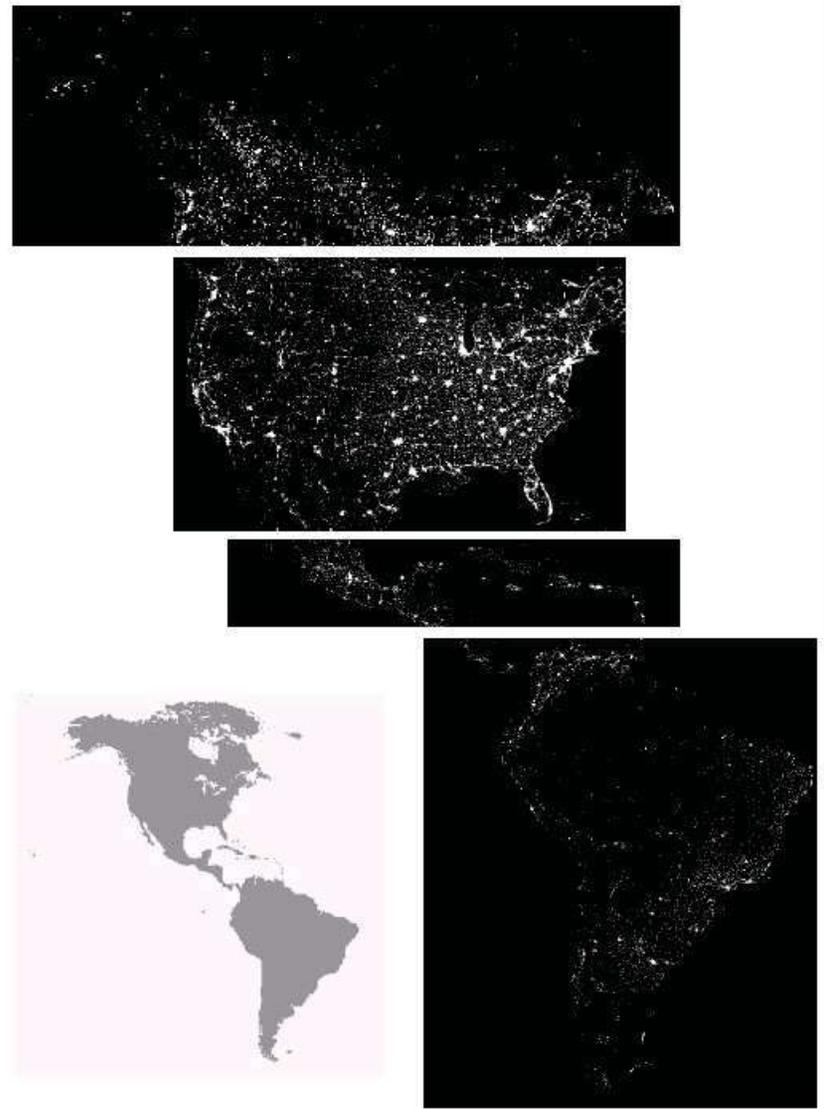
- Digital image processing techniques are used extensively to manipulate satellite imagery
- Terrain classification
- Meteorology



# 4-Examples:

## GIS

- *Night-Time Lights of the World* data set
  - Global inventory of human settlement
  - Not hard to imagine the kind of analysis that might be done using this data



# Radar Image

**FIGURE 1.16**  
Spaceborne radar  
image of  
mountains in  
southeast Tibet.  
(Courtesy of  
NASA.)

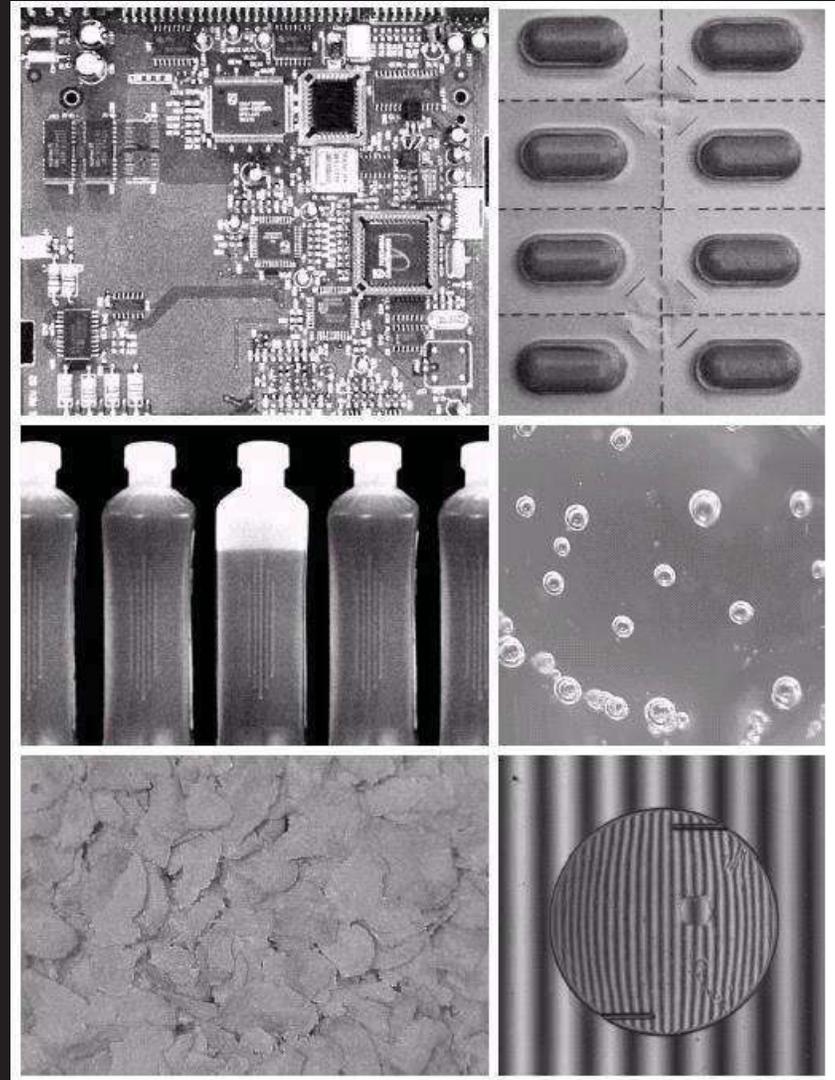
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# 4-Examples:

## Industrial Inspection

- Human operators are expensive, slow and unreliable
- Make machines do the job instead
- Industrial vision systems are used in all kinds of industries
- Can we trust them?

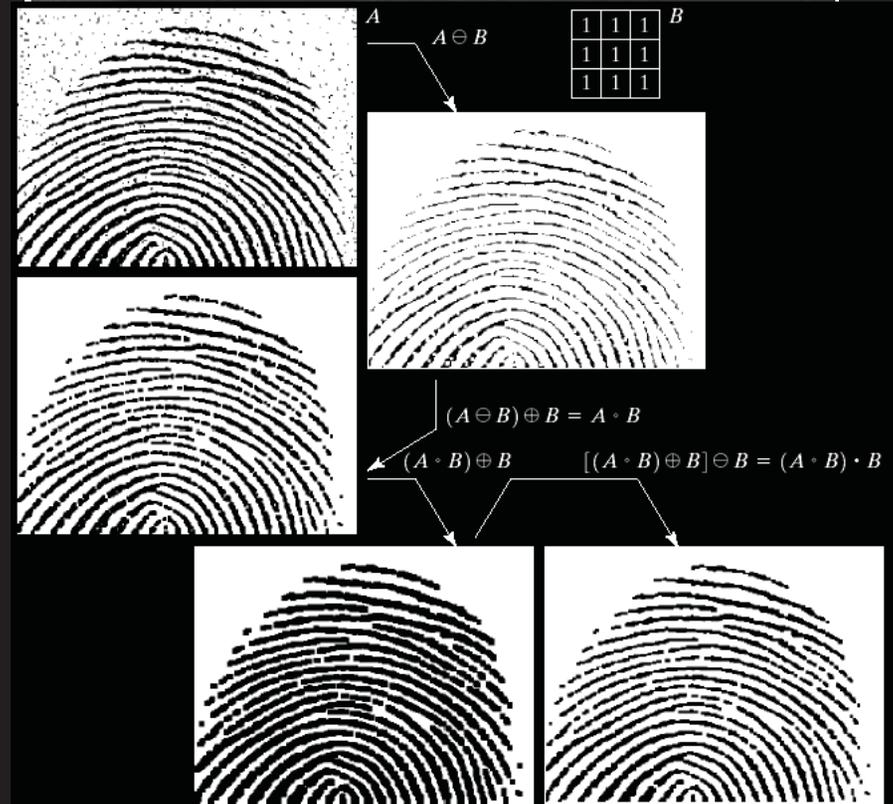


# 4-Examples:

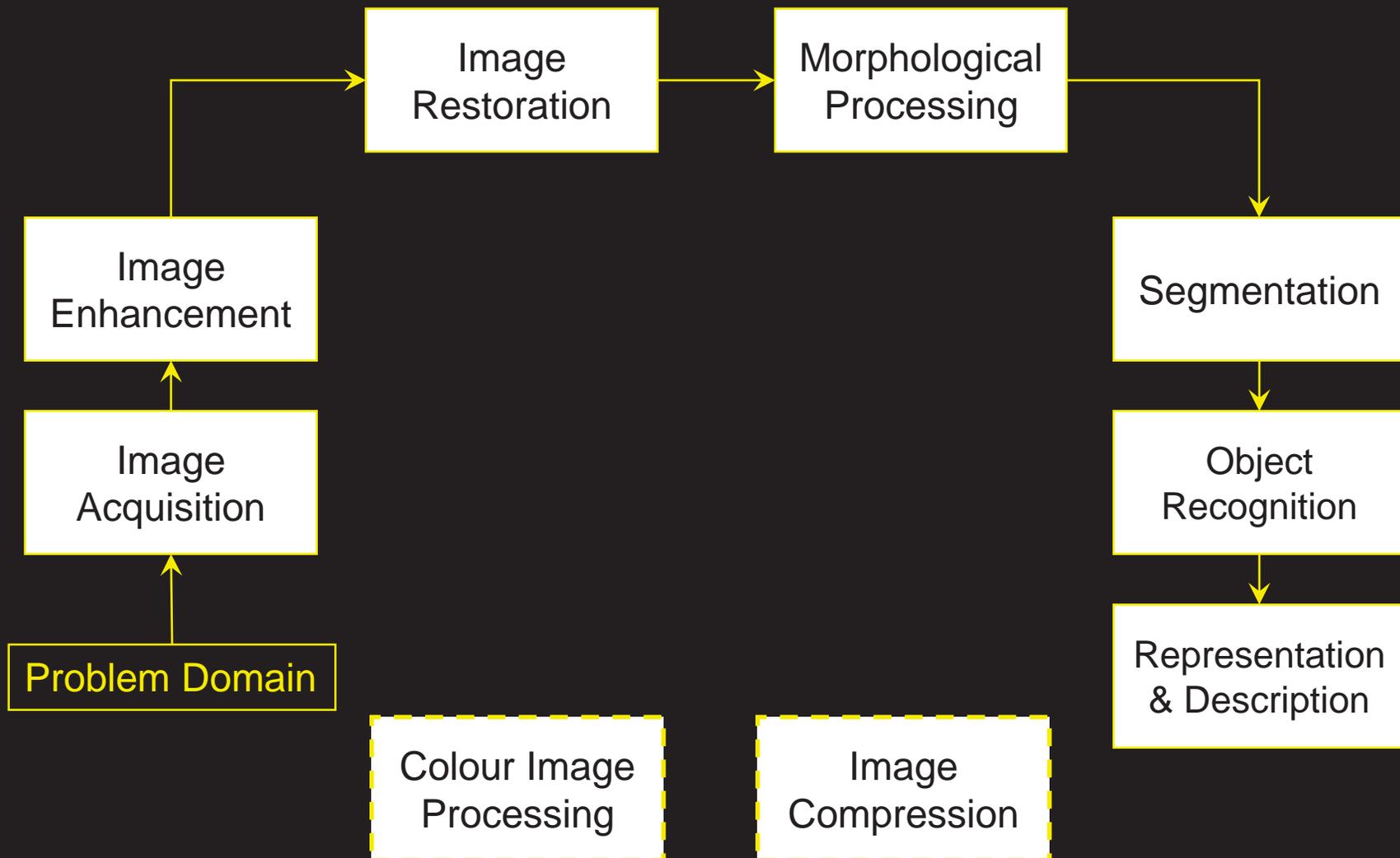
## Law Enforcement

Image processing techniques are used extensively by law enforcers:

- Number plate recognition for speed cameras/automated toll systems
- Fingerprint recognition
- Enhancement of CCTV images

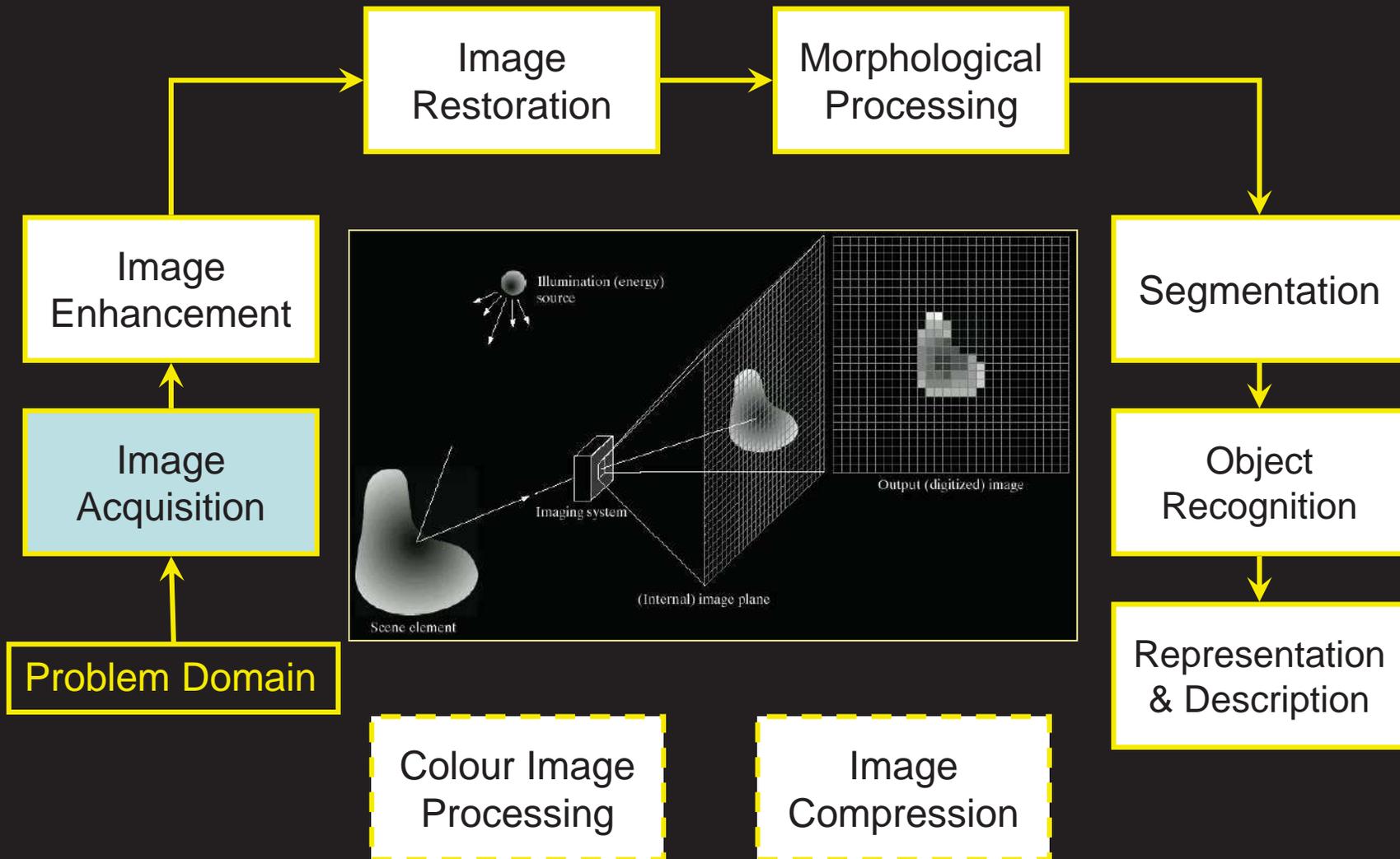


# 5-Key Stages in Digital Image Processing



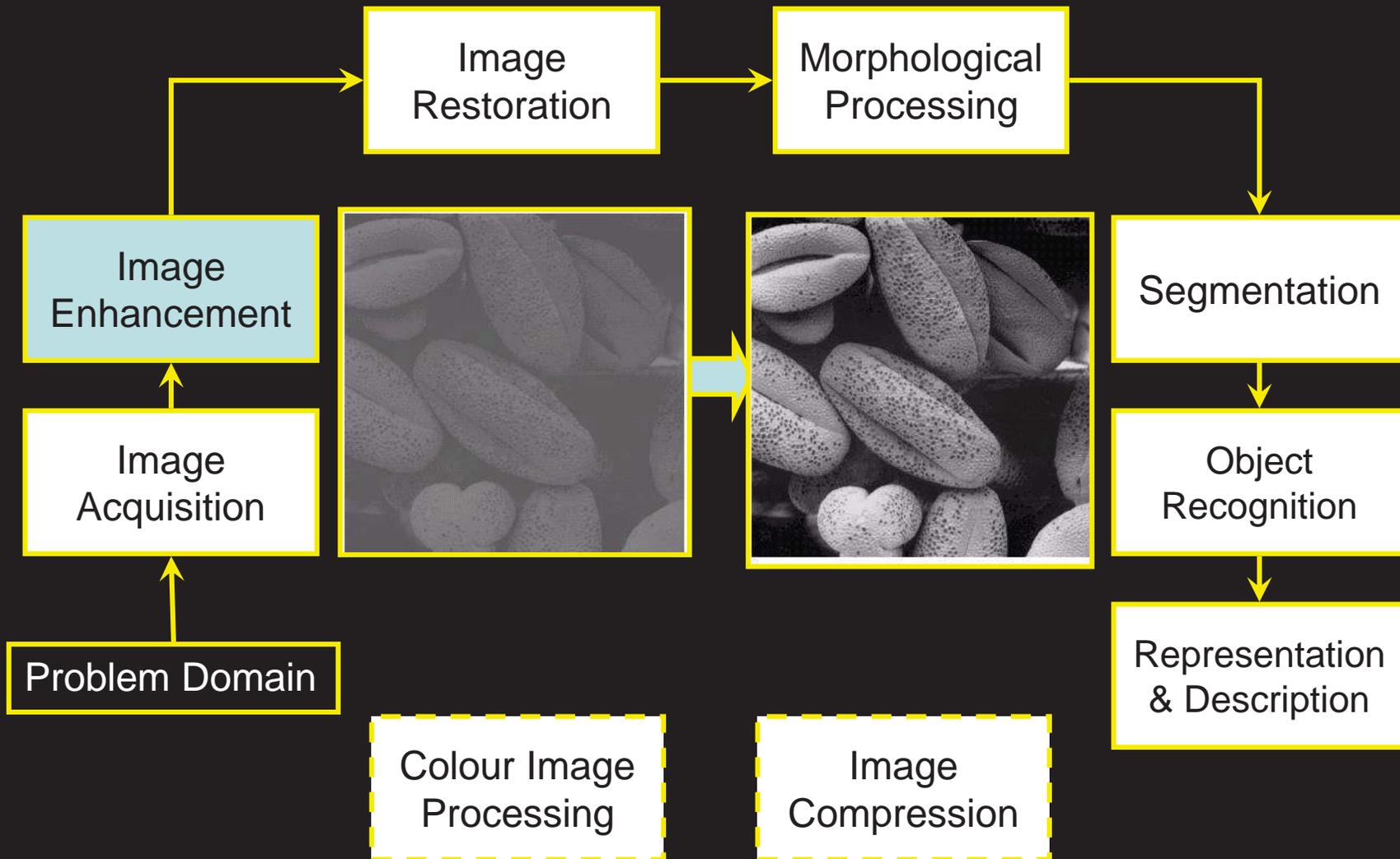
# 5-Key Stages in Digital Image Processing

## Image Acquisition



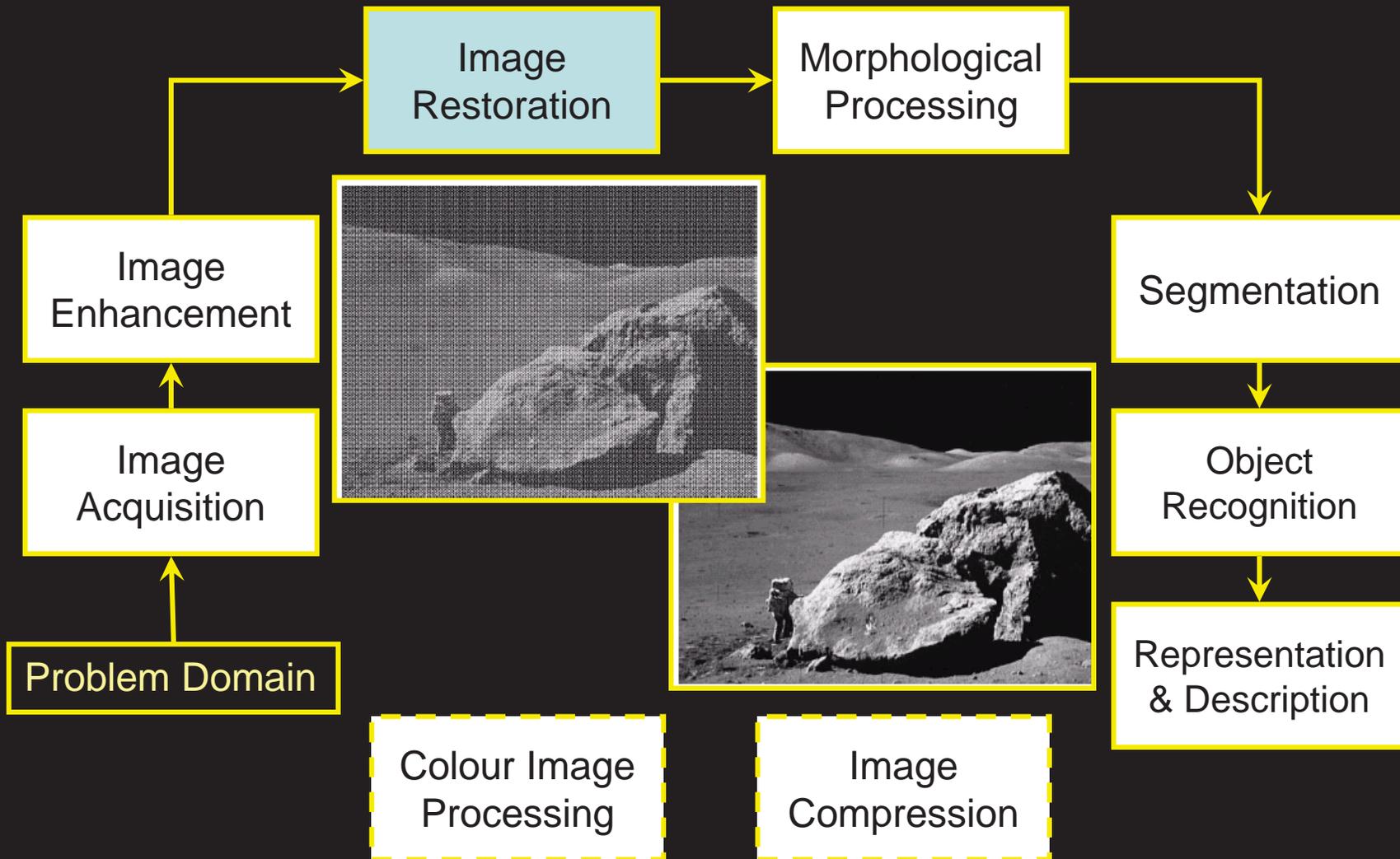
# 5-Key Stages in Digital Image Processing

## Image Enhancement



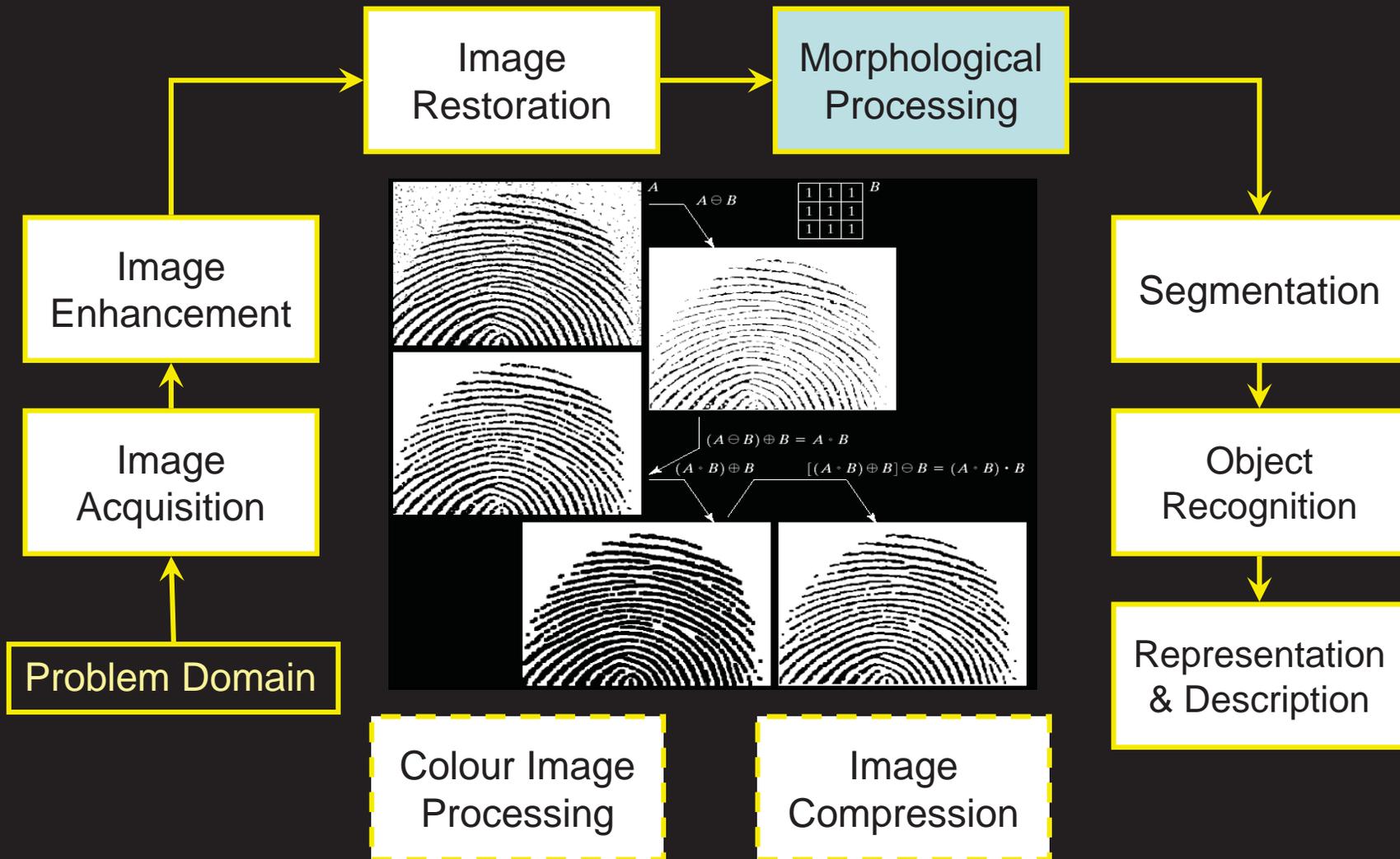
# 5-Key Stages in Digital Image Processing

## Image Restoration



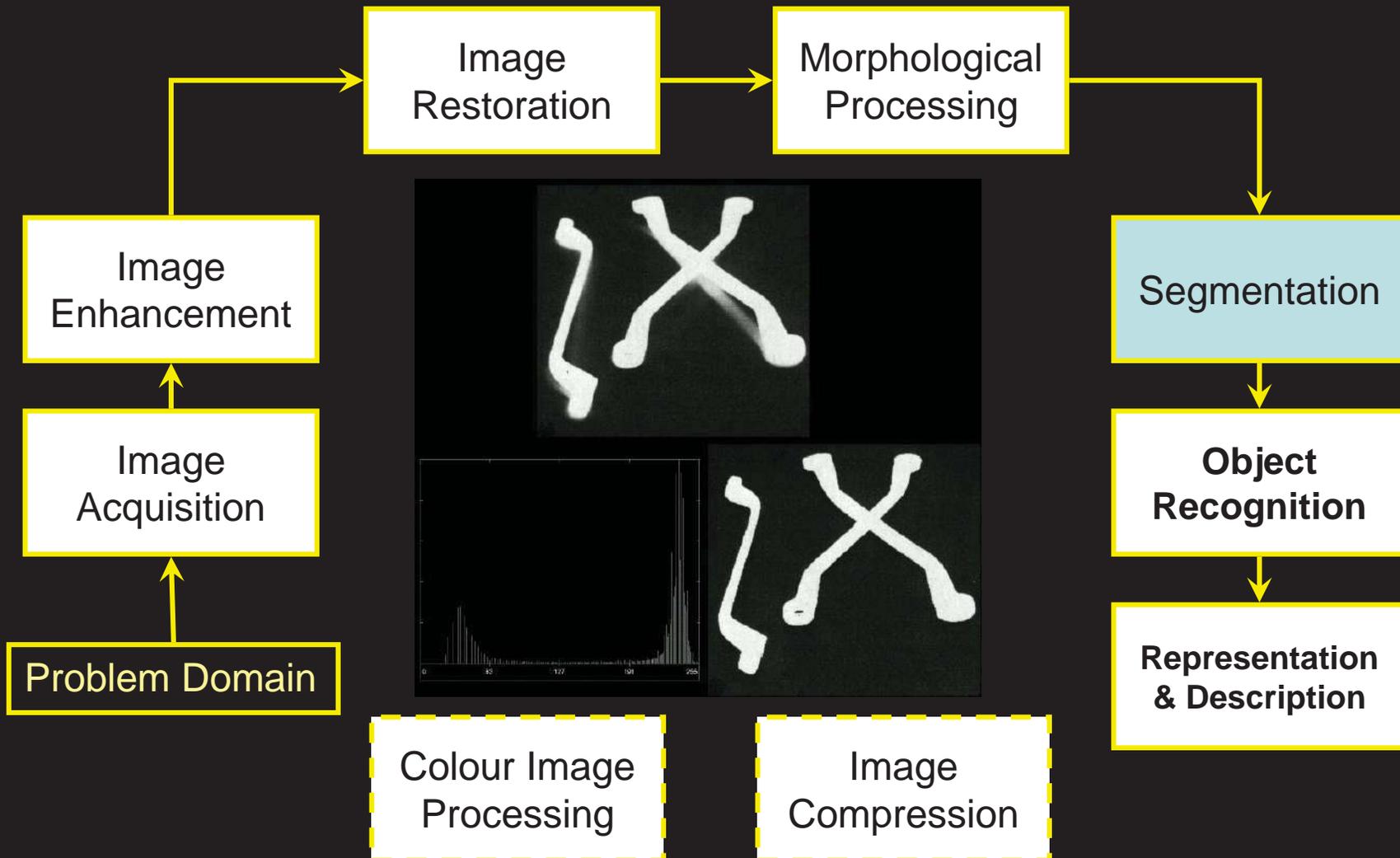
# 5-Key Stages in Digital Image Processing

## Morphological Processing



# 5-Key Stages in Digital Image Processing

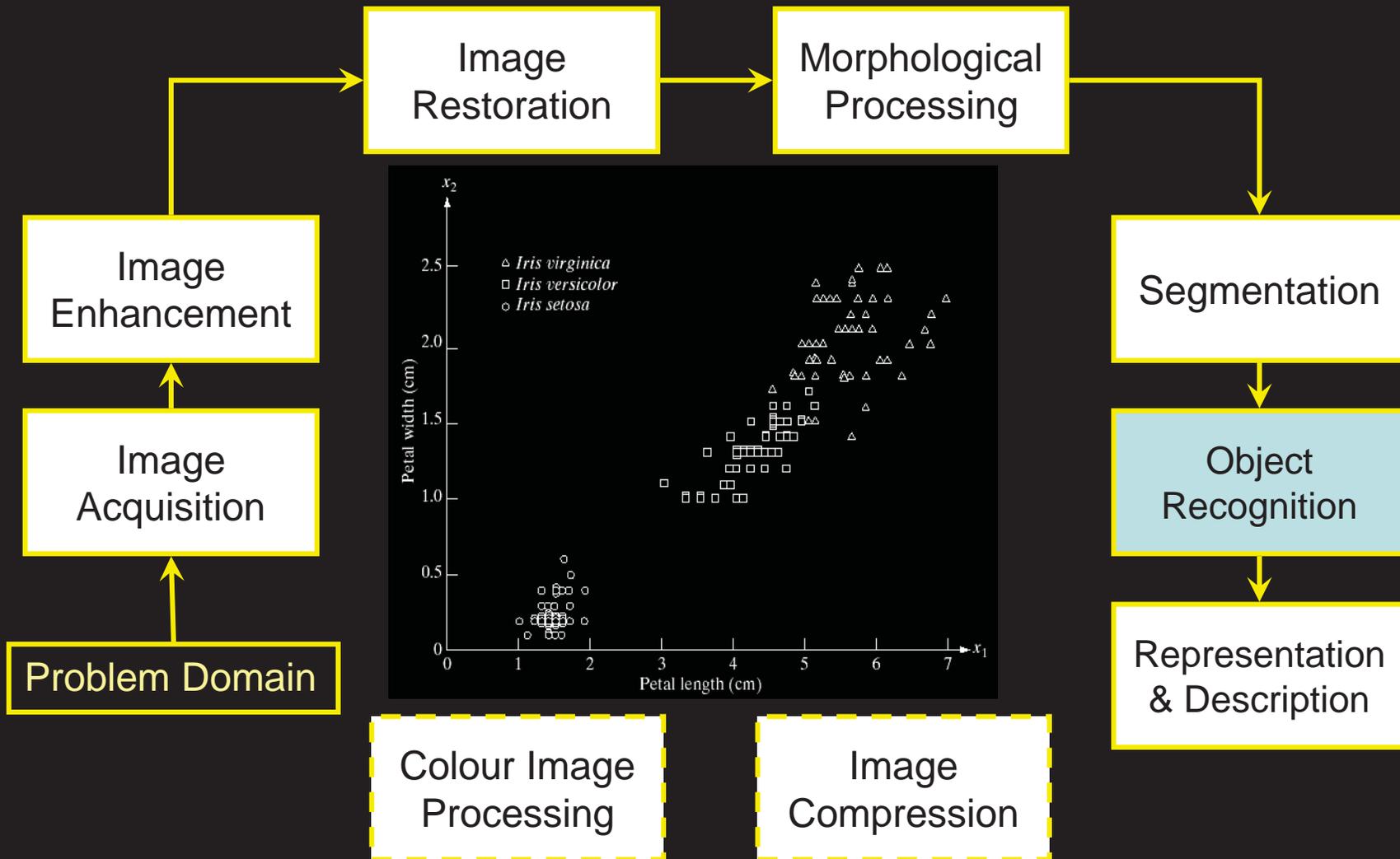
## Segmentation



# 5-Key Stages in Digital Image Processing

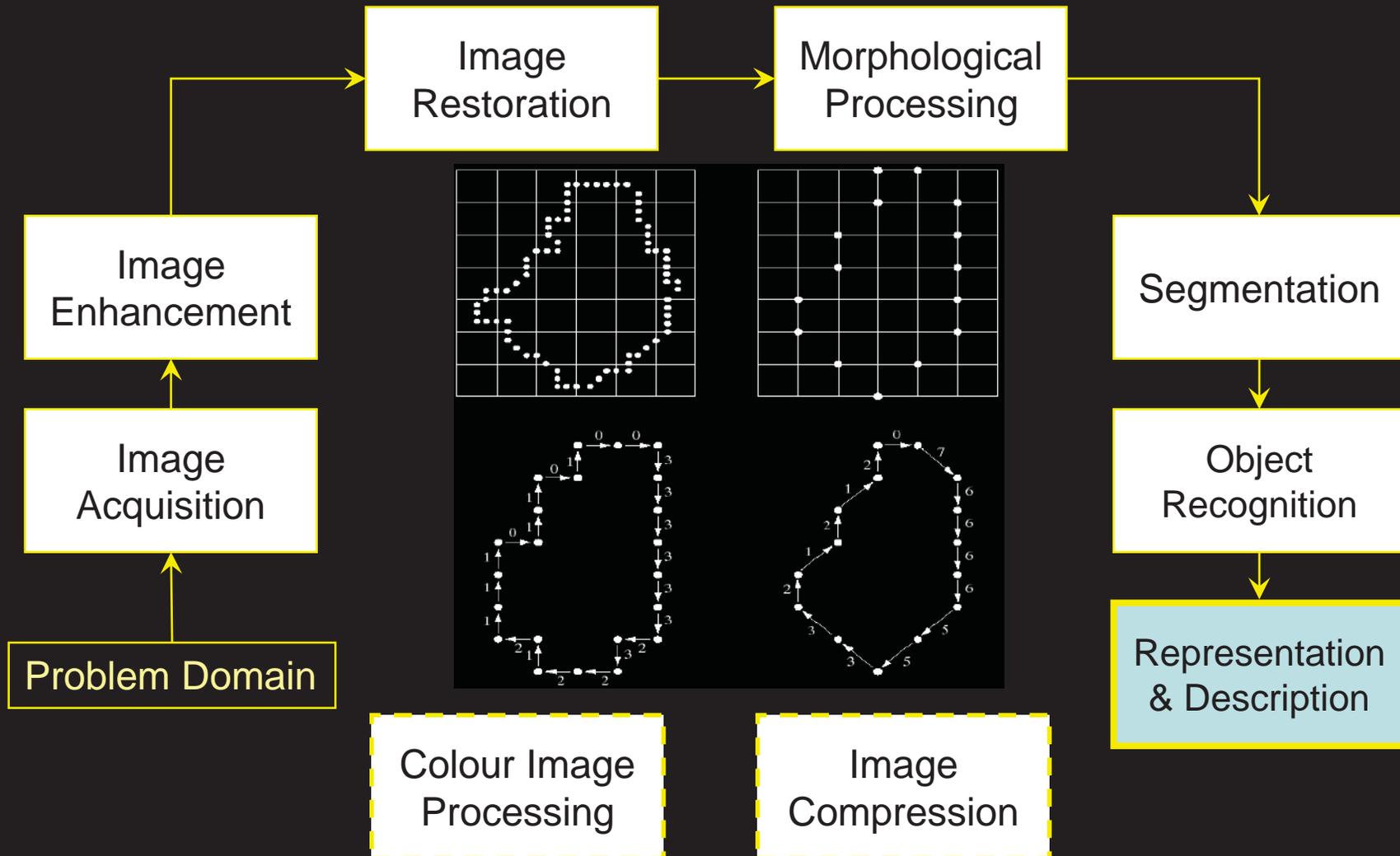
## Object Recognition

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



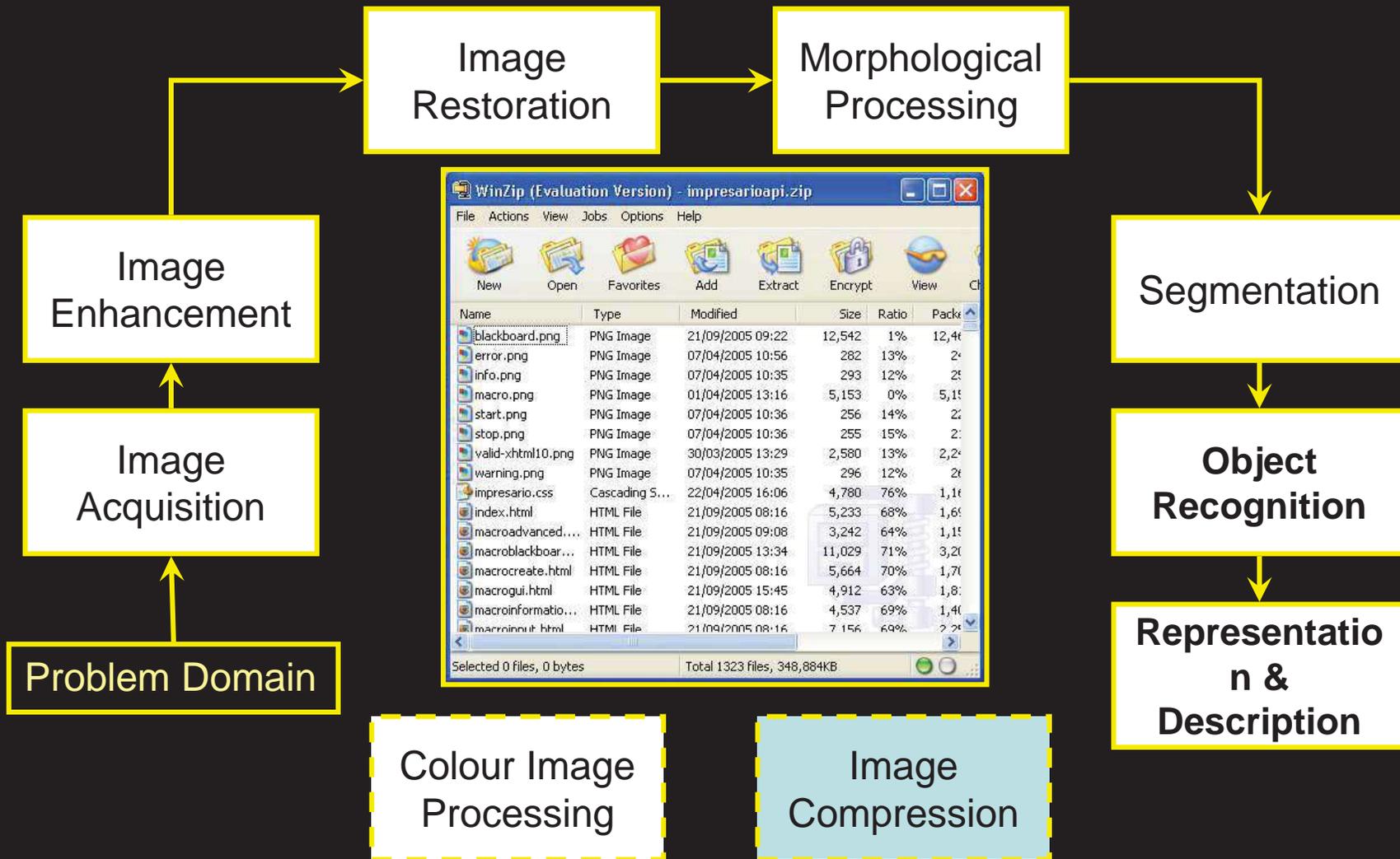
# 5-Key Stages in Digital Image Processing

## Representation & Description



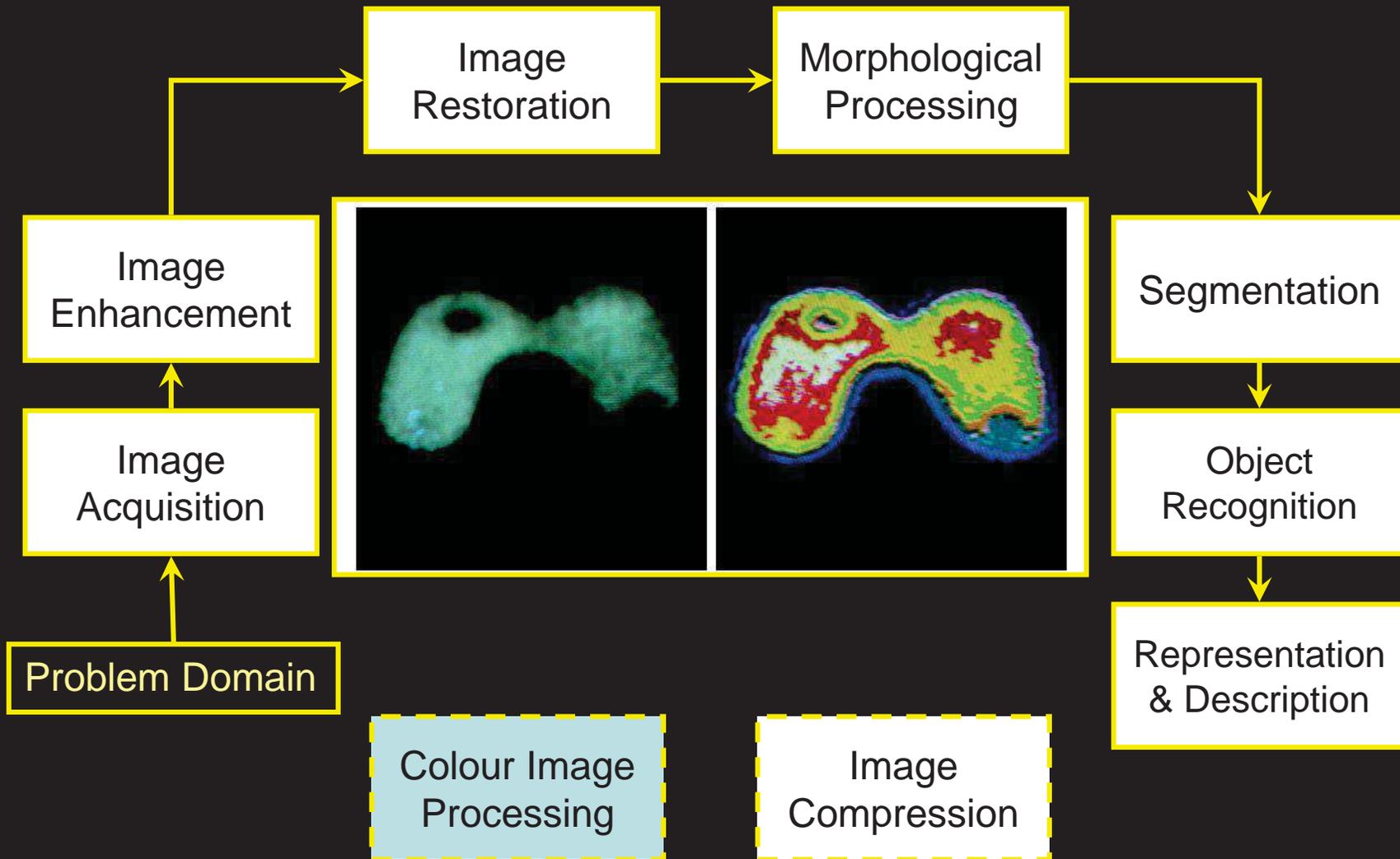
# 5-Key Stages in Digital Image Processing

## Image Compression



# 5-Key Stages in Digital Image Processing

## Colour Image Processing



Thank you

Any Questions ?

**END**

**Of Lecture 1**

# Homework

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ



# *Aerial Photography & Photogrammetry*

## Lecture 1

# Aerial Photography

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[faiselgm73@gmail.com](mailto:faiselgm73@gmail.com)

*2017-2019*

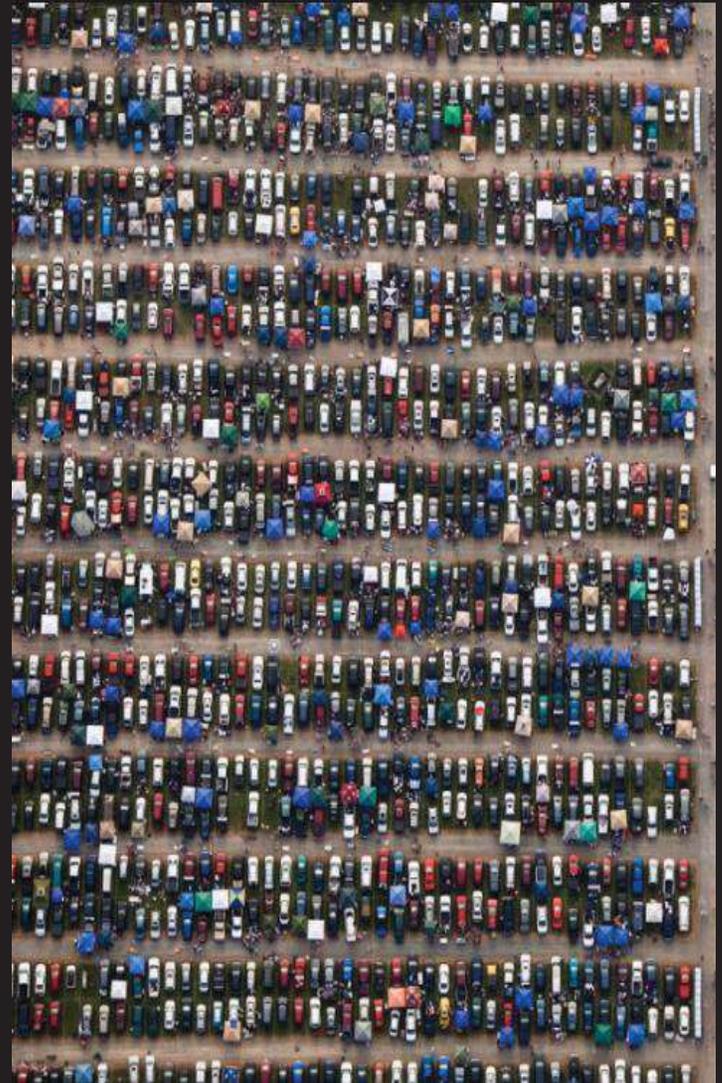
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# Reading Chapters

*"Aerial Photography and Image Interpretation", by David ..., 2012.*

	<i>Material</i>	<i>Chapter Sections</i>	<i>Page</i>	<i>Exercises page</i>	<b>LABORATORY EXERCISE page</b>
<u>1</u>	<b>Introduction</b>	all	1	23	83
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<u>4</u>	<b>Scale of a Vertical Aerial Photograph</b>		68		101
<u>5</u>	<b>Horizontal Measurements - Distance, Bearings and Areas</b>		86		127
<u>6</u>	<b>Vertical Measurements</b>		105		127
<u>7</u>	<b>Acquisition of Aerial Photography</b>		131		154

1. Introduction: What is Aerial Photography?
2. Historical Aerial Photography
3. Collecting and Processing the imagery
4. Making Use of Aerial Photography
5. Satellite Imagery and Colour Infrared



# Aerial Photography

- Aerial photography is the production of photographic images from balloons, helicopters or airplanes; it's used primarily for mapping.
- In 1855, French balloonist Gaspar Felix Tournachon patented the first aerial photography process, though it took three years to produce the first image.
- Early experiments included using pigeons equipped with automatic cameras and using biplanes in World War I to capture images of enemy trenches.
- Aerial photography was successfully commercialized by Sherman Fairchild for aerial surveys of land and cities after WorldWar1 and has been used in government and civil applications ever since.

One of the smaller models of aerial camera, dated 1907, kept in Duetsches Museum, Germany.

Source: Curran, (1988).



Pigeon photo

# Satellite Imagery

- The term "satellite imagery" may refer to a number of types of digitally transmitted images taken by **artificial satellites orbiting** the Earth.
- The United States launched the first satellite imaging system in **1960** to **spy** on the Soviet Union. Since then, in addition to military applications, satellite imagery has been used for mapping, environmental monitoring, archaeological surveys and weather prediction.
- Governments, large corporations and educational institutions make the most use of these images.

## Advantages

### Aerial Photography

- Aerial photography is still a **better choice** for most business and personal commercial uses than satellite imagery.
- Aerial photography **costs less** and, in some cases, it's more **up-to-date**, as many available satellite maps are more than a year old and don't necessarily reflect recent changes or developments.
- Individuals and small companies can more easily **hire** an aerial photographer and have more input in the process.
- **Resolution** and clarity is likely to be **higher** as well, making images easier to understand and often eliminating the need for special analysis.

### Satellite Imagery

- It can be used to **track weather systems**, especially dangerous storms like **hurricanes**, with great accuracy.
- Satellites **circle the Earth**, so their imaging activity can be repeated easily.
- It also allows for **much greater areas** of coverage and, because all information is digital, it can be easily integrated with software.
- In some cases, **cloud cover** does not affect results.

# **Aerial photography**

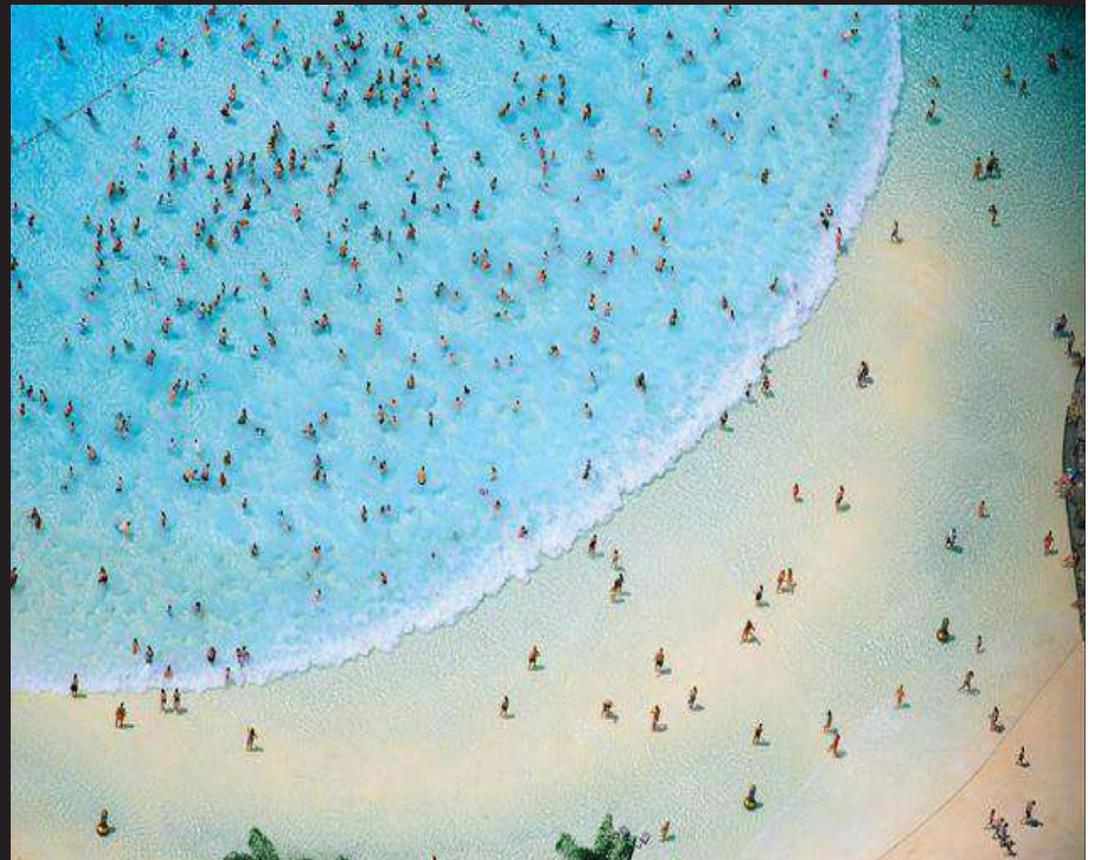
is the taking of photographs  
of the ground from an  
elevated position.

# 1- What is Aerial Photography

A type of remote sensing involving the capture of images from an elevated position above the Earth's surface, using a camera in not supported by a ground based structure .

A snapshot of the Earth at a particular instant in time.

Contains mass data and can be interpreted for a range of purposes.

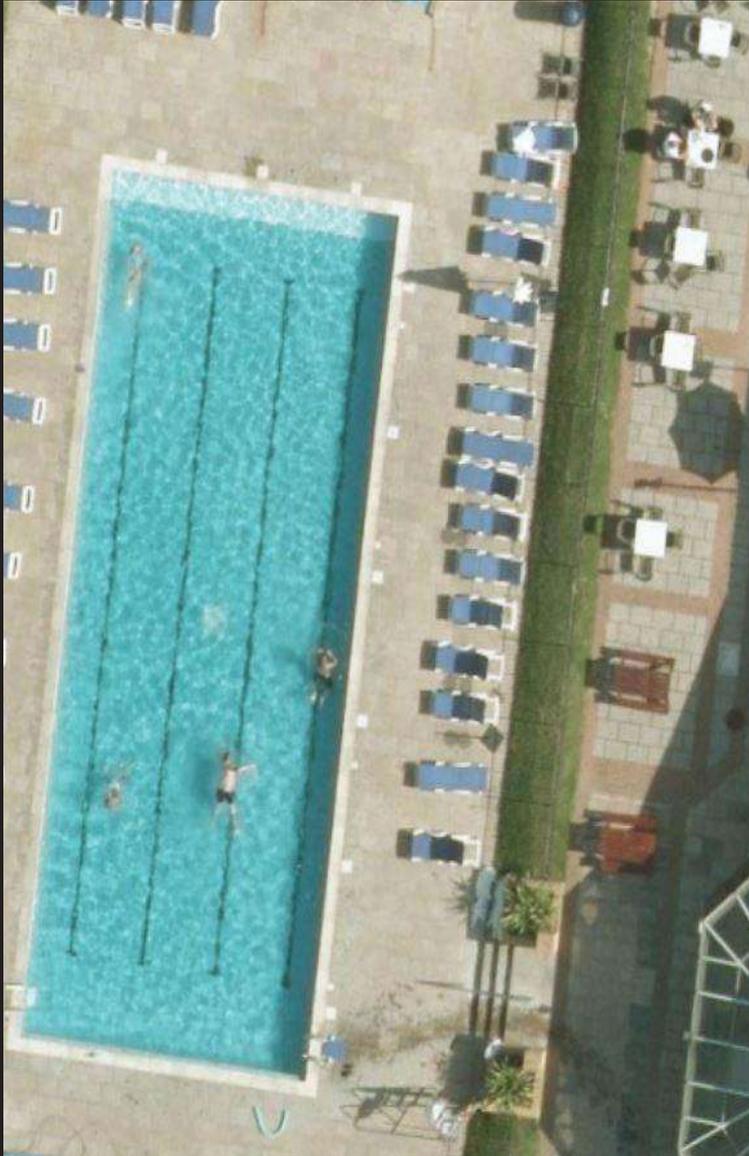


Can be collected from a range of platforms.

- Aircraft
- Helicopters
- Blimps
- Rockets
- Kites
- UAV/Drones
- Balloons
- Pigeons



# Aerial Photography



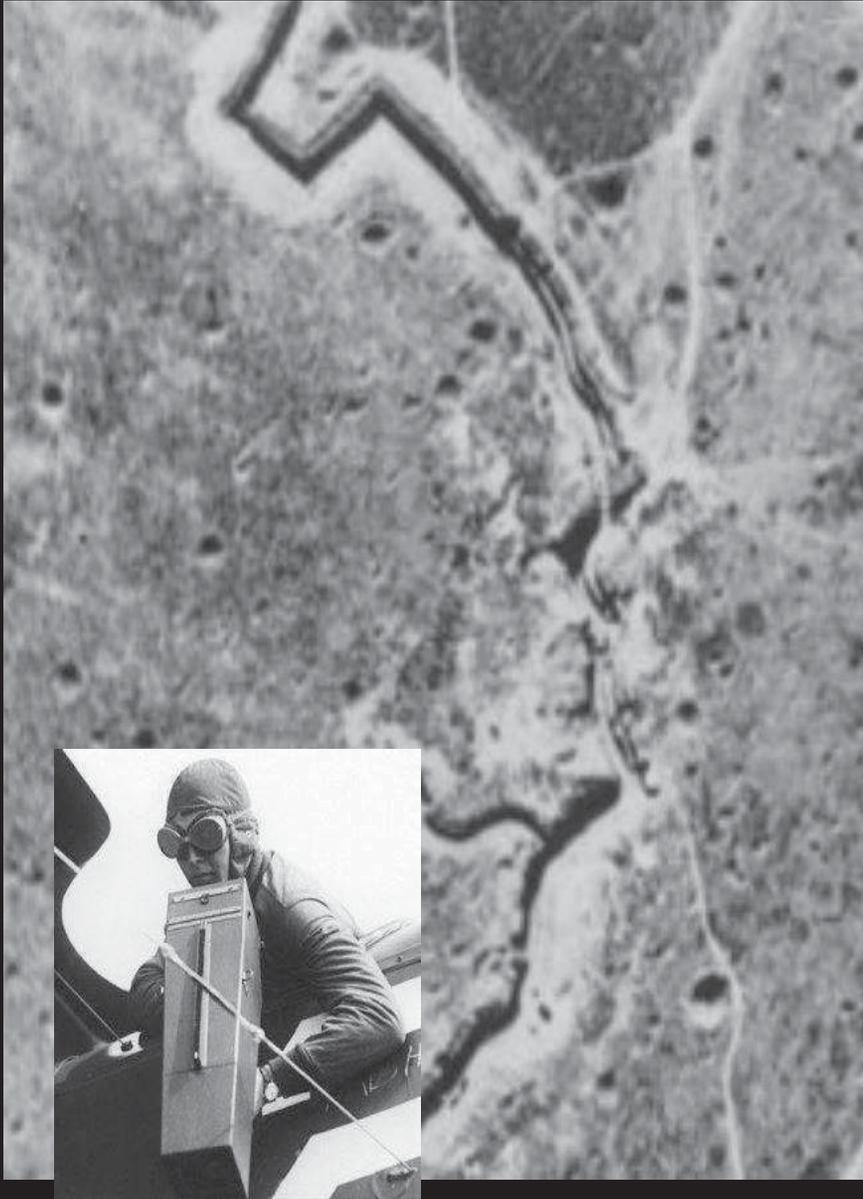
1. The level of details is determined by the resolution of the imagery
2. This is the Ground Sample Distance (GSD).
3. The standard resolution is **12.5 cm**, available over most of Iraq.
4. Imagery can be captured up to any resolution required – 5cm Enfield

# Historic Aerial Photography



1. Oldest Surviving Aerial Photograph “Boston as the Eagle and Wild Geese See it” – 1860
2. Captured from a balloon at 1200 feet.
3. **First** Aerial Photograph was **captured over Paris** in 1858 which is 45 years before the Wright Brothers pioneered powered flight .
4. The same technique was used for **reconnaissance** in the American Civil War

# Historic Aerial Photography



1. First extensive use of Aerial Photography was during the WorldWar1.
2. Many of the techniques still used for capture and image processing today to were pioneered during WWI.
3. Aerial Photographs were used to identify enemy positions, troop movements and improvements to trenches
4. Image – German Lines at 8,000ft

# Historic Aerial Photography



Commercial surveying began after the First World War with a British Company called Aerofilms.

Vertical Imagery captured was typically used to aid in the creation of mapping

Aerial Reconnaissance was used extensively during the Second World War to identify targets such as V1 and V2 sites

Image – Sword Beach at 1,000ft

# Capturing Aerial Photography



- Getmapping capture aerial photography from fixed wing twin-engine turboprop aircraft
- All Aircraft are based at Oxford but are able to deploy from other airports if the weather is favourable
- 2 man crew consist of a survey pilot trained to fly in a certain way to collect the imagery and a camera operator

# Capturing Aerial Photography

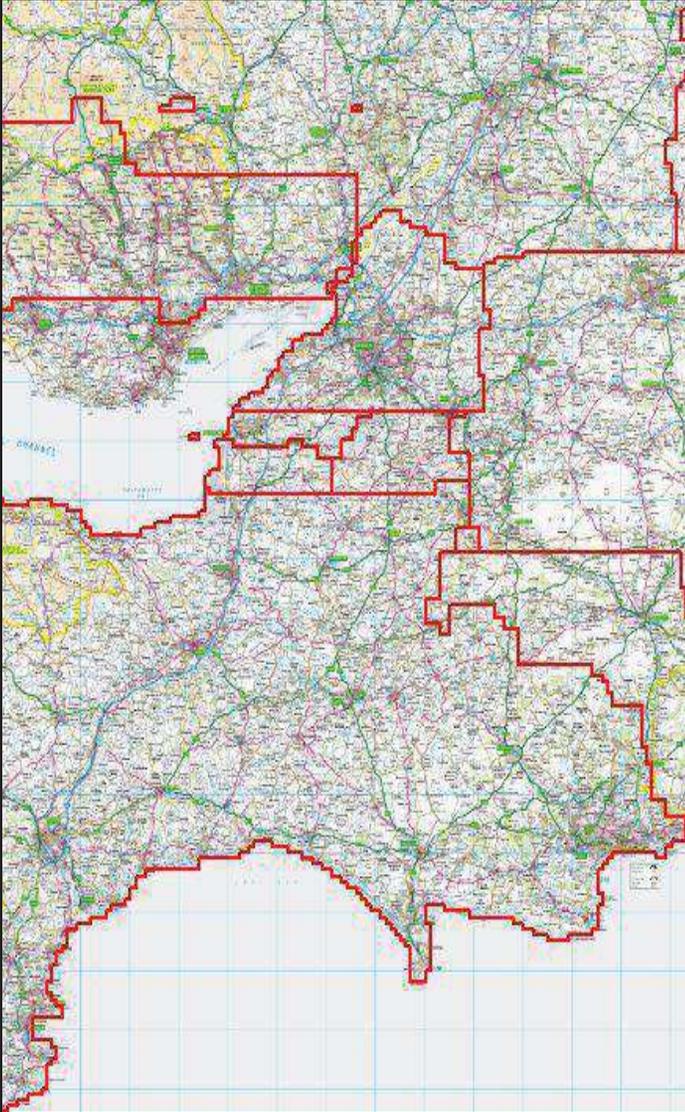


GPS IMU measures X, Y, Z position of aircraft and pitch, yaw, roll for each frame captured (**Aircraft principal axes**)



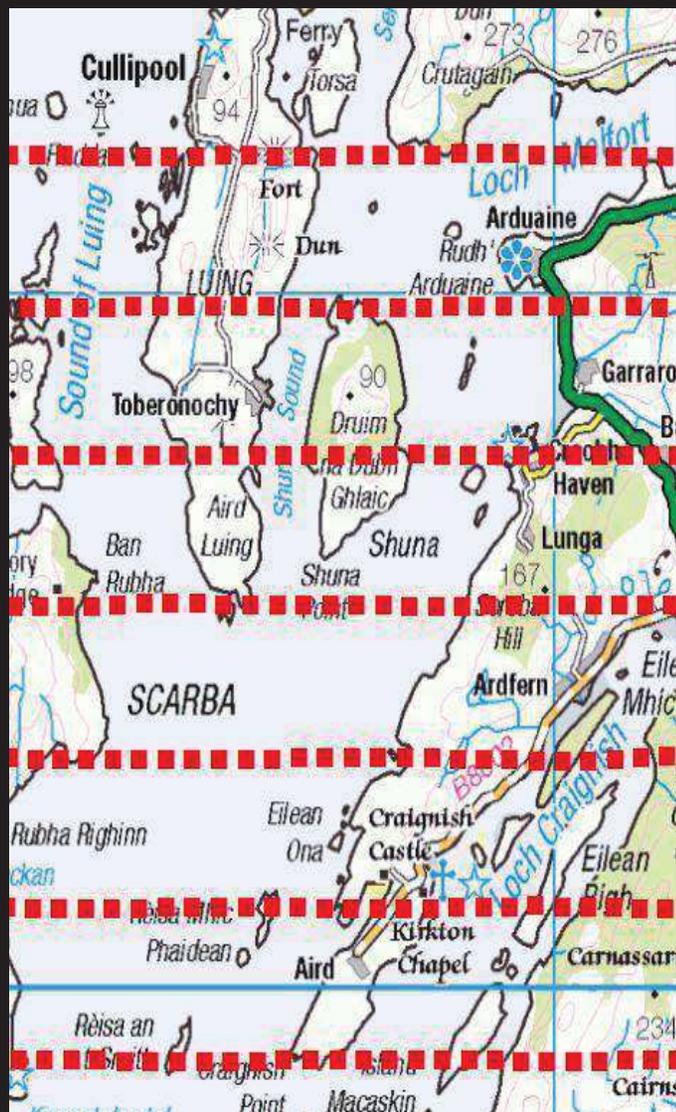
- **Getmapping** make use of the latest in digital camera technology  
<http://www.getmapping.com/support/aerial-photography>
- Capture height 1,000-7,000m
- 260 mega pixel camera
- Captures imagery through hole in the base of fuselage
- Sits with a gyroscopic mount which compensates for the motion of the plane

# Capturing Aerial Photography



- The flying season in the UK is between April and October
- Areas are captured on 3-5 year cycle.
- Targets areas are determined by ages of existing imagery and customer requirements
- Flying target blocks are established and non standard flight plans (NSFs) are submitted for approval to ATC
- Typically 50,000sqkm of new capture per year

# Capturing Aerial Photography



- Imagery is captured by specialist survey pilots
- Capture in flight-lines running W-E or N-S depending the shape of the block
- Each “frame” overlaps the previous one by 60% or 80% and flight lines overlap by 30% or 40%.
- Stereo capture allows for ortho-rectification and AT
- All imagery is captured to specification laid out by RICS

# Issues with Capturing Data

Weather



# Issues with Capturing Data

Weather



# Issues with Capturing Data



Air Traffic Restrictions

Military Exercises



Volcanic Ash Cloud

# Post Processing



- After capture data processing takes approximately some months.
- Processing has several stages
  1. Colour Balancing
  2. Aerial Triangulation
  3. Orthorectification
  4. Mosaicing
  5. Edge-matching

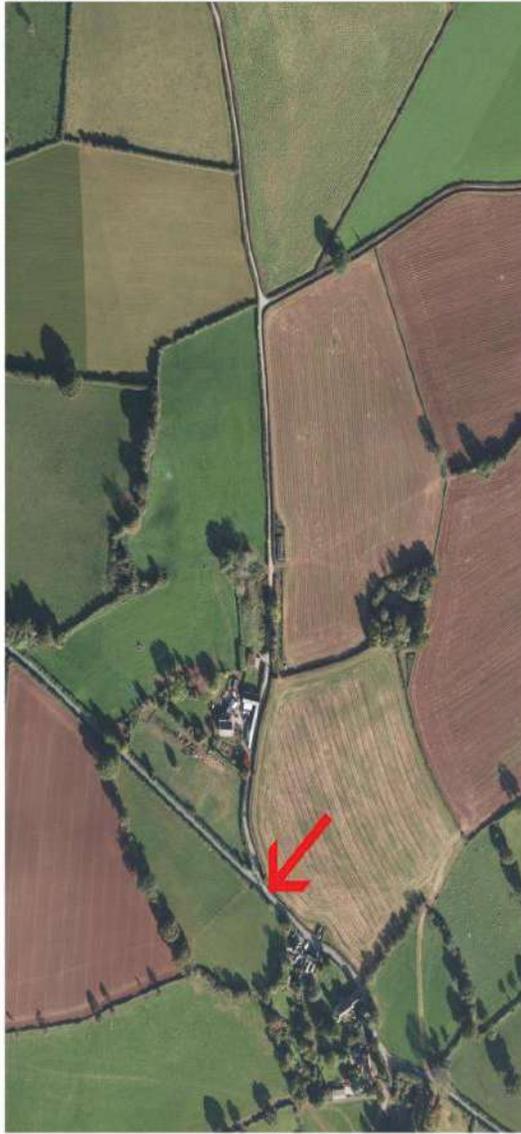
# Post Processing



- All data is checked to ensure no issues like cloud, haze.
- Colour balance done first to ensure imagery is homogenous and adheres to **RICS specification**
- Ground control points are then collected
- **Aerial Triangulation** uses **IMU data** in conjunction with ground control to ensure each frame is positioned correctly.

# Post Processing

Overview



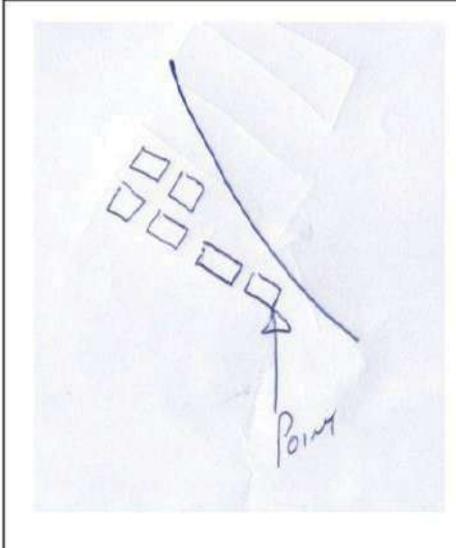
Detailed View



**GEOsense**  
www.geosense.co.uk

Project: Monmouth  
Point: SO4110\_10  
Frame: 9222 396  
Date: 28/01/2010

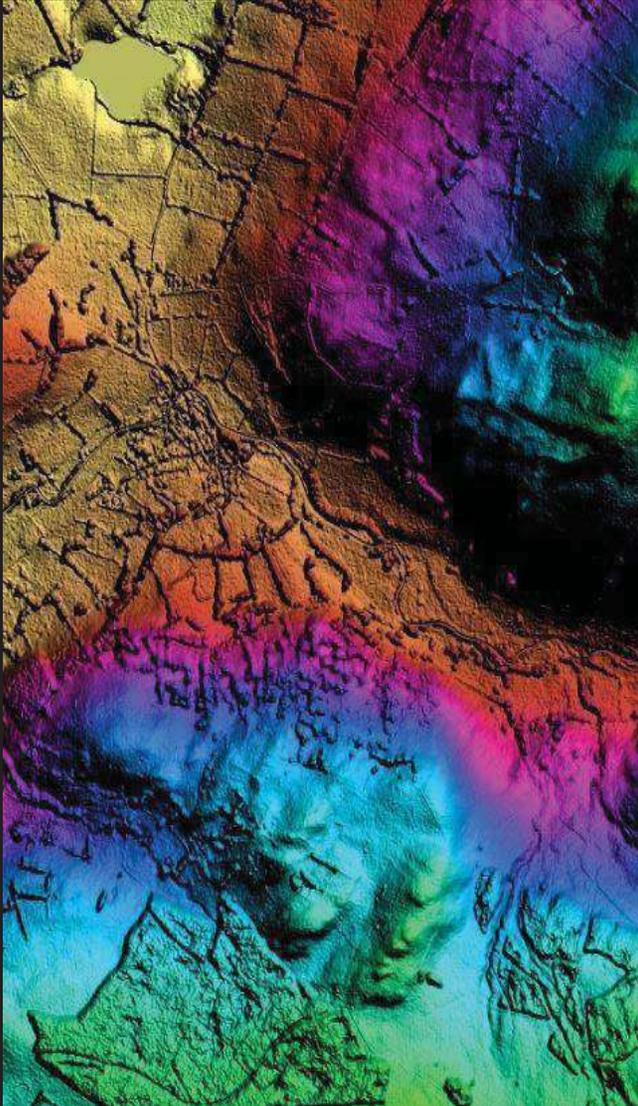
Sketch



Photo



# Post Processing



- Every frame needs to go through the process of Ortho-rectification
- Ortho-rectifies involves draping the imagery over the terrain then flattening to create a 2D image.
- First a **DSM** or **DTM** is created which is used to **orthorectify** the imagery using the **AT results**
- All imagery is created in British National Grid projection

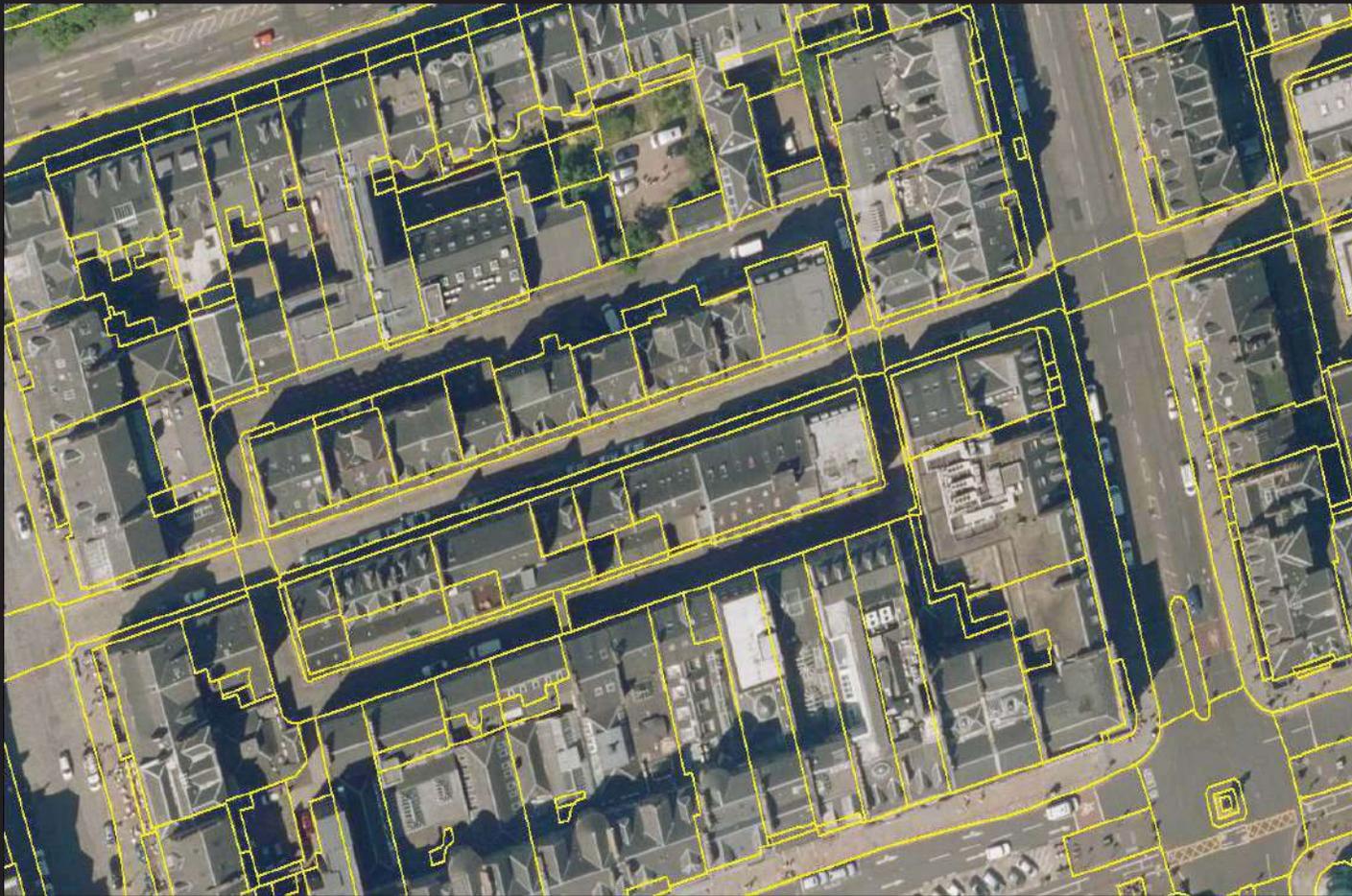
# Post Processing



- All Orthorectified frames in a flying block are combined in to a single image called a mosaic.
- The mosaic is then split in to tiles 1km x 1km
- Edge-matching done to ensure the mosaic is seamless.
- Time consuming process involving drawing around existing ground features.

# Uses of Aerial Photography

## Map Making



# Uses of Aerial Photography

## Infrastructure Planning



# Uses of Aerial Photography

## Event Planning



# Uses of Aerial Photography

Land Management



# Uses of Aerial Photography

## Archaeology



# Uses of Aerial Photography

## Change Detection



# Uses of Aerial Photography

## Media



## Uses of Aerial Photography



- Ecology
- Online Mapping Tools - Google/Bing
- 3D modelling
- Simulation
- Surveillance

# Colour Infrared Imagery



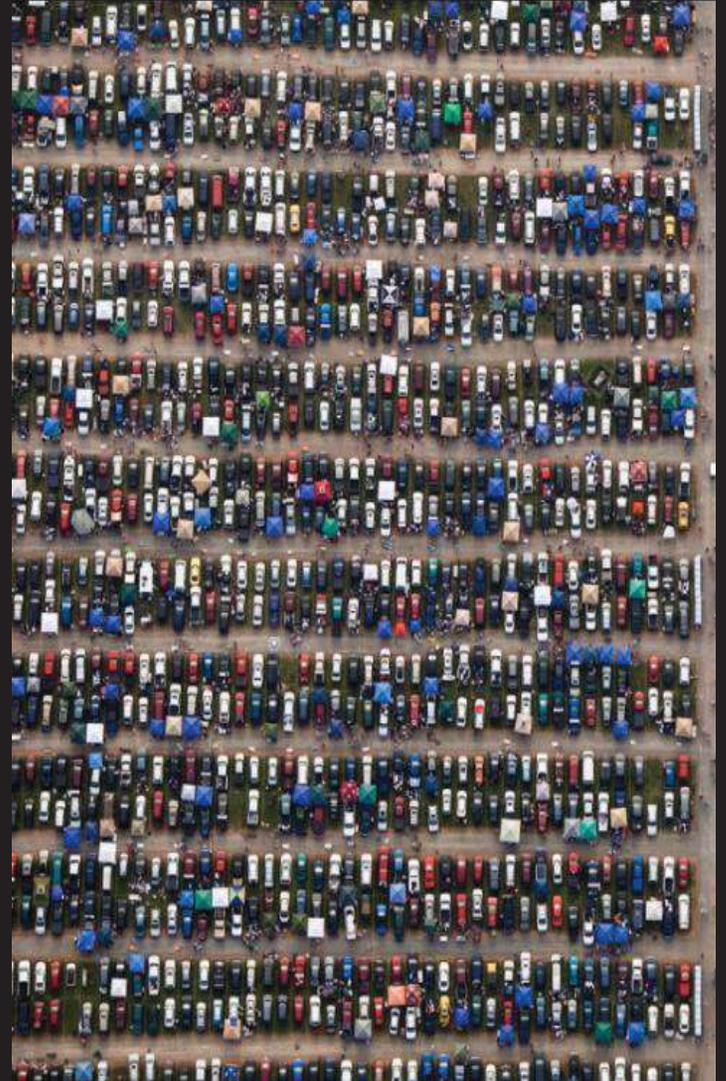
## Satellite Photography

- Use similar camera technology to
- Aerial Photography
- Altitude 700km
- Capture about 50-60cm resolution
- Capture through Atmosphere
- 



## Summery

- Definition of aerial photography
- History of Aerial Photography
- How aerial photography is collected
- How it is processed
- How it is used





*Thank you*

Any Questions ?



**END**

**of Lecture**

**Lecture 2-a**

# **Image Enhancement**

## **(Histogram Processing)**

*Lecture:*

*Faisal Ghazi Mohammed*

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*2020-2021*

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# *1-What Is Image Enhancement?*

Image enhancement is the process of making images more useful

The reasons for doing this include:

1. Highlighting interesting detail in images
2. Removing noise from images
3. Making images more visually appealing

- Image enhancement can be considered as one of the fundamental processes in image analysis.
- The goal of contrast enhancement is to improve the quality of an image to become more suitable for a particular application.
- Till today, numerous image enhancement methods have been proposed for various applications and efforts have been directed to further increase the quality of the enhancement results and minimize the computational complexity and memory usage.

- In this lecture, an image enhancement methods based on Histogram Equalization (HE) was studied.
- This lecture presents an exhaustive review of these studies and suggests a direction for future developments of image enhancement methods.
- Each method shows the owned advantages and drawbacks.
- In future, this work will give the direction to other researchers in order to propose new advanced enhancement techniques.

- In an image processing context, the histogram of an image normally refers to a histogram of the pixel intensity values. This histogram is a graph showing the number of pixels in an image at each different intensity value found in that image.
- For an 8-bit grayscale image there are 256 different possible intensities, and so the histogram will graphically display 256 numbers showing the distribution of pixels amongst those grayscale values.
- Histograms can also be taken of color images either individual histogram of red, green and blue channels can be taken, or a 3-D histogram can be produced, with the three axes representing the red, blue and green channels, and brightness at each point representing the pixel count.
- The exact output from the operation depends upon the implementation it may simply be a picture of the required histogram in a suitable image format, or it may be a data file of some sort representing the histogram statistics

## Point processing

1. **Histogram processing**
  1. Histogram equalization
  2. Histogram matching (specification)
2. **Simple gray level transformations**
  1. Image negatives
  2. Log transformations
  3. Power-law transformations
  4. Contrast stretching
  5. Gray-level slicing
  6. Bit-plane slicing
3. **Arithmetic/logic operations**
  1. Image averaging

## Mask processing (spatial filters)

1. **Smoothing filters (blur details)**
  1. Average, weighted average
  2. Order statistics (e.g. median)
2. **Sharpening filters (highlight details)**
  1. Unsharp masking
  2. High-boost filters
  3. Derivative filters
    1. The Laplacian
    2. The Gradient

## Frequency domain filters

1. **Smoothing filters (blur details)**
  1. Ideal lowpass filter
  2. Butterworth lowpass
  3. Gaussian lowpass
2. **Sharpening filters (highlight details)**
  1. Unsharp masking
  2. High-boost filters
  3. Derivative filters - The Laplacian
  4. Ideal highpass filter
  5. Butterworth highpass filter
  6. Gaussian highpass filter
3. **Homomorphic filtering**

## *2-Different kind of Image Enhancement*

### Spatial & Frequency Domains

There are two broad categories of image enhancement techniques:

1. **Spatial** domain techniques

- Direct manipulation of image pixels

2. **Frequency** domain techniques

- Manipulation of Fourier transform or wavelet transform of an image

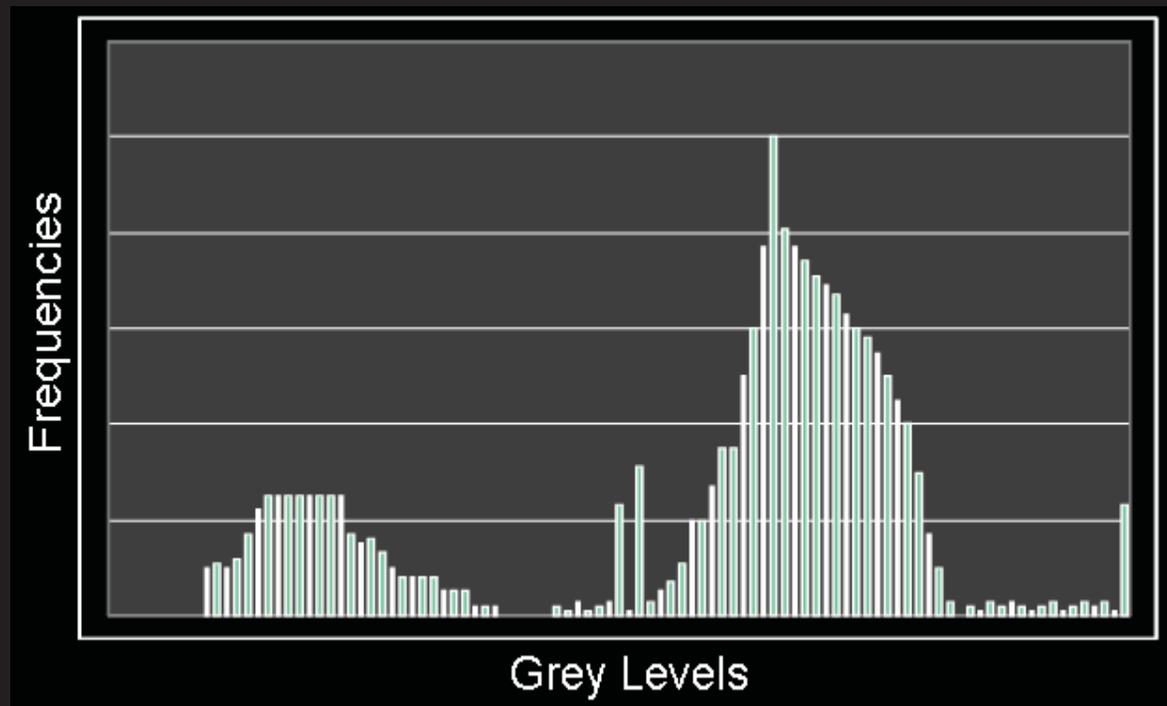
For the moment we will **concentrate** on techniques that operate in the **spatial** domain

# 3- Histogram Processing

## Image Histograms

The histogram of an image shows us the distribution of grey levels in the image

Massively useful in image processing, especially in segmentation



# 3- Histogram Processing

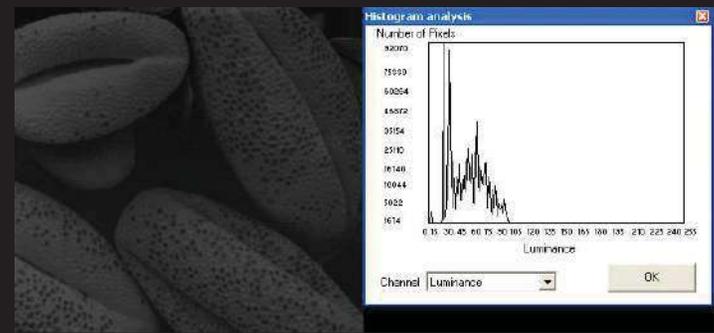
## Histogram Examples

A selection of images and their histograms

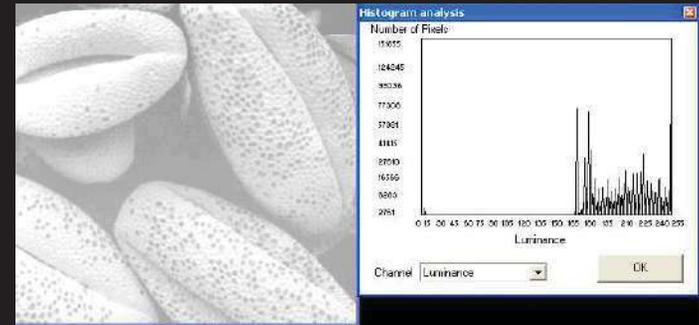
Notice the relationships between the images and their histograms

Note that the high contrast image has the most evenly spaced histogram

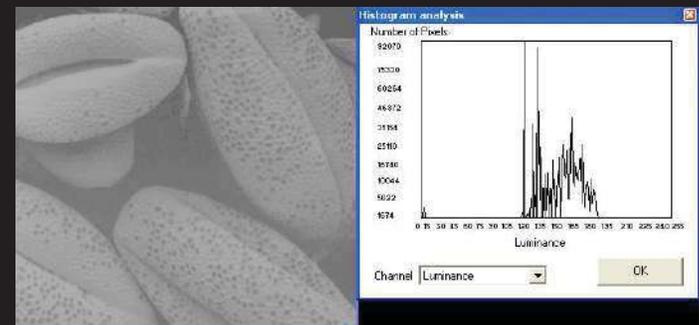
Dark image



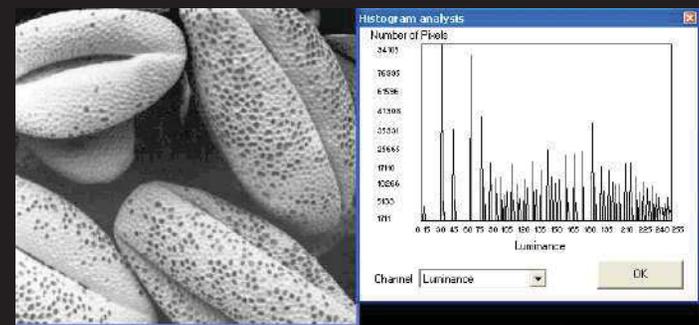
Bright image



Low contrast image

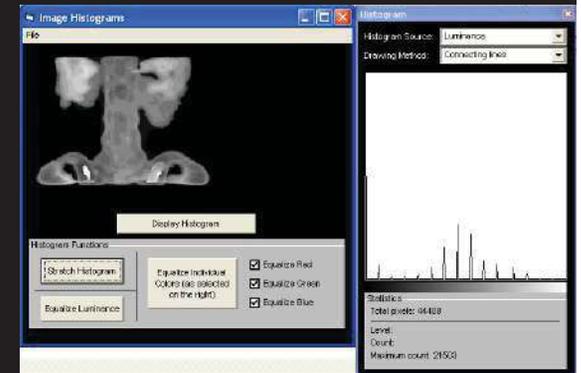


High-contrast image



# Example: Histogram\Contrast Stretching Algorithm

Stretch Histogram - if an image histogram doesn't reach from 0 to 255, make it.



## Algorithm Steps:

1. Find Min and Max graylevels for all pixels within image
2. Calculate Range, [Max-Min]
3. Force all pixels to a new "stretched" value using the following formula:

$$\text{NewGray} = 255 * (\text{OldGray} - \text{Min}) / (\text{Range})$$

# Example: Histogram Equalization Algorithm

Equalize Histogram - attempt to Re-distribute values across the brightness spectrum with roughly the same amount of pixels at each brightness level

## Algorithm Steps:

- 1- First, tally the amount of each color (i.e. build the histogram)

$$\text{Histogram}(\text{Pixel-Value}) = \text{Histogram}(\text{Pixel-Value}) + 1$$

- 2- Compute scaling factor

$$\text{ScaleFactor} = 255 / (\text{iWidth} * \text{iHeight})$$

- 3- Handle pixel values as following (CDF) :

$$\text{Histogramm}(0) = \text{Histogramm}(0) * \text{scaleFactor}$$

x : 1 , 255 Do :

$$\text{NewHistogram}(x) = \text{Histogram}(x - 1) + (\text{scaleFactor} * \text{Histogramm}(x))$$

- 4- Integrate all the look-up values and Draw NewHistogram :

x : 1 , 255 Do :

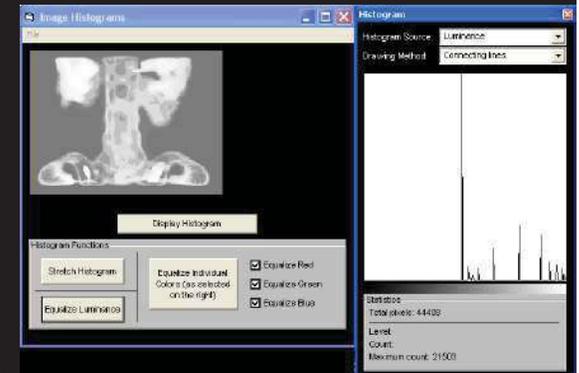
$$\text{NewHistogram}(x) = \text{integer}(\text{NewHistogram}(x))$$

$$\text{If Histogram}(x) > 255 \text{ Then Histogram}(x) = 255$$

- 5- Apply the equalized values and Draw picture

For all picture pixels in the range x,y Do:

$$\text{NewPixel}(x,y) = \text{NewHistogram}(\text{OldPixel}(x,y))$$



# Example: Histogram Equalization Algorithm

Intensity	0	1	2	3	4	5	6	7
Number of pixels	10	20	12	8	0	0	0	0

$$p(0) = 10/50 = 0.2$$

$$p(1) = 20/50 = 0.4$$

$$p(2) = 12/50 = 0.24$$

$$p(3) = 8/50 = 0.16$$

$$p(r) = 0/50 = 0, \quad r = 4, 5, 6, 7$$

$$T(r) = \text{round} \left( 7 \sum_{i=0}^r p(i) \right)$$

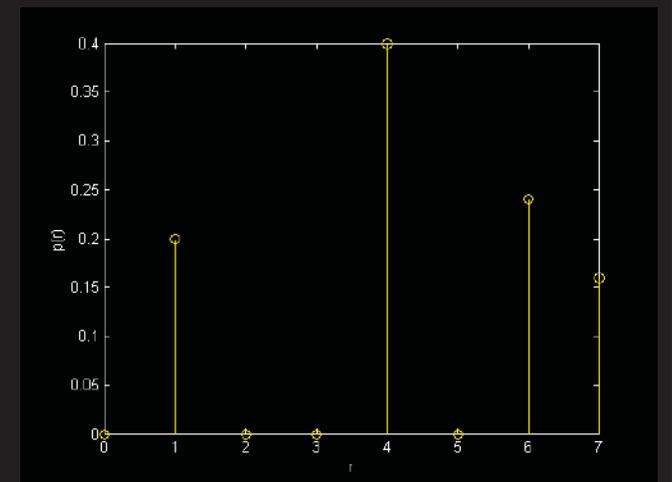
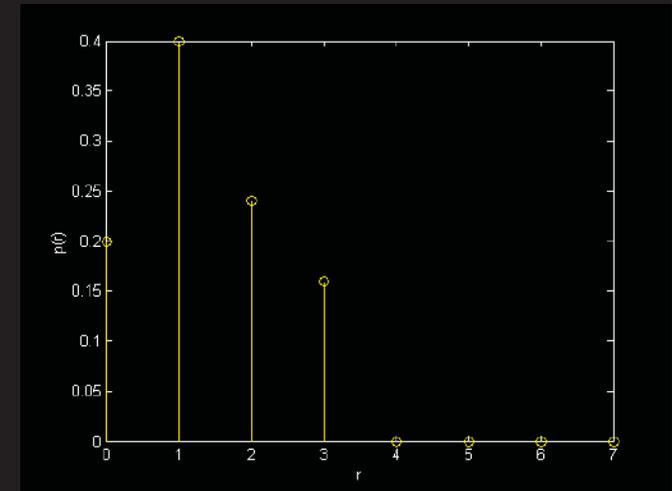
$$T(0) = \text{round} (7 * p(0)) = \text{round} (7 * 0.2) = 1$$

$$T(1) = \text{round} (7 * (p(0) + p(1))) = \text{round} (7 * 0.6) = 4$$

$$T(3) = \text{round} (7 * (p(0) + p(1) + p(2) + p(3))) = 7$$

$$T(r) = 7, \quad r = 4, 5, 6, 7$$

Intensity	0	1	2	3	4	5
Number of pixels	1	4	?	?	?	?



# Example: Histogram Equalization Algorithm

$$T(r) = \text{round} \left( 7 \sum_{i=0}^r p(i) \right)$$

- Consider an 8-level 64x64 image with grayvalues (0, 1, ..., 7).
- The normalized grayvalues are (0, 1/7, 2/7, ..., 1).
- The normalized histogram is given in the table:

$i$	$r_i$	$n_i$	$p(r_i) = n_i/n$
0	0	790	0.19
1	1/7	1023	0.25
2	2/7	850	0.21
3	3/7	656	0.16
4	4/7	329	0.08
5	5/7	245	0.06
6	6/7	122	0.03
7	1	81	0.02

# Example: Histogram Equalization Algorithm

Applying the previous transformation, we have (after rounding off to nearest graylevel):

Notice that there are only five distinct graylevels -- (1/7, 3/7, 5/7, 6/7, 1) in the output image. We will relabel them as ( $s_0, s_1, \dots, s_4$ ).

$$s_0 = T(r_0) = \sum_{i=0}^0 p_m(r_i) = p_m(r_0) = 0.19 \rightarrow 1/7$$

$$s_1 = T(r_1) = \sum_{i=0}^1 p_m(r_i) = p_m(r_0) + p_m(r_1) = 0.44 \rightarrow 3/7$$

$$s_2 = T(r_2) = \sum_{i=0}^2 p_m(r_i) = p_m(r_0) + p_m(r_1) + p_m(r_2) = 0.65 \rightarrow 5/7$$

$$s_3 = T(r_3) = \sum_{i=0}^3 p_m(r_i) = p_m(r_0) + p_m(r_1) + \dots + p_m(r_3) = 0.81 \rightarrow 6/7$$

$$s_4 = T(r_4) = \sum_{i=0}^4 p_m(r_i) = p_m(r_0) + p_m(r_1) + \dots + p_m(r_4) = 0.89 \rightarrow 6/7$$

$$s_5 = T(r_5) = \sum_{i=0}^5 p_m(r_i) = p_m(r_0) + p_m(r_1) + \dots + p_m(r_5) = 0.95 \rightarrow 1$$

$$s_6 = T(r_6) = \sum_{i=0}^6 p_m(r_i) = p_m(r_0) + p_m(r_1) + \dots + p_m(r_6) = 0.98 \rightarrow 1$$

$$s_7 = T(r_7) = \sum_{i=0}^7 p_m(r_i) = p_m(r_0) + p_m(r_1) + \dots + p_m(r_7) = 1.00 \rightarrow 1$$

## Example: Histogram Equalization Algorithm

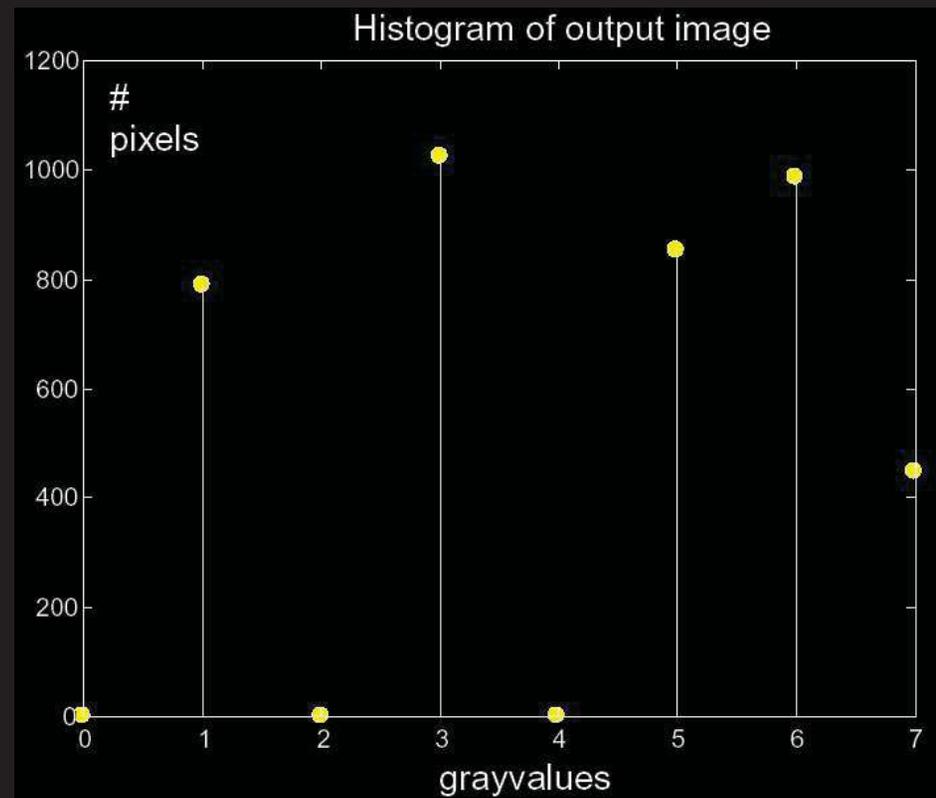
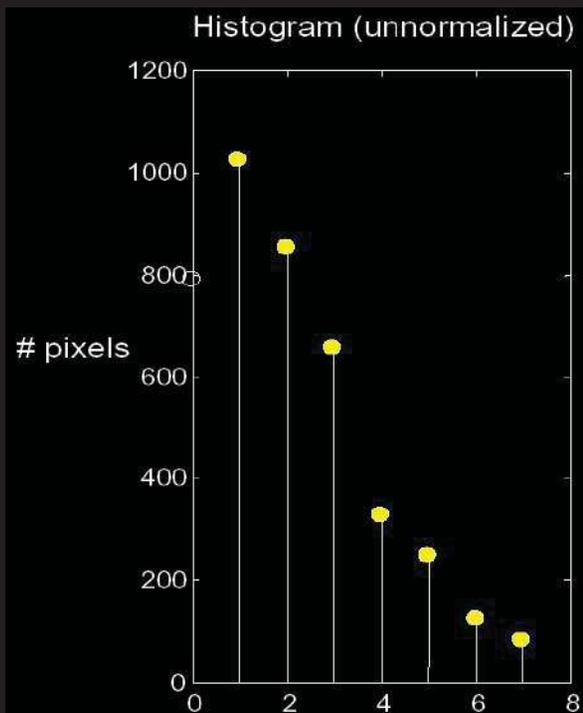
With this transformation, the output image will have histogram

$$T(r) = \text{round} \left( 7 \sum_{i=0}^r p(i) \right)$$

$i$	$s_i$	$n_i$	$p(s_i) = n_i/n$
0	1/7	790	0.19
1	3/7	1023	0.25
2	5/7	850	0.21
3+4	6/7	985	0.24
5+6+7	1	448	0.11

# Example: Histogram Equalization Algorithm

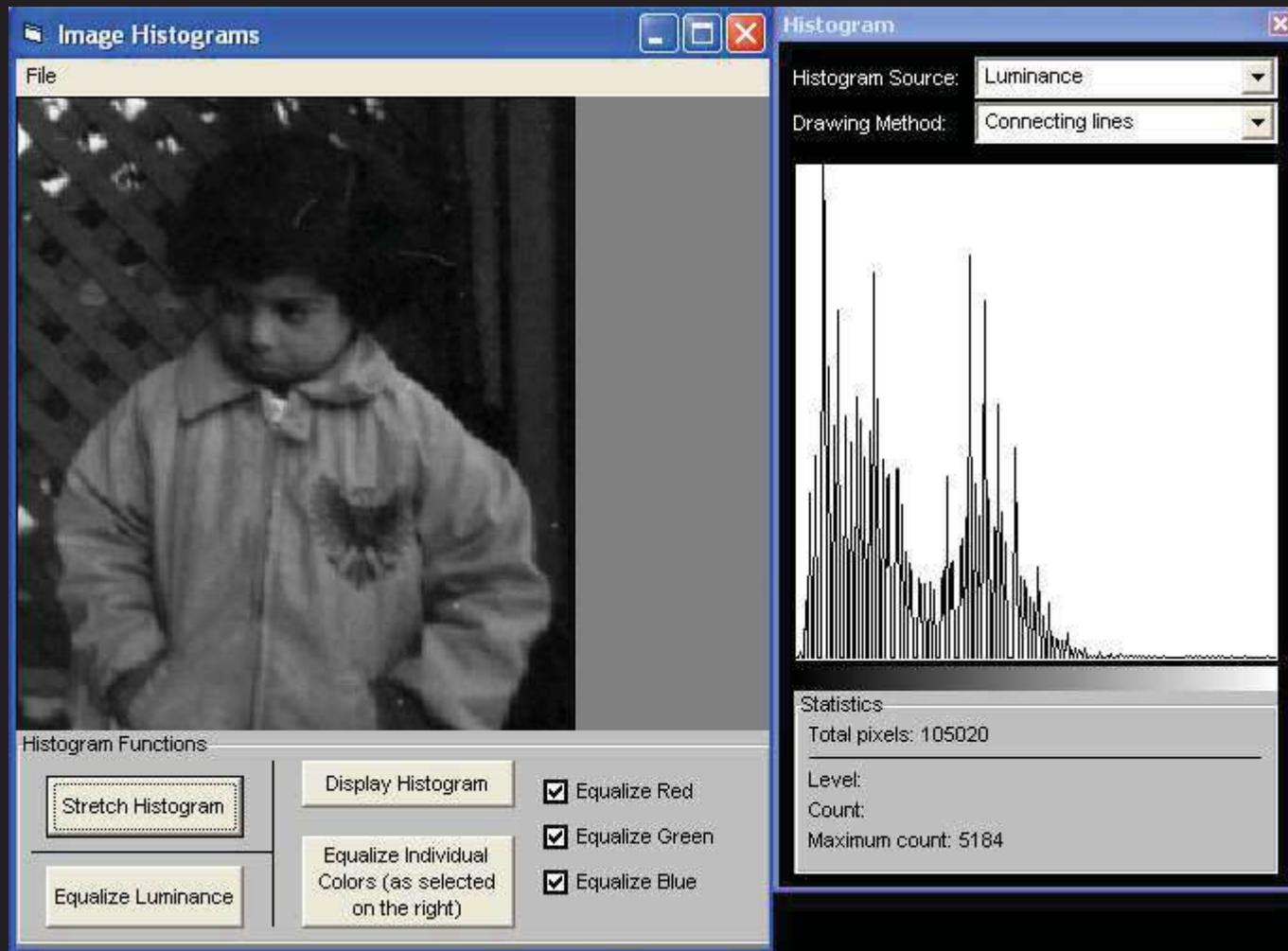
Note that the histogram of output image is only approximately, and not exactly, uniform. This should not be surprising, since there is no result that claims uniformity in the **discrete** case.



The screenshot displays two windows from an image processing application. The 'Image Histograms' window on the left shows a grayscale image of a young child in a light-colored jacket. Below the image is a 'Histogram Functions' panel with several options: 'Stretch Histogram', 'Equalize Luminance' (highlighted with a dashed border), 'Display Histogram', 'Equalize Individual Colors (as selected on the right)', and three checked checkboxes for 'Equalize Red', 'Equalize Green', and 'Equalize Blue'. The 'Histogram' window on the right shows a histogram plot with 'Connecting lines' as the drawing method. Below the plot is a 'Statistics' section with the following data: Total pixels: 105020, Level: (blank), Count: (blank), and Maximum count: 5184.

Before

# Example: Contrast Stretching

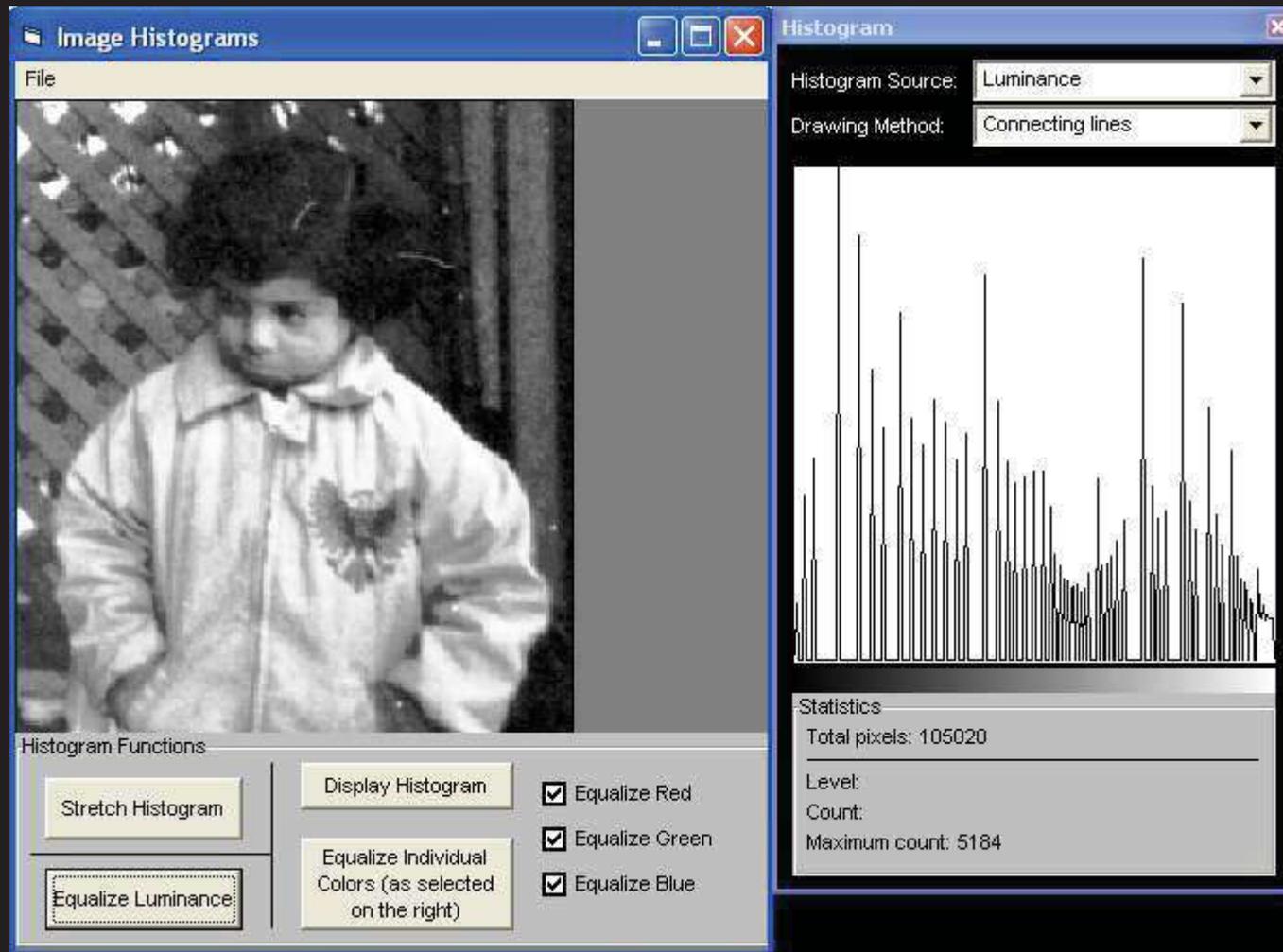


The screenshot displays two windows from an image processing application. The 'Image Histograms' window on the left shows a grayscale image of a child in a jacket. Below the image is a 'Histogram Functions' panel with several buttons: 'Stretch Histogram' (highlighted with a dashed border), 'Equalize Luminance', 'Display Histogram', and 'Equalize Individual Colors (as selected on the right)'. The 'Equalize Individual Colors' section has three checked checkboxes: 'Equalize Red', 'Equalize Green', and 'Equalize Blue'. The 'Histogram' window on the right shows a histogram plot with 'Luminance' selected as the source and 'Connecting lines' as the drawing method. Below the plot is a 'Statistics' section with the following data:

Statistics	
Total pixels:	105020
Level:	
Count:	
Maximum count:	5184

After

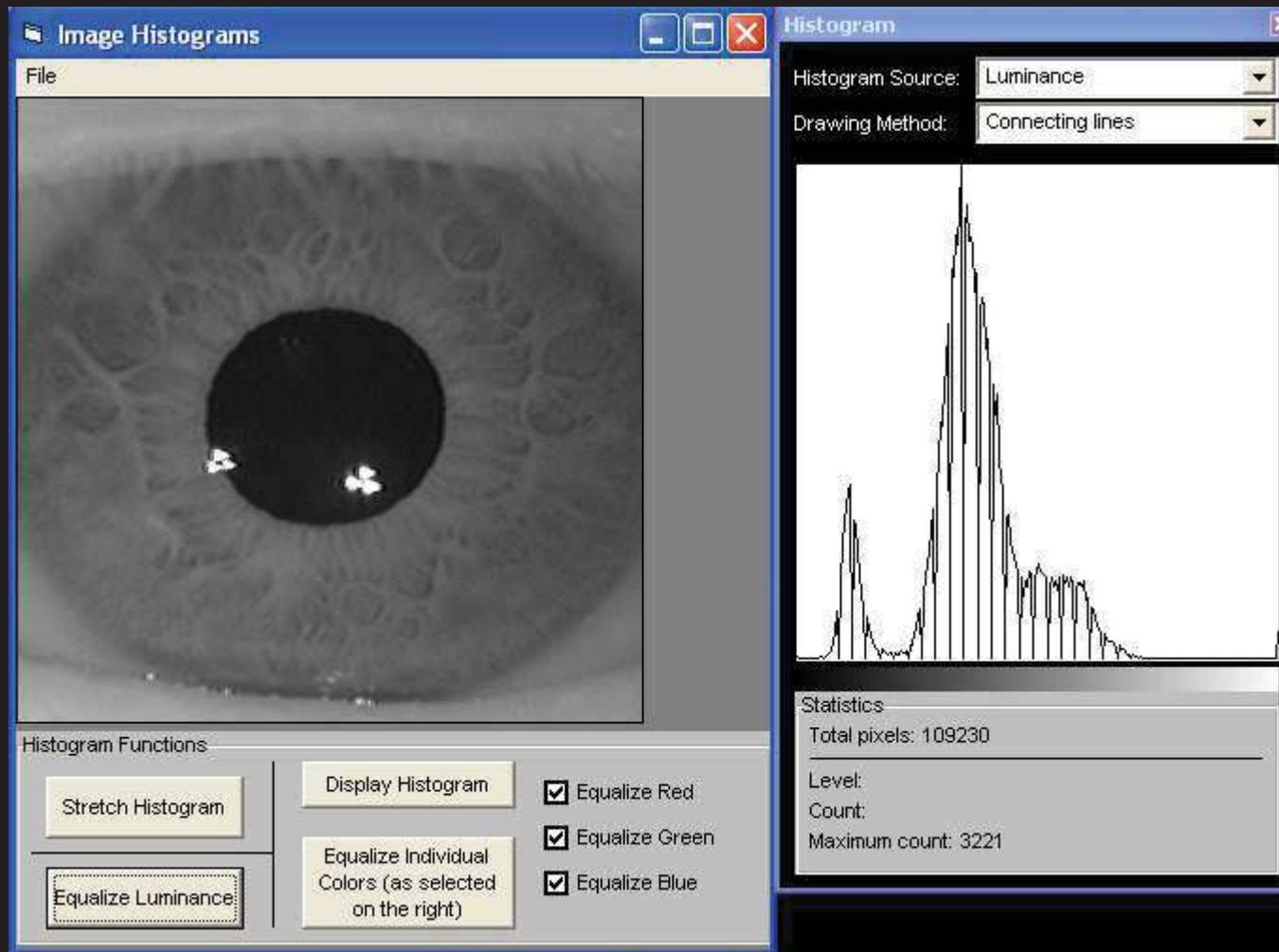
# Example: Histogram Equalization



After

# Example: Histogram Equalization

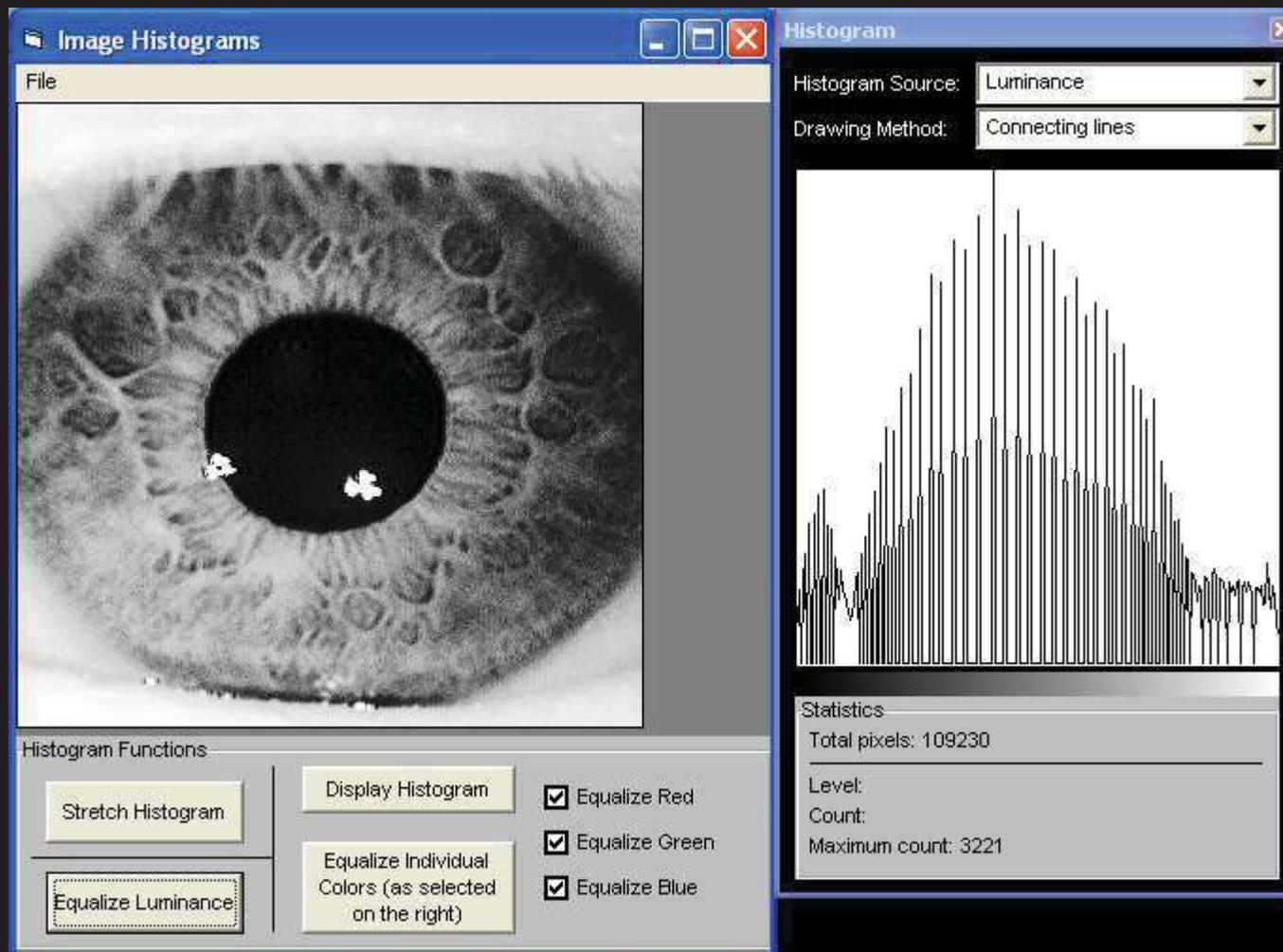
## Application: Iris Recognition



Before

# Example: Histogram Equalization

## Application: Iris Recognition



After

# Histogram Equalization

- Histogram equalization may **not** always produce desirable results,
- particularly if the given histogram is very narrow. It can produce false edges and regions.
- It can also increase image “graininess” and “patchiness.”

**Quiz** - Apply global histogram equalization to the image  $f(x,y)$ . Let  $T_{out}(r)$  denote the resulting (equalized) histogram of pixel values  $s$  taking values in  $[0, 7]$

0	1	3	4
1	2	2	3
1	3	4	4
3	2	5	2

$f(x, y)$

$$T(r) = \text{round} \left( 7 \sum_{i=0}^r p(i) \right)$$

$i$	$r_i$ (normalized)	$P(r_i)$ $= r_i/n$	$T_{out}(r_i) =$ $7 \times P(r_i)$
0			
1			
2			
3			
4			
5			
6			
7			

**Thank you**

**Any Questions ?**

END

## Quiz - Solution

Apply global histogram equalization to the image  $f(x, y)$ . Let  $p_{out}(s)$  denote the resulting (equalized) histogram of pixel values  $s$  taking values in  $[0, 7]$

0	1	3	4
1	2	2	3
1	3	4	4
3	2	5	2

$f(x, y)$

$i$	$\hat{n}_i$ (normalized)	$p(s_i)$ $= \hat{n}_i/n$	$p_{out}(s_i) =$ $7 \times p(s_i)$
0	1/16	1/16	0
1	3/16	4/16	2
2	4/16	8/16	4
3	4/16	12/16	5
4	3/16	15/16	7
5	1/16	16/16	7
6	0/16	16/16	7
7	0/16	16/16	7

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

## Lecture 2-b

# *Image Enhancement* Noise Distribution

*Lecturer:*

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*2021-2023*

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# Contents

In this lecture we will look at image restoration techniques used for noise removal

1. What is image restoration?
2. Noise and images
3. Noise (Image Degradation) Models
4. Noise removal using spatial domain filtering
5. Periodic noise

# *Introduction* Goals & Examples

## Noise Removal:

- There are many sources of noise in images, and special techniques (and sequences of algorithms) can be developed to remove specific kinds of noise.



Original



SAR speckle noise

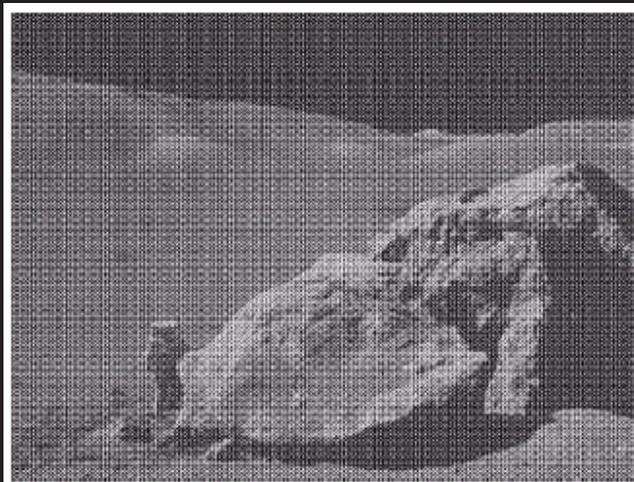


Enhancement

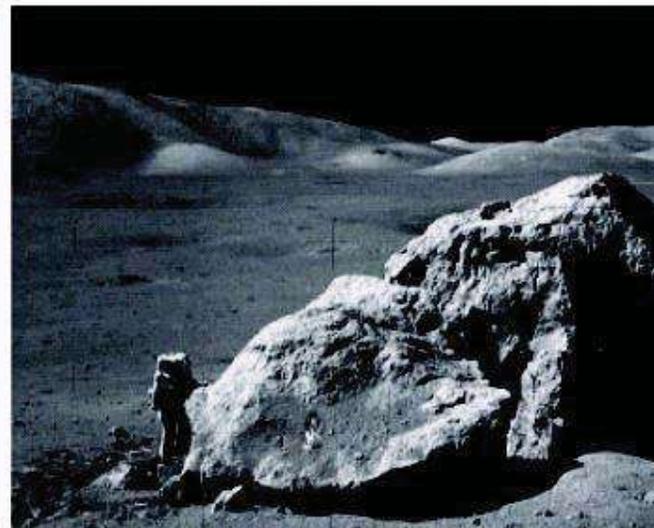
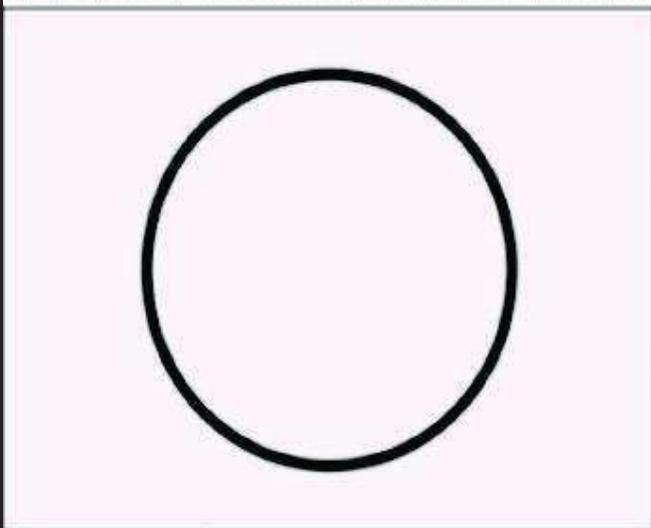
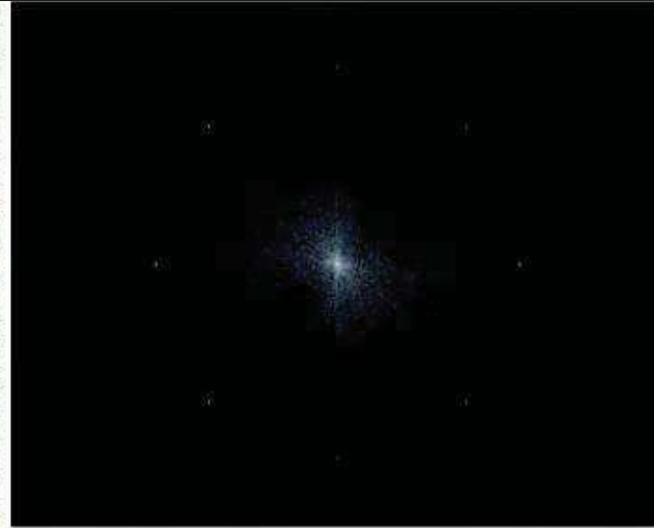
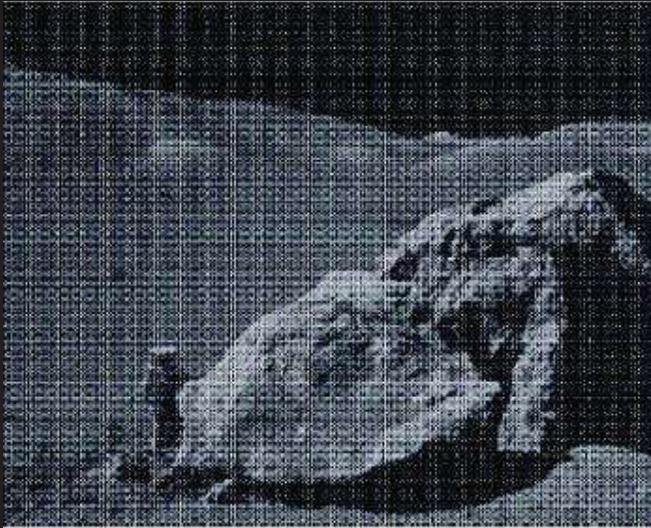
# 1- What is Image Restoration?

Image restoration attempts to **restore** images that have been degraded

- **Identify** the degradation process and attempt to reverse it.
- **Similar** to image enhancement, but more objective

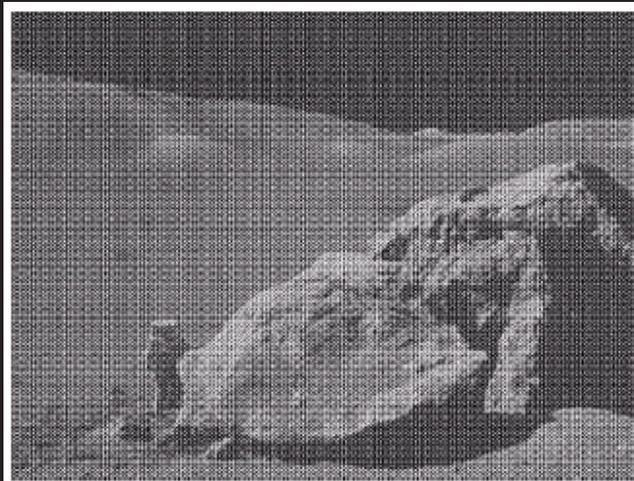


# 1- What is Image Restoration?



# 1- What is Image Restoration?

- **Image restoration**: recover an image that has been degraded by using a **prior** knowledge of the degradation phenomenon.
- **Model** the degradation and applying the **inverse** process in order to recover the original image.



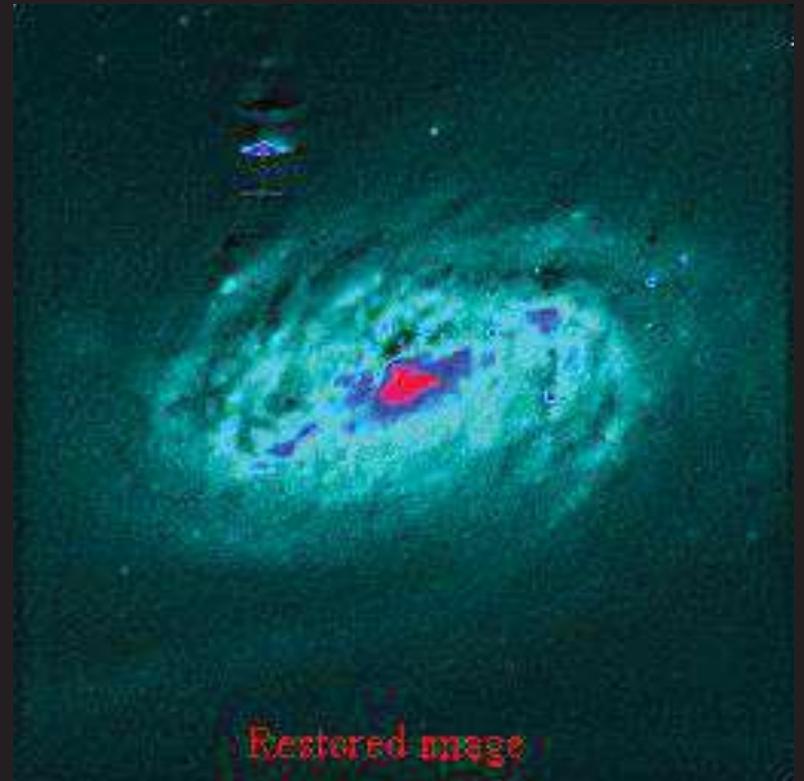
# Application (I): Astronomical Imaging

## The Story of Hubble Space Telescope (HST)

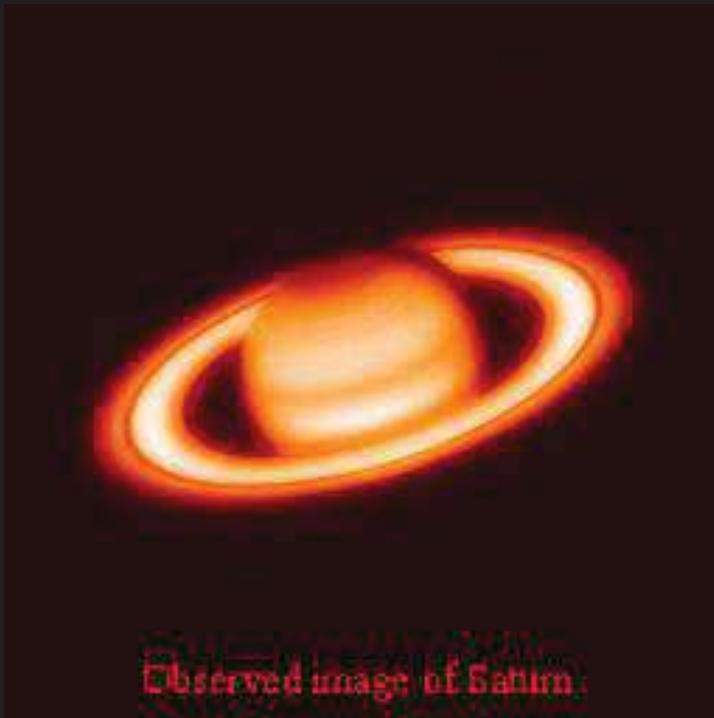
- HST Cost at Launch (1990): \$1.5 billion
- Main mirror imperfections due to human errors
- Got repaired in 1993



# Restoration of HST Images



# Another Example



# The Real (Optical) Solution



Before the repair



After the repair

# Application (II): Law Enforcement Restoration Example



12-5996 New York, NY, Statue of Liberty with Stinson  
Aerials Only Gallery 508-295-5551(C) (E)

*Blurring due to uniform motion*



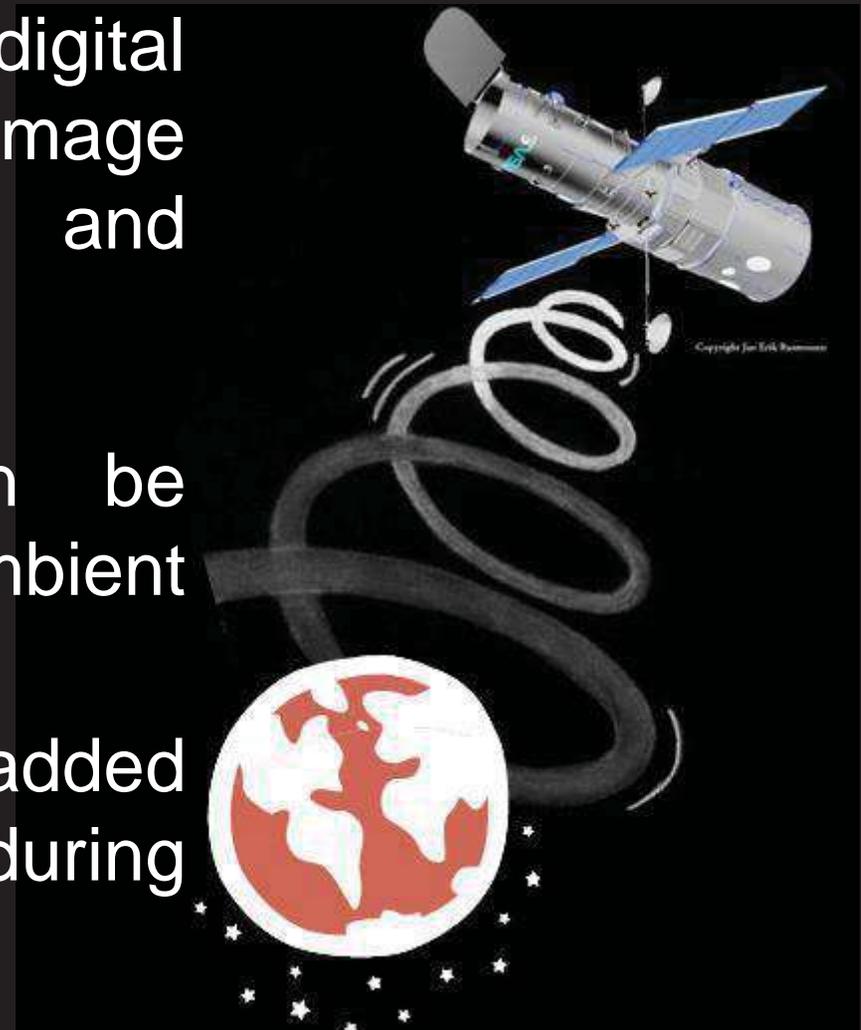
*Blurring due to uniform motion*



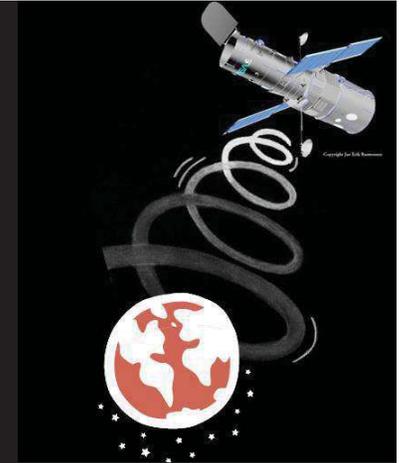
## 2- Noise and Images

The sources of noise in digital images arise during image **acquisition** (digitization) and **transmission**

- **Imaging sensors** can be affected by ambient conditions
- **Interference** can be added to an image during transmission



## 2- Noise and Images



### Image acquisition

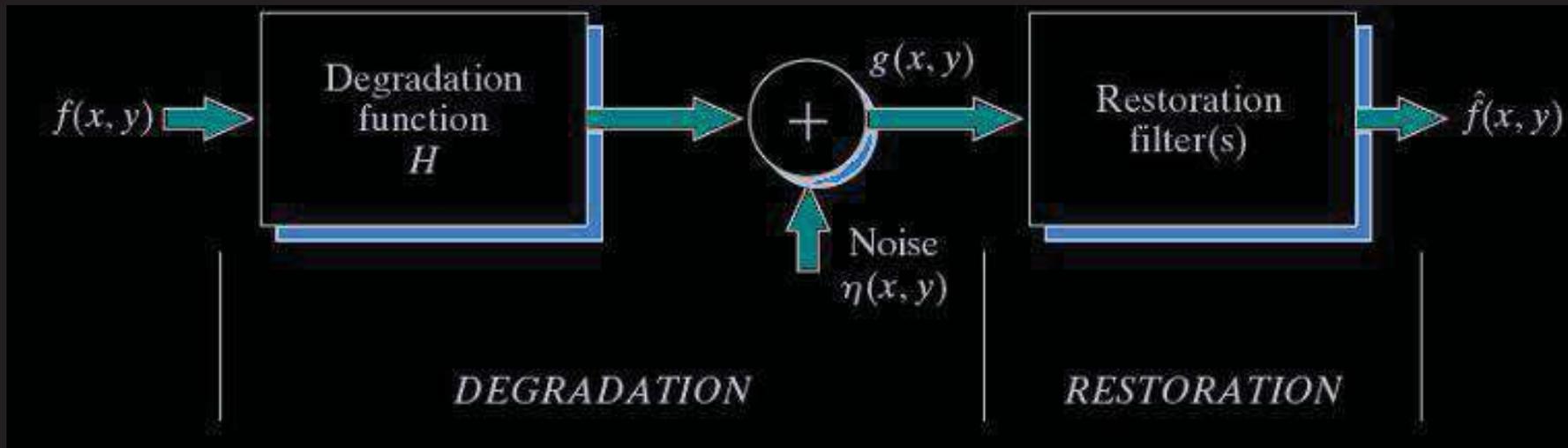
e.g., light levels, sensor temperature, etc.

### Transmission

e.g., lightning or other atmospheric disturbance in wireless network

# 3- Noise (Image Degradation) Models

## A Model of Image Degradation/Restoration Process



Degradation model:

$$g(x, y) = f(x, y) * h(x, y) + \eta(x, y)$$

where  $h(x, y)$  is a system that causes image distortion and  $\eta(x, y)$  is noise.

# 3- Noise (Image Degradation) Models

## A Model of Image Degradation/Restoration Process

### 1- Spatial variant degradation model

$$g_{(x,y)} = \sum \sum h_{(x,y,m,n)} f_{(x,y)} + \eta_{(x,y)}$$

### 2- Spatial-invariant degradation model

$$g_{(x,y)} = \sum \sum h_{(x-m,y-n)} f_{(m,n)} + \eta_{(x,y)}$$

#### 2-1 Frequency domain representation

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

### 3- Noise (Image Degradation) Models

#### A Model of Image Degradation/Restoration Process

As we see before, we considered a noisy image to be modelled as follows:

$$g_{(x,y)} = f_{(x,y)} + \eta_{(x,y)}$$

where  $f(x, y)$  is the original image pixel,  $\eta(x, y)$  is the noise term and  $g(x, y)$  is the resulting noisy pixel

If we can **estimate** the model the **noise** in an image is based on this will **help us** to figure out **how to restore** the image

### 3- Noise (Image Degradation) Models

Noise cannot be predicted but can be approximately described in statistical way using the probability density function (PDF)

Gaussian noise:

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2 / 2\sigma^2}$$

Rayleigh noise:

$$p(z) = \begin{cases} \frac{2}{b} (z - a) e^{-(z-a)^2 / b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

Erlang (Gamma) noise:

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

### 3- Noise (Image Degradation) Models

Exponential noise:  $p(z) = ae^{-az}$

Uniform noise:

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

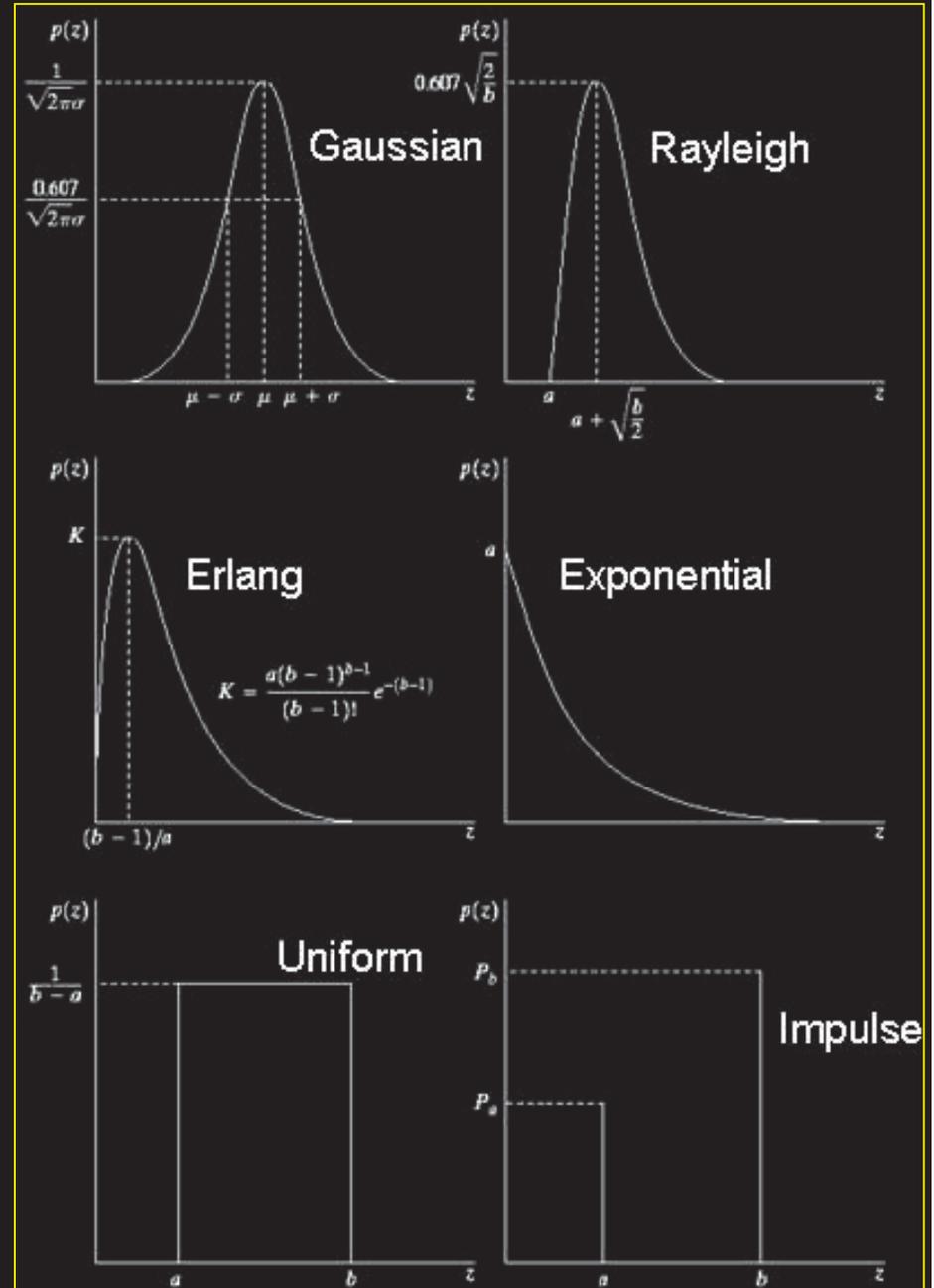
Impulse (salt & pepper) noise:

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

# 3- Noise (Image Degradation) Models

There are many different models for the image noise term  $\eta(x, y)$ :

- Gaussian
  - Most common model
- Rayleigh
- Erlang
- Exponential
- Uniform
- Impulse
  - *Salt and pepper* noise

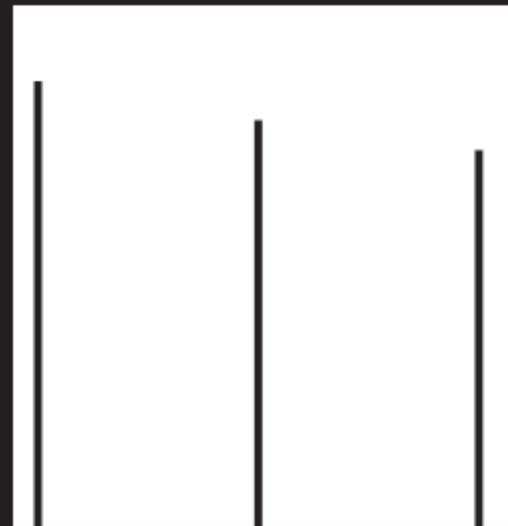
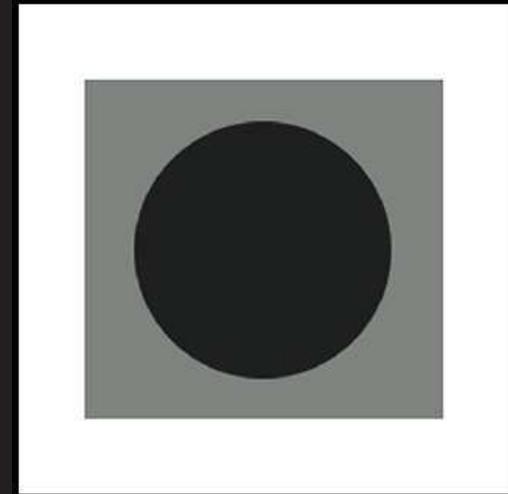


# 3- Noise (Image Degradation) Models

## Noise Example

The test pattern to the right is ideal for demonstrating the addition of noise

The following slides will show the result of adding noise based on various models to this image

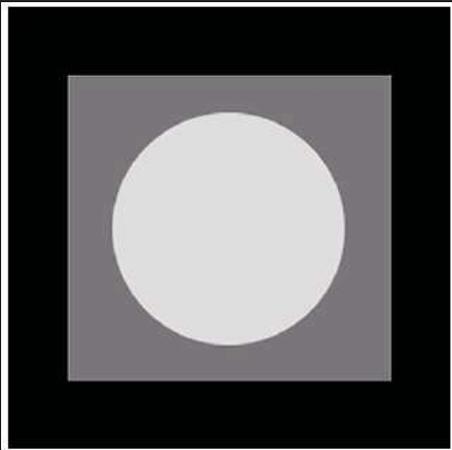


# 3- Noise (Image Degradation) Models

## Image Degradation with Additive Noise

$$g(x, y) = f(x, y) + \eta(x, y)$$

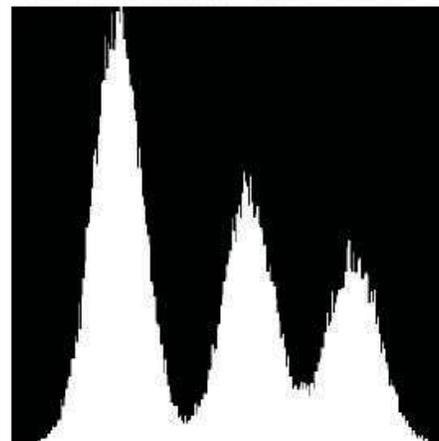
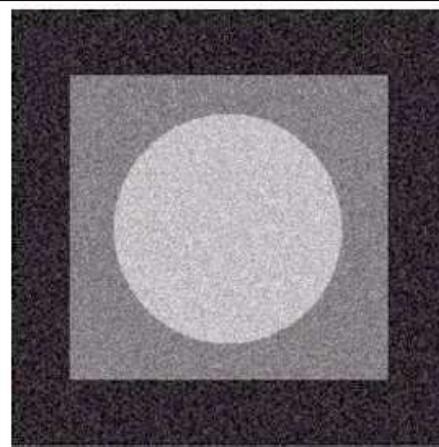
Degraded images



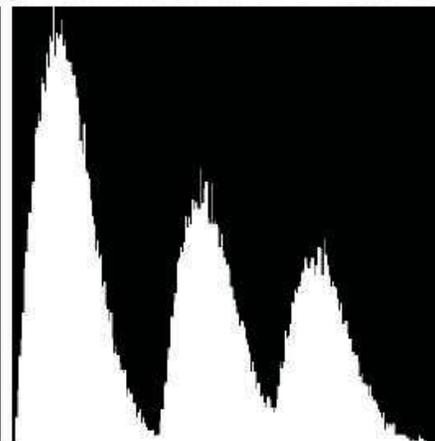
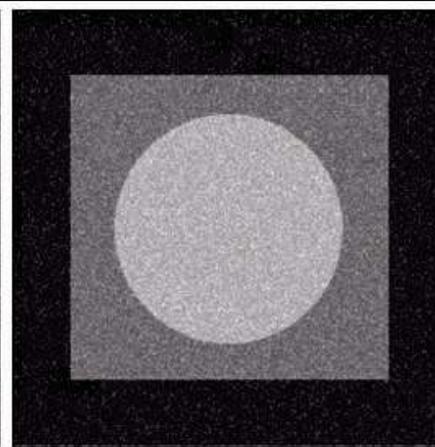
Original image



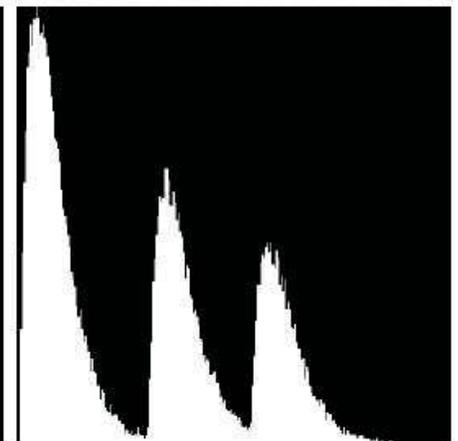
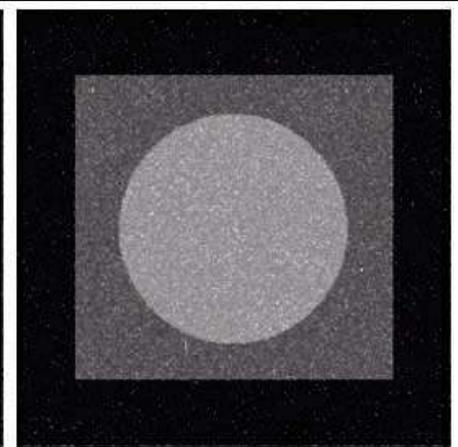
Histogram



Gaussian



Rayleigh



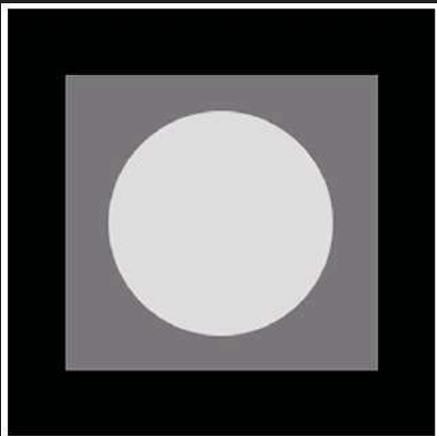
Gamma

# 3- Noise (Image Degradation) Models

## Image Degradation with Additive Noise

$$g(x, y) = f(x, y) + \eta(x, y)$$

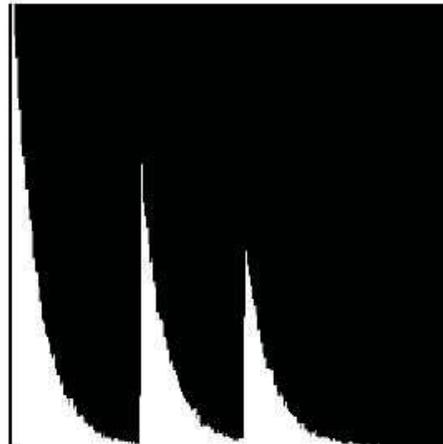
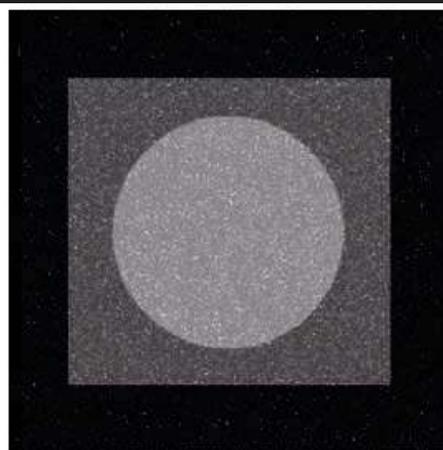
Degraded images



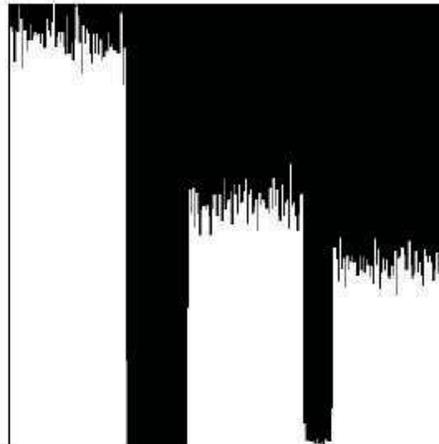
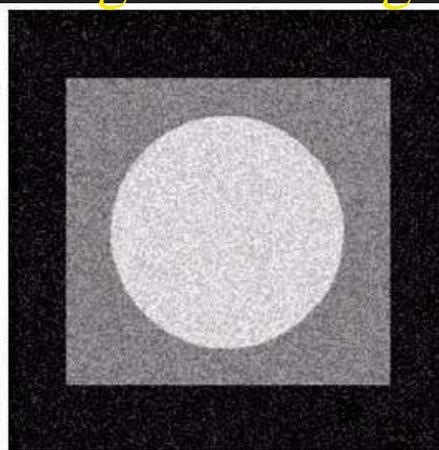
Original image



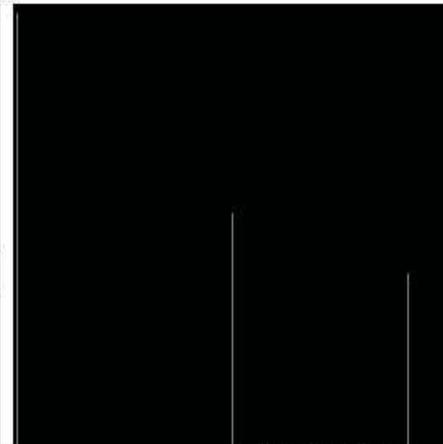
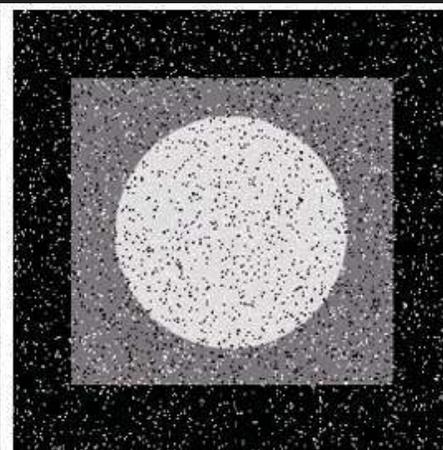
Histogram



Exponential

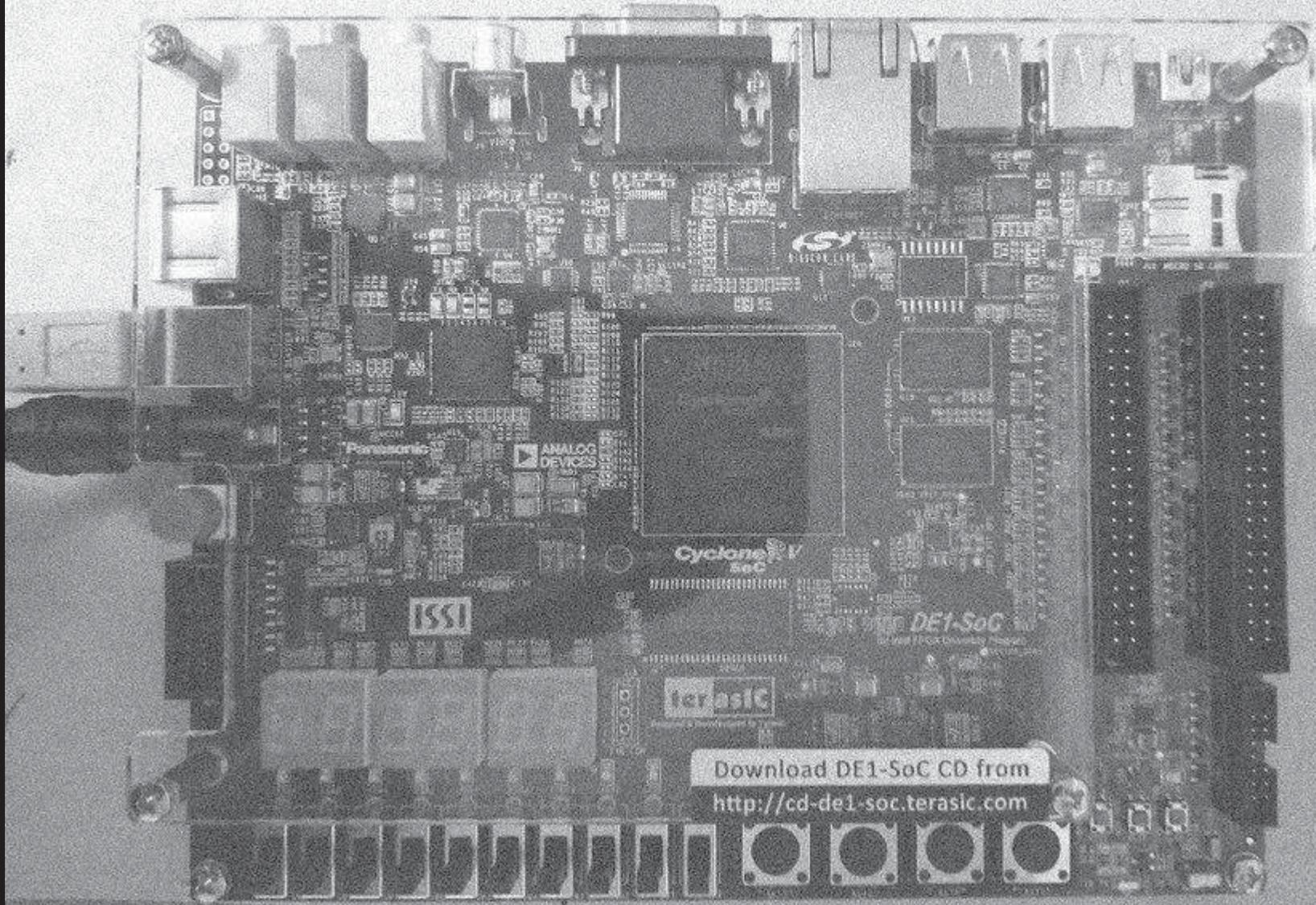


Uniform

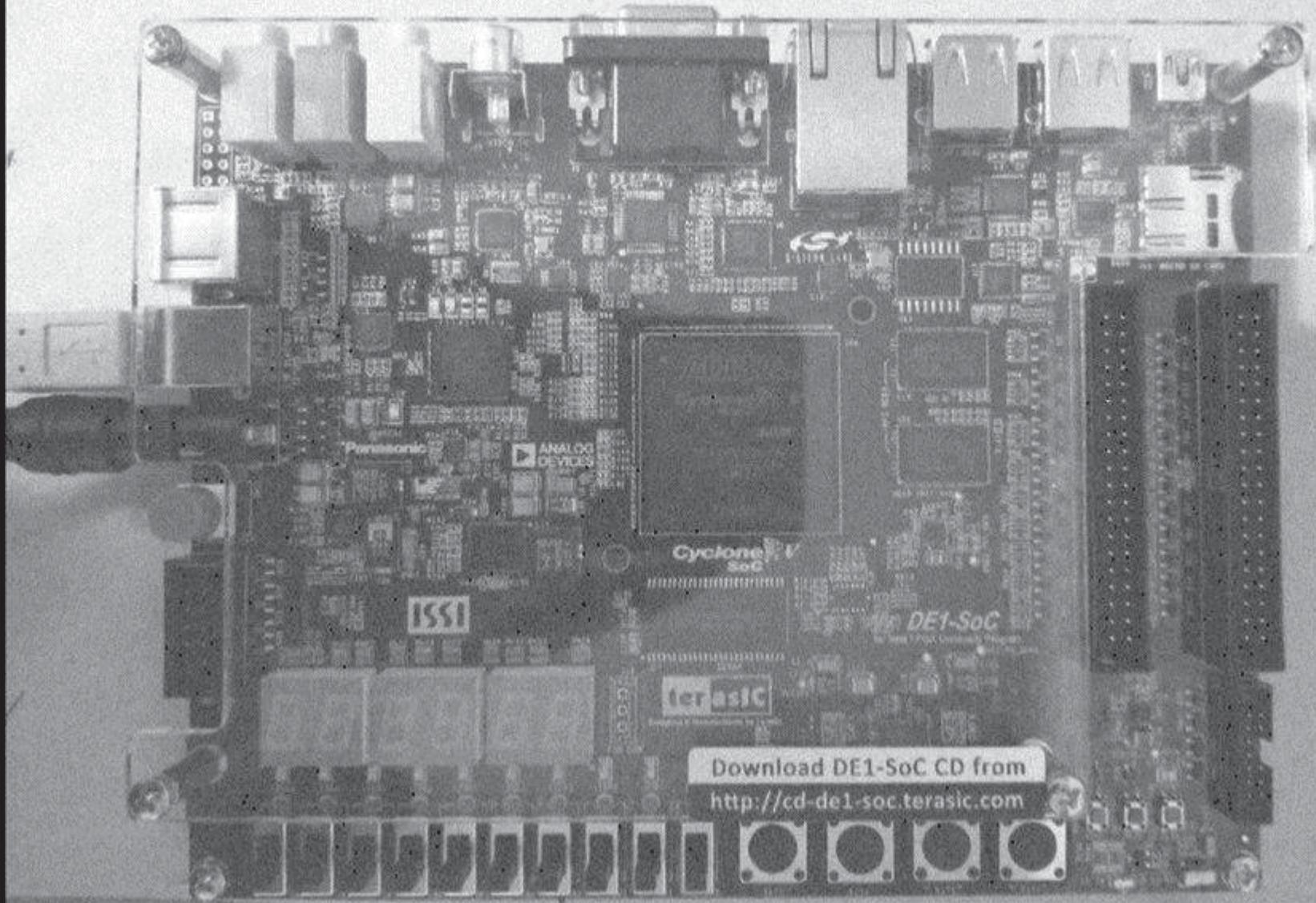


Salt & Pepper

Input image with added gaussian noise



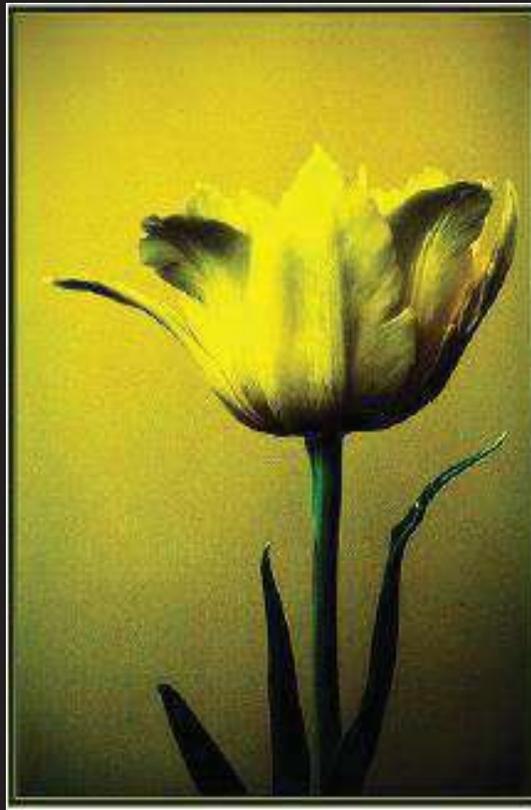
Output image after filter



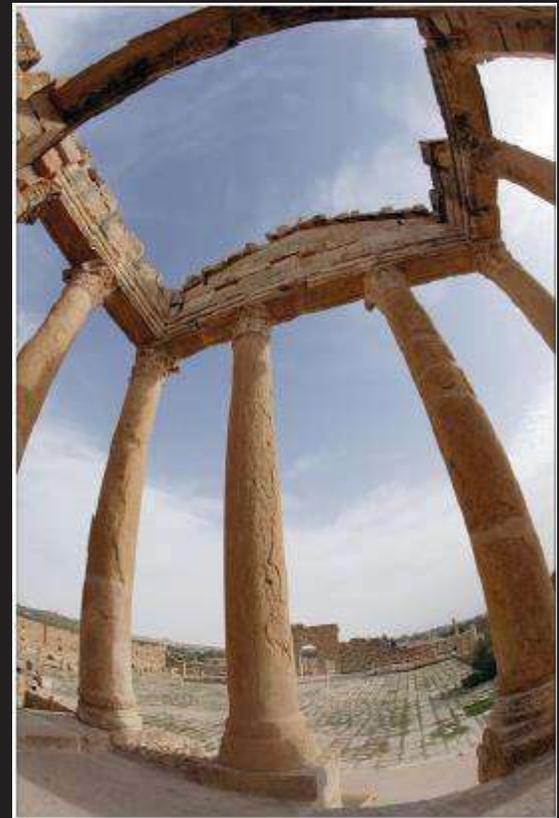
*Who said distortion is a bad thing?*



**blur ...**



**noise ...**



**geometric ...**

Thank you

Any Questions ?

**END**

**Of Lecture**

# Homework

# LSI Degradation Models

- **Motion Blur**

- Due to camera panning or fast motion

$$h(i, j) = \begin{cases} 1 & ai + bj = 0, i_{\min} \leq i \leq i_{\max} \\ 0 & \textit{otherwise.} \end{cases}$$

- **Atmospheric turbulence blur**

- Due to long exposure time through atmosphere

$$h(i, j) = K \cdot \exp\left(-\frac{i^2 + j^2}{2\sigma^2}\right)$$

- **Hufnagel and Stanley**

$$h(i, j) = \exp\left(-k \cdot (i^2 + j^2)^{5/6}\right)$$

- **Uniform out-of-focus blur:**

$$h(i, j) = \begin{cases} \frac{1}{\pi R^2} & i^2 + j^2 \leq R^2 \\ 0 & \textit{otherwise.} \end{cases}$$

- **Uniform 2D Blur**

$$h(i, j) = \begin{cases} \frac{1}{L^2} & -L/2 \leq i, j \leq L/2 \\ 0 & \textit{otherwise.} \end{cases}$$

# Turbulence Blur Examples

a b  
c d

**FIGURE 5.25**

Illustration of the  
atmospheric  
turbulence model.

(a) Negligible  
turbulence.

(b) Severe  
turbulence,  
 $k = 0.0025$ .

(c) Mild  
turbulence,  
 $k = 0.001$ .

(d) Low  
turbulence,  
 $k = 0.00025$ .

(Original image  
courtesy of  
NASA.)

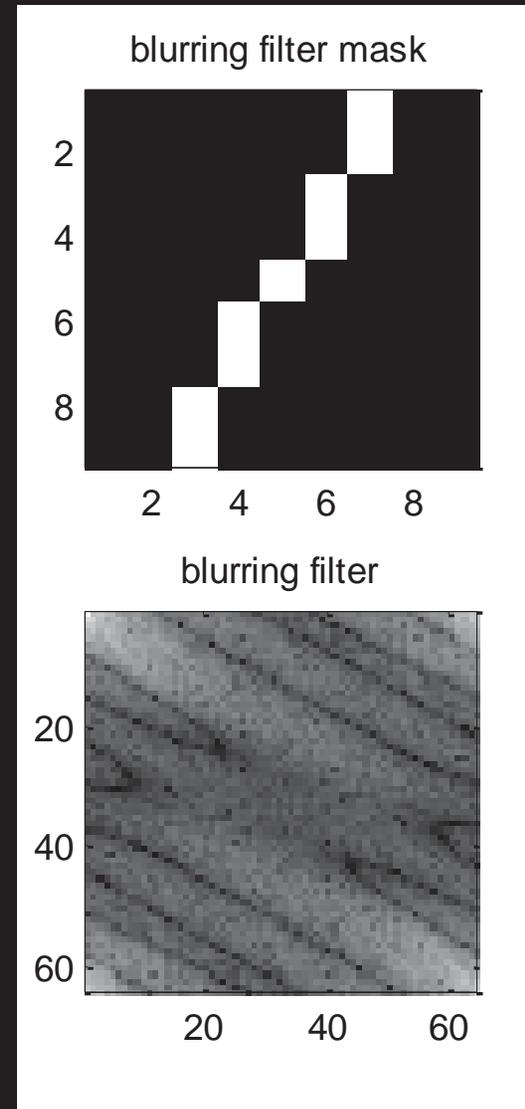
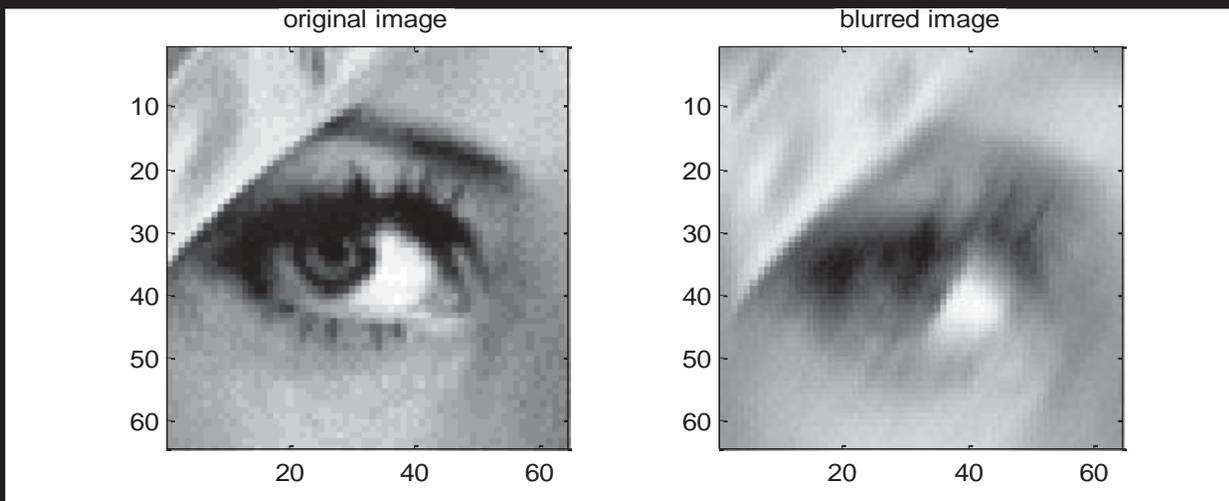


$$h(i, j) = \exp\left(-k \cdot (i^2 + j^2)^{5/6}\right)$$

# Motion Blur

- Often due to camera panning or fast object motion.
- Linear along a specific direction.

Blurdemo.m



# Adaptive Filters

The filters discussed so far are applied to an entire image without any regard for how image characteristics vary from one point to another

The behaviour of **adaptive filters** changes depending on the characteristics of the image inside the filter region

We will take a look at the **adaptive median filter**

# Adaptive Median Filtering

The median filter performs relatively well on impulse noise as long as the spatial density of the impulse noise is not large

The adaptive median filter can handle much more spatially dense impulse noise, and also performs some smoothing for non-impulse noise

The key insight in the adaptive median filter is that the filter size changes depending on the characteristics of the image

# Adaptive Median Filtering

Remember that filtering looks at each original pixel image in turn and generates a new filtered pixel

First examine the following notation:

- $Z_{min}$  = minimum grey level in  $\mathcal{S}_{xy}$
- $Z_{max}$  = maximum grey level in  $\mathcal{S}_{xy}$
- $Z_{med}$  = median of grey levels in  $\mathcal{S}_{xy}$
- $Z_{xy}$  = grey level at coordinates  $(x, y)$
- $S_{max}$  = maximum allowed size of  $\mathcal{S}_{xy}$

# Adaptive Median Filtering

Level A:  $A1 = z_{med} - z_{min}$

$$A2 = z_{med} - z_{max}$$

If  $A1 > 0$  and  $A2 < 0$ , Go to level B

Else increase the window size

If window size  $\leq S_{max}$  repeat level A

Else output  $z_{med}$

Level B:  $B1 = z_{xy} - z_{min}$

$$B2 = z_{xy} - z_{max}$$

If  $B1 > 0$  and  $B2 < 0$ , output  $z_{xy}$

Else output  $z_{med}$

## Lecture 2-c

# Image Enhancement

## Spatial Averaging..

*Lecturer:*

*Dr. Faisal Ghazi Mohammed*

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[faiselgm73@gmail.com](mailto:faiselgm73@gmail.com)

*2020-2021*

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# Contents

In this lecture we will look at spatial filtering techniques:

1. Neighbourhood operations
2. What is spatial filtering?
3. Smoothing operations
4. What happens at the edges?
5. Correlation and convolution

- Mean filters

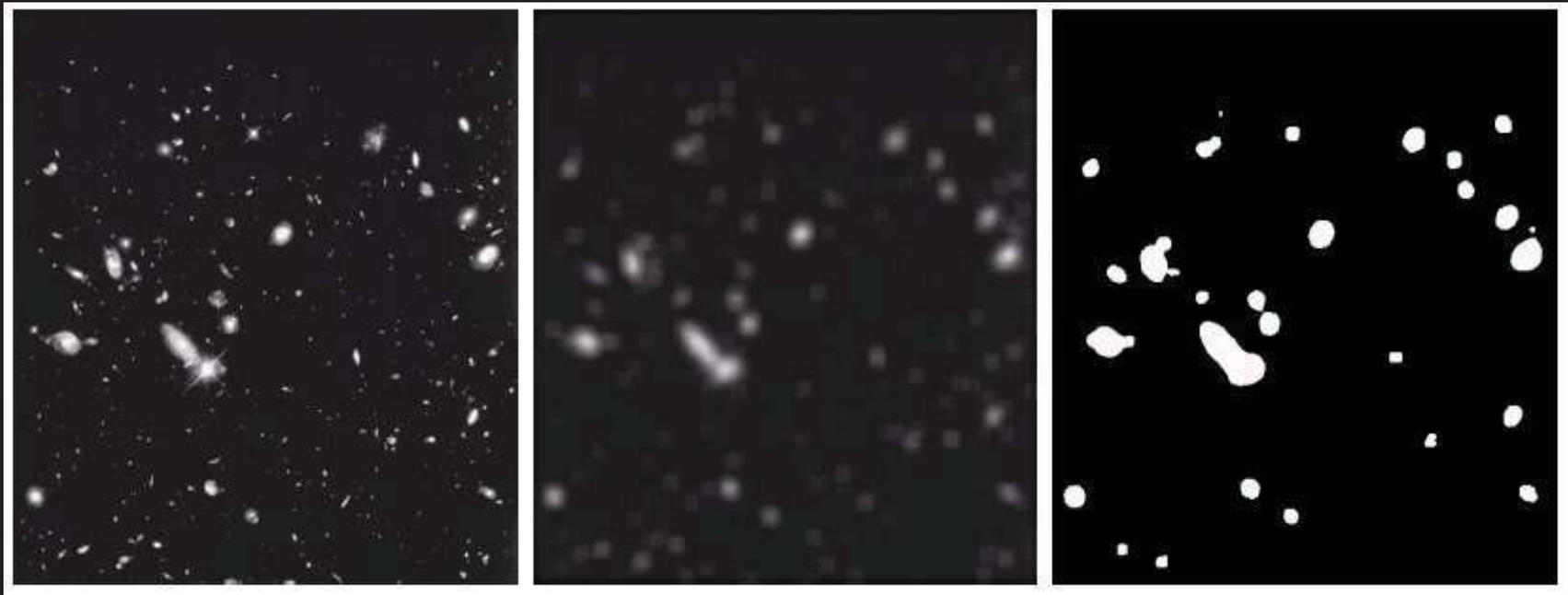
1. Arithmetic mean (Spatial Averaging)
2. Geometric mean
3. Harmonic mean
4. Directional Smoothing

- Order statistics filters

1. Median
2. Max and Min

# Smoothing Example

By smoothing the original image we get rid of lots of the finer detail which leaves only the gross features for thresholding



Original Image

Smoothed Image

Thresholded Image

# 1-Mean filters - Spatial Averaging

Its is a very simple one and is calculated as follows:

$$v(m, n) = \sum \sum_{(k, l) \in W} a(k, l) y(m - k, n - l)$$

Each pixel is replaced by a weighted average of it's neighborhood pixels that is,

- $y(m, n)$  and  $v(m, n)$  are the input and output images ,respectively.
- $W$  is suitably chosen window .
- $a(k, l)$  are the filter weights .

# 1-Mean filters - Spatial Averaging

A common class of spatial averaging filters has all equal weights

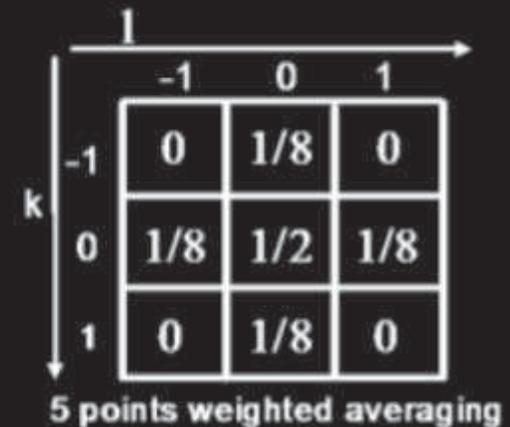
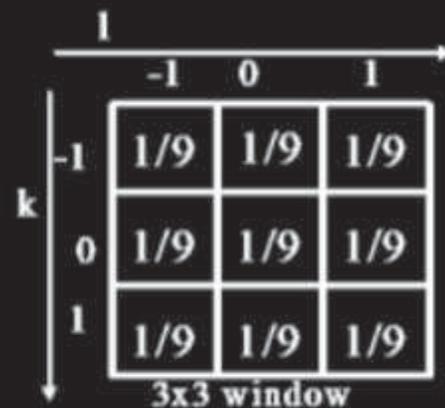
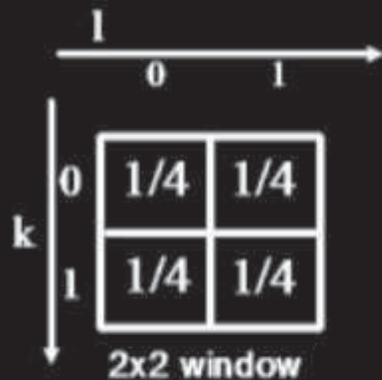
$$v(m, n) = \frac{1}{N} \sum_{(k,l) \in W} y(m - k, n - l)$$

Used for **Noise smoothing**, low-pass filtering and **subsampling** of images.

$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$

# 1-Mean filters - Spatial Averaging

Examples of spatial averaging masks



# 1-Mean filters - Spatial Averaging

$$v(m, n) = \frac{1}{N} \sum_{(k,l) \in W} y(m - k, n - l)$$

$x-1,y-1$	$x-1,y$	$x-1,y+1$
$x,y-1$	$x,y$	$x,y+1$
$x+1,y-1$	$x+1,y$	$x+1,y+1$

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

# 1-Mean filters - Spatial Averaging

$x-1,y-1$	$x-1,y$	$x-1,y+1$
$x,y-1$	$x,y$	$x,y+1$
$x+1,y-1$	$x+1,y$	$x+1,y+1$

## 2-Mean filters - Geometric Mean

- The geometric mean filter is an image filtering process meant to smooth and reduce noise of an image
- It is based on the **mathematic geometric mean**.
- The output image  $g(x,y)$  of a geometric mean is given by

$$g(x,y) = \left[ \prod_{i,j \in S} s(i,j) \right]^{\frac{1}{mn}}$$

## 2-Mean filters - Geometric Mean

- Where  $s(x,y)$  is the original image, and the filter mask is  $m$  by  $n$  pixels.
- Each pixel of the output image at point  $(x,y)$  is given by the product of the pixels within the geometric mean mask raised to the power of  $1/mn$ .

$$g(x, y) = \left[ \prod_{i,j \in S} s(i, j) \right]^{\frac{1}{mn}} = \left( \prod_{i=1}^n x_i \right)^{\frac{1}{n}} = \sqrt[n]{x_1 x_2 \cdots x_n}$$

# Mean filters - Geometric Mean

- For instance, the geometric mean of two numbers, say 2 and 8, is just the square root of their product, that is,

$$\sqrt{2 \cdot 8} = 4$$

- As another example, the geometric mean of the three numbers 4, 1, and 1/32 is the cube root of their product (1/8), which is 1/2, that is,

$$\sqrt[3]{4 \cdot 1 \cdot 1/32} = 1/2.$$

- The geometric mean applies only to positive numbers.[3]

## 2-Mean filters - Geometric Mean

- For **example**, using a mask size of **3×3**, pixel  $(x,y)$  in the output image will be the product of  $S(x,y)$  and all **8** of its surrounding pixels raised to the **1/9th** power.
- Using the following original image with pixel  $(x,y)$  at the center:

5	16	22
6	3	18
12	3	15

Gives the result of:

$$(5*16*22*6*3*18*12*3*15)^{(1/9)} = 8.77.$$

## 2-Mean filters - Geometric Mean

### Application

- The geometric mean filter is most widely used to filter out Gaussian noise.
- In general it will help smooth the image with less data loss than an arithmetic mean filter.[1]
- For more details see ([appendix 1](#))

## 3-Mean filters - Harmonic Mean

- In mathematics, the harmonic mean is one of several kinds of average, and in particular, one of the Pythagorean means. Typically, it is appropriate for situations when the average of rates is desired.

$$g(x, y) = \frac{mn}{\sum_{i,j \in S} \frac{1}{s(i, j)}}$$

- Works **well** for **salt** noise, but **fails** for pepper noise
- Also does well for other kinds of noise such as **Gaussian** noise

## 3-Mean filters - Harmonic Mean

- The harmonic mean can be expressed as the reciprocal of the arithmetic mean of the reciprocals of the given set of observations.
- As a simple **example**, the harmonic mean of 1, 4, and 4 is

$$\left( \frac{1^{-1} + 4^{-1} + 4^{-1}}{3} \right)^{-1} = \frac{3}{\frac{1}{1} + \frac{1}{4} + \frac{1}{4}} = \frac{3}{1.5} = 2.$$

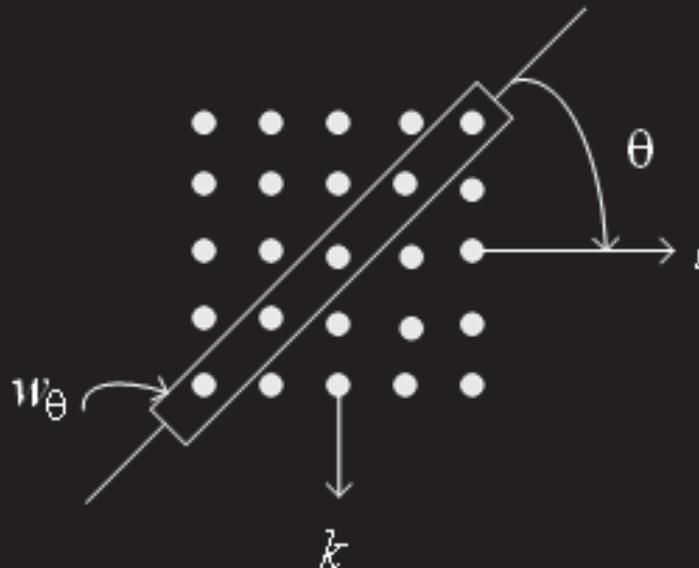
## 4- Directional Smoothing

- To protect edges from blurring while smoothing.
- Spatial averages  $v(m, n, \theta)$  are calculated in **several directions**, and the direction giving the smallest changes before and after filtering is selected.

$$v(m, n; \theta) = \frac{1}{N_\theta} \sum_{(k, l) \in \mathcal{W}_\theta} y(m - k, n - l)$$

## 4- Directional Smoothing

- And a direction  $\theta$  is found such that
- $\left| v(m, n) - v(m, n; \theta^k) \right|$  is minimum
- Then  $v(m, n) = v(m, n; \theta^k)$  gives the desired result.



# Noise Removal Examples

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Original Image

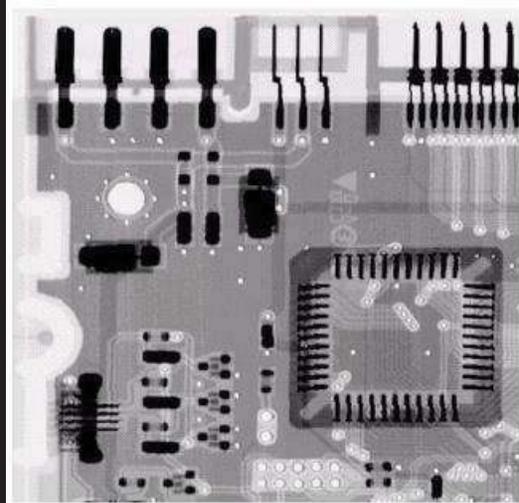
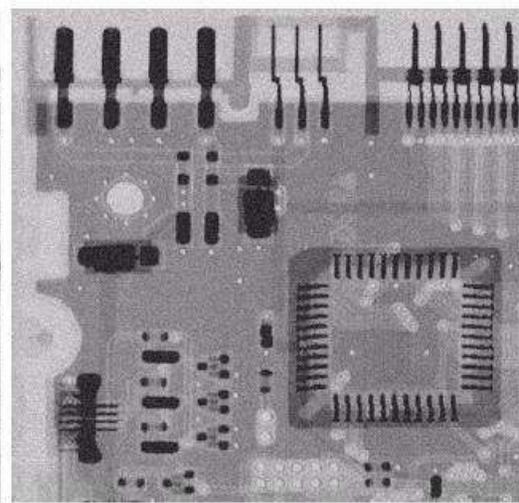
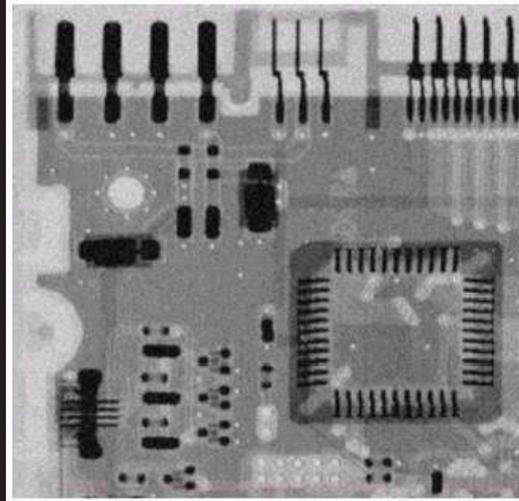


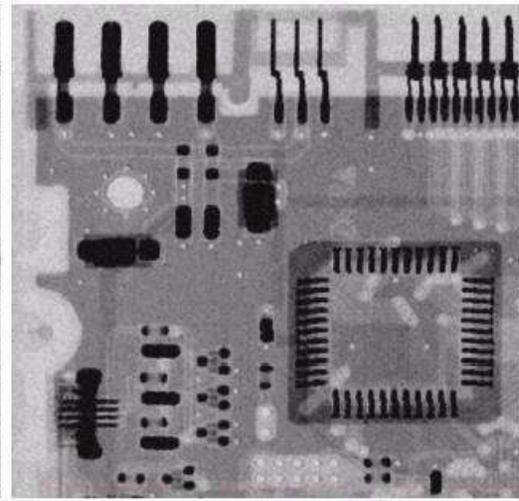
Image Corrupted By Gaussian Noise



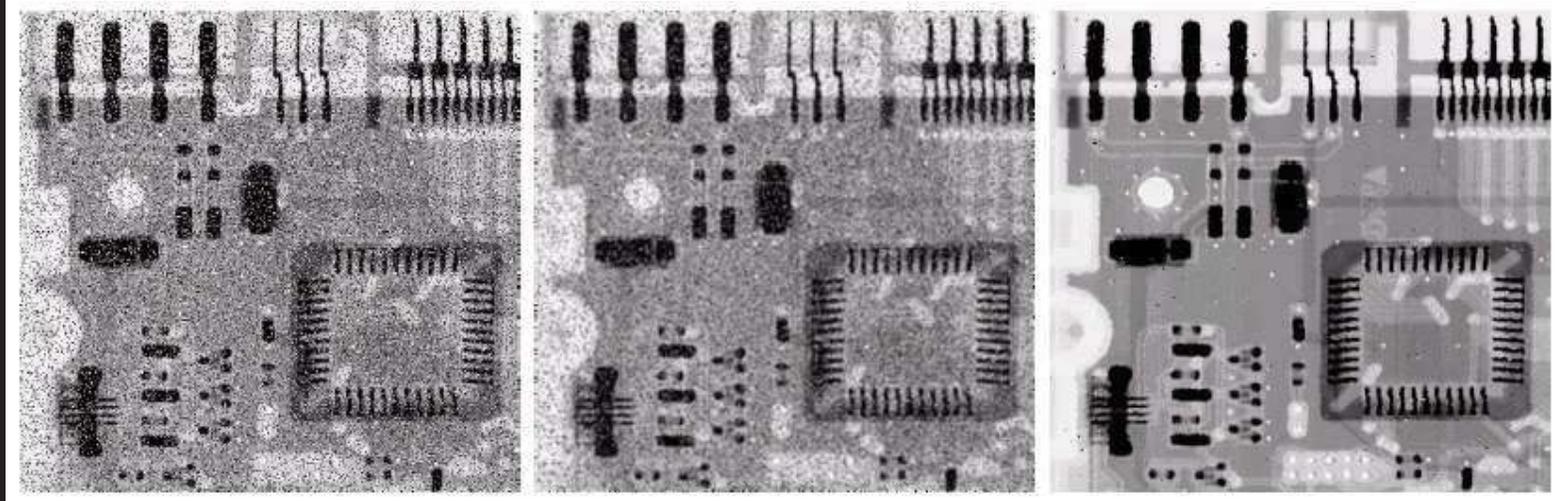
After A 3\*3 Arithmetic Mean Filter



After A 3\*3 Geometric Mean Filter



# Averaging Filter Vs. Median Filter Example



- Filtering is often used to remove noise from images
- Sometimes a median filter works better than an averaging filter

# Order Statistics Filters - Median

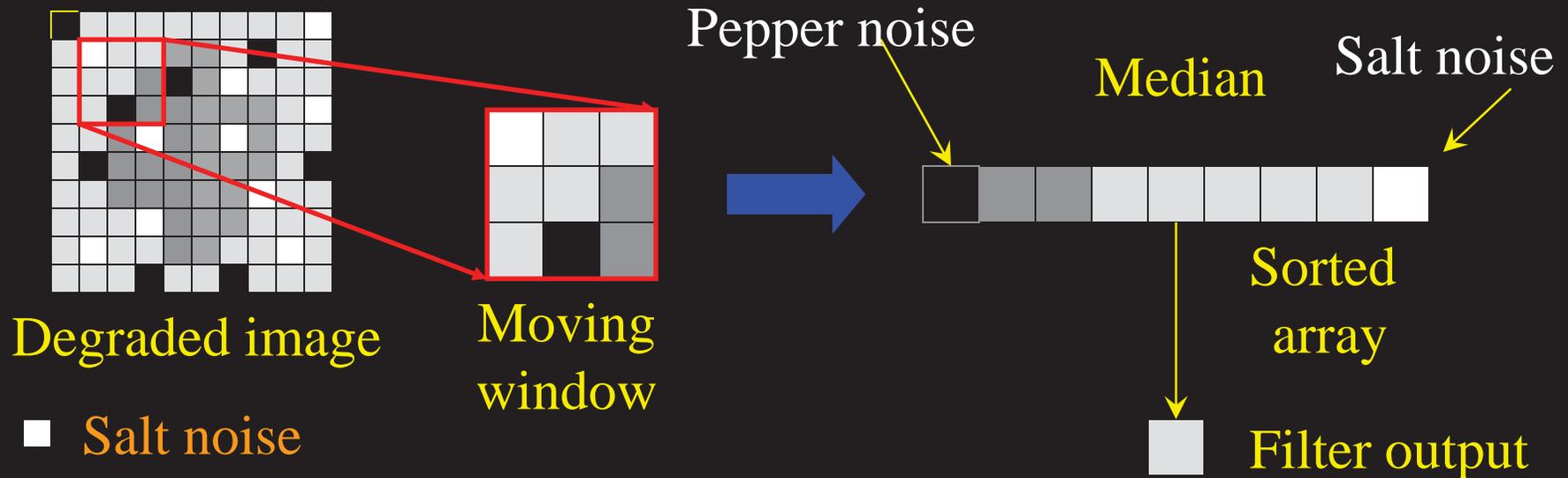
$$\hat{f}(x, y) = \underset{(s,t) \in S_{xy}}{\text{median}}\{g(s, t)\}$$

Excellent at noise removal, **without** the **smoothing** effects that can occur with other smoothing filters

Particularly good when **salt** and **pepper** noise is present

# Order Statistics Filters - Median

A median filter is good for removing impulse, isolated noise



Degraded image

Moving window

■ Salt noise

□ Pepper noise

Normally, impulse noise has high magnitude and is isolated. When we sort pixels in the moving window, noise pixels are usually at the ends of the array.

Therefore, it's rare that the noise pixel will be a median value.

# Order Statistics Filters – Max & Min

**Max Filter:**

$$g(x,y) = \text{Max} \{ s(i, j) \}_{i,j \in S}$$

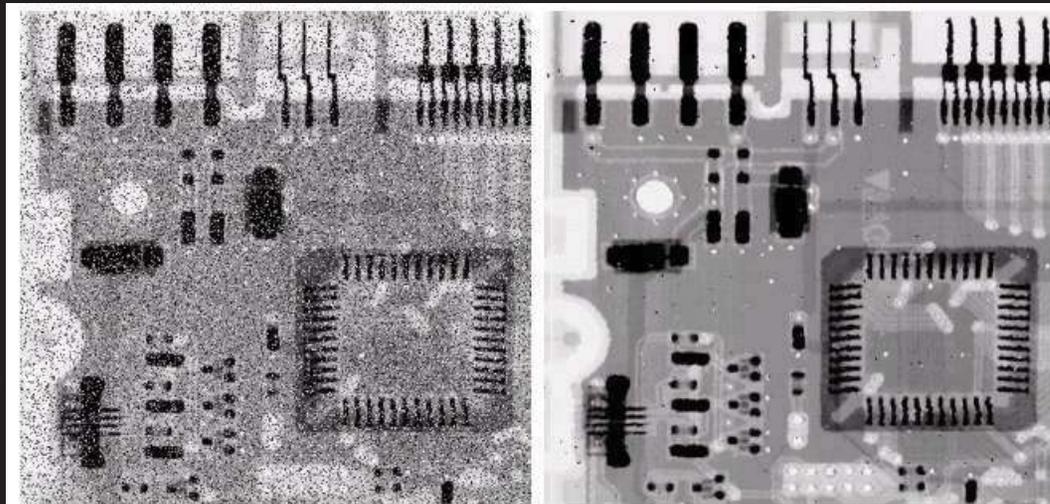
**Min Filter:**

$$g(x,y) = \text{Min} \{ s(i, j) \}_{i,j \in S}$$

**Max** filter is good for **pepper** noise and **Min** is good for **salt** noise

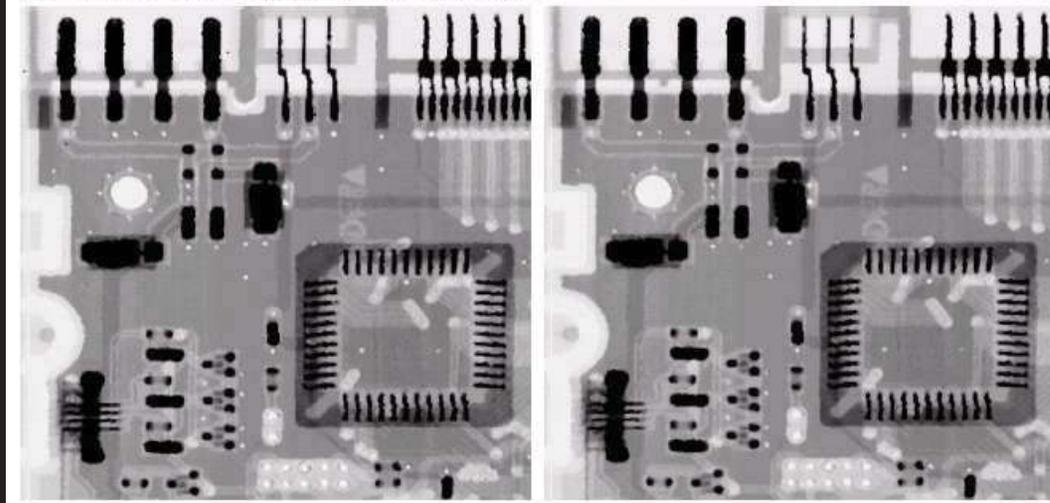
# Noise Removal Examples

Image Corrupted By **Salt And Pepper** Noise



Result of **1** Pass With A **3\*3 Median** Filter

Result of **2** Passes With A **3\*3 Median** Filter

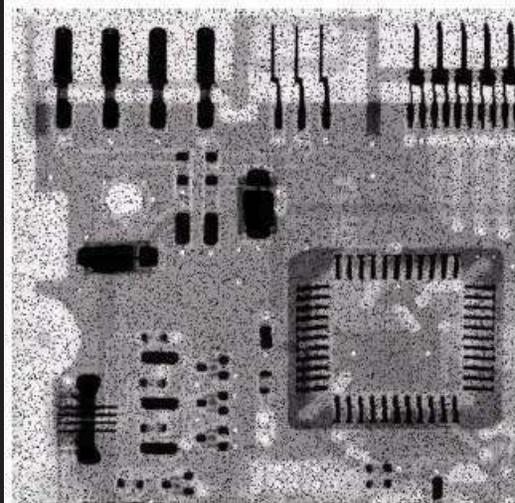


Result of **3** Passes With A **3\*3 Median** Filter

# Noise Removal Examples

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Image Corrupted By **Pepper** Noise



Result Of Filtering Above With A 3\*3 **Max** Filter

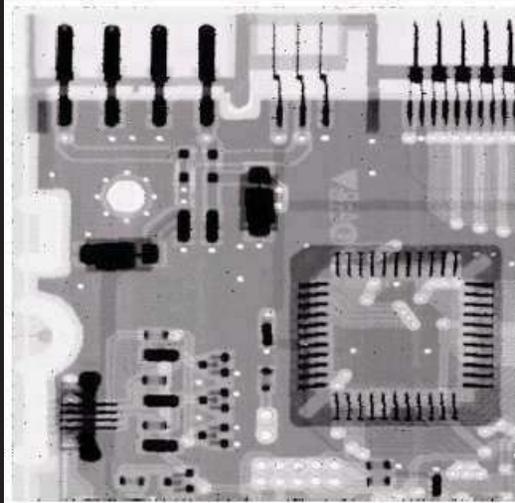
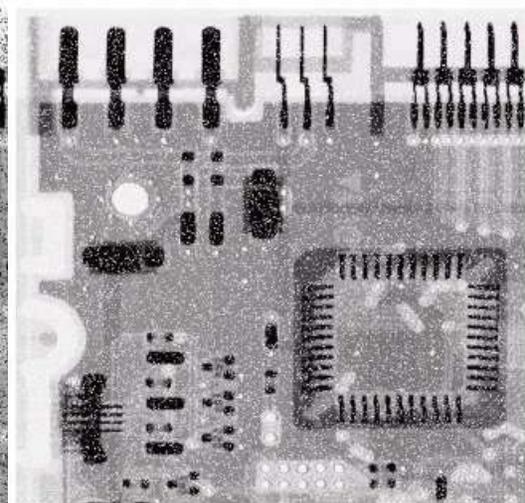
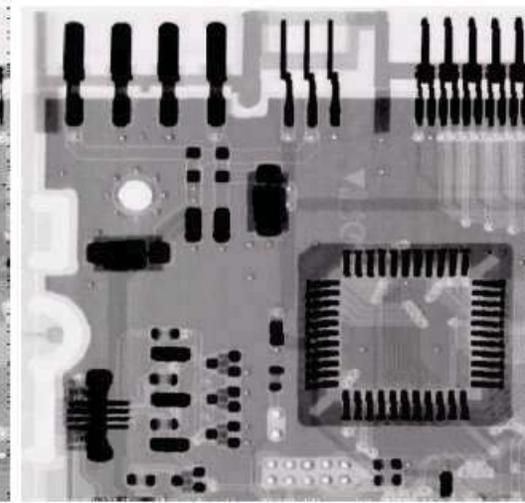


Image Corrupted By **Salt** Noise

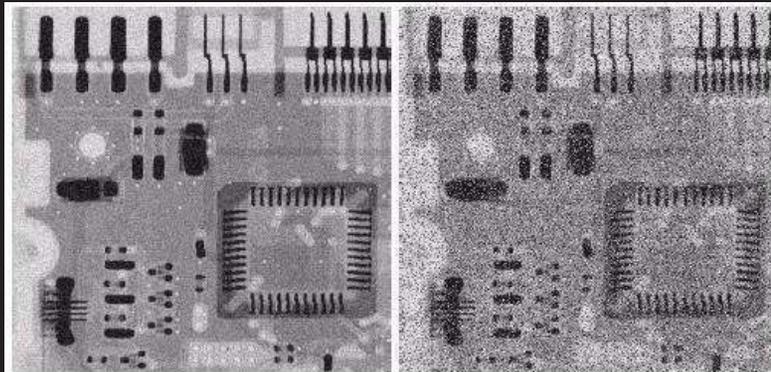


Result Of Filtering Above With A 3\*3 **Min** Filter

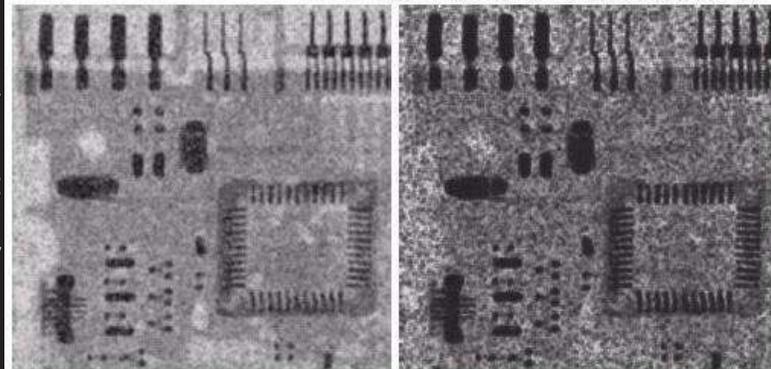


# Noise Removal Examples

Image  
Corrupted  
By **Uniform**  
Noise



Filtered By  
 $5 \times 5$  **Arithmetic**  
**Mean** Filter



Filtered By  
 $5 \times 5$  **Median**  
Filter

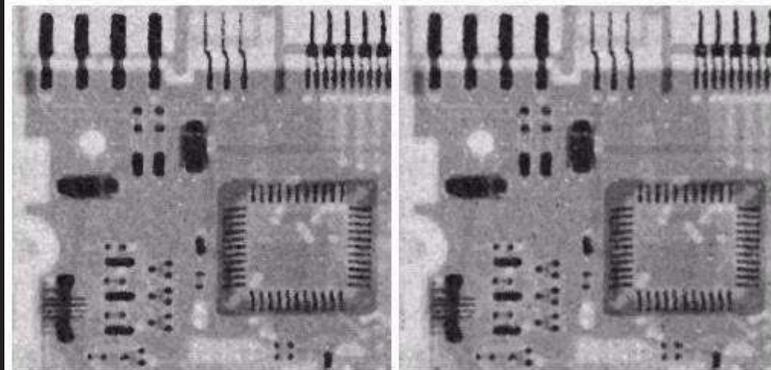
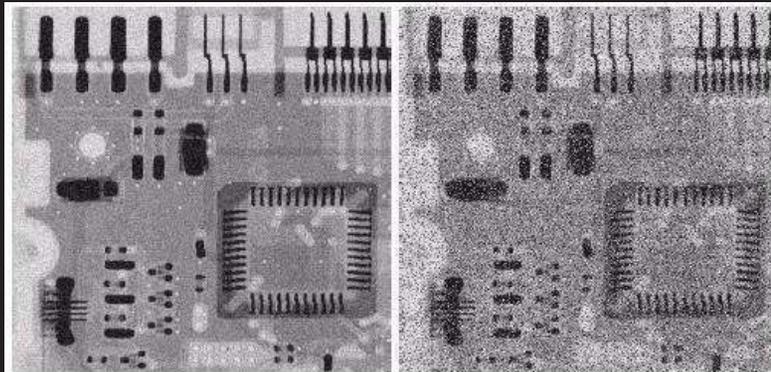
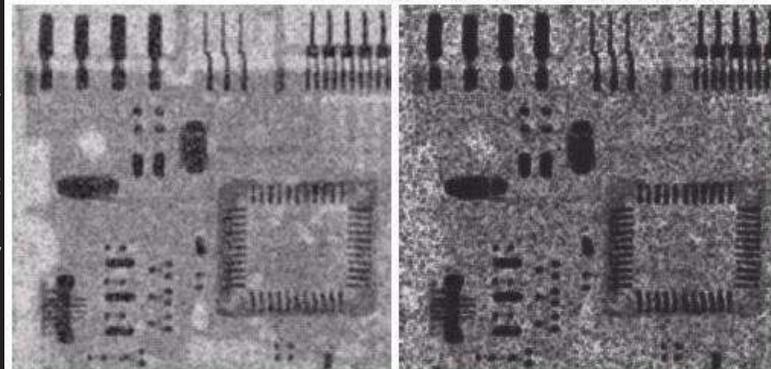


Image Further  
Corrupted  
By **Salt** and  
**Pepper** Noise



Filtered By  
 $5 \times 5$  **Geometric**  
**Mean** Filter



**Thank you**

**Any Questions ?**

END

# Homework

# Appendix 1

Mean filters - **Geometric Mean**

# Mean filters - Geometric Mean

- In mathematics, the geometric mean is a mean or average, which indicates the central tendency or typical value of a set of numbers by using the product of their values (as opposed to the arithmetic mean which uses their sum).
- The geometric mean is defined as the  $n$ th root of the product of  $n$  numbers, i.e., for a set of numbers  $x_1, x_2, \dots, x_n$ , the geometric mean is defined as

$$\left( \prod_{i=1}^n x_i \right)^{\frac{1}{n}} = \sqrt[n]{x_1 x_2 \cdots x_n}$$

# Mean filters - Geometric Mean



# Mean filters - Geometric Mean

- For instance, the geometric mean of two numbers, say 2 and 8, is just the square root of their product, that is,

$$\sqrt{2 \cdot 8} = 4$$

- As another example, the geometric mean of the three numbers 4, 1, and 1/32 is the cube root of their product (1/8), which is 1/2, that is,

$$\sqrt[3]{4 \cdot 1 \cdot 1/32} = 1/2.$$

- The geometric mean applies only to positive numbers.[3]

# Mean filters - Geometric Mean

For more details, please visit the site:

[https://en.wikipedia.org/wiki/Geometric\\_mean](https://en.wikipedia.org/wiki/Geometric_mean)

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

# *Aerial Photography & Photogrammetry*

## Lecture 3-1

# Geometry of a Vertical Aerial Photograph

*Lecturer: Faisel Ghazi Mohammed*

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# Reading Chapters

*“Elements of Photogrammetry with Applications in GIS”, by Förstner etl, 2016.*

	<b>Material</b>	<b>Chapter Sections</b>	<b>Page</b>	<b>Exercises page</b>	<b>LABORATORY exercises Page</b>
<u>1</u>	Introduction		1	23	83
<u>2</u>	<b>Geometry of a Vertical Aerial Photograph</b>		27	43	83
<u>3</u>	Principles of Stereoscopic Vision		44		83
<u>4</u>	Scale of a Vertical Aerial Photograph		68		101
<u>5</u>	Horizontal Measurements - Distance, Bearings and Areas		86		127
<u>6</u>	Vertical Measurements		105		127
<u>7</u>	Acquisition of Aerial Photography		131		154

# OBJECTIVES

After a thorough understanding of this chapter, you will be able to:

1. Identify different types of aerial photographs—whether they are vertical, high, or low oblique, or horizontal—and sketch the shapes of the ground area covered by each type.
2. Give precise definitions for camera focal length and angle of coverage and classify narrow-, normal-, wide-, and super-wide-angle lenses according to focal length and angle of coverage.

# OBJECTIVES

After a thorough understanding of this chapter, you will be able to:

3. Identify on an aerial photograph or sketch the fiducial marks, coordinate axes, and the three different photo centers on an “unintentionally tilted” vertical aerial photograph.
4. State the difference between photo distortion and photo displacement.

# OBJECTIVES

After a thorough understanding of this chapter, you will be able to:

5. List the type of distortion or displacement that radiates from the three photo centers and know how to remove or avoid them.
6. List four other types of distortion or displacement.
7. Define ratioed and rectified prints and explain how each is obtained.
8. Compute the unknown variable given the equation for image displacement due to relief and any four of the five variables involved.
9. State five inferences that can be made from the image displacement equation for topography and solve problems based on these inferences.

# Chapter Two: **Geometry of a Vertical Aerial Photograph**

## **2.1 CLASSIFICATION OF PHOTOGRAPHS**

2.1.1 Advantages of Vertical as Compared

2.1.2 Advantages of Oblique as Compared

2.2 FOCAL LENGTH AND ANGLE OF COVERAGE

## **2.2 The Coordinate Axes 31**

## **2.3 THE THREE PHOTO CENTERS**

2.3.1 Principal Point

2.3.2 Nadir

2.3.3 Isocenter

## **2.4 Distortion and Displacement**

2.5.1 Lens Distortion

2.5.2 Tilt Displacement

2.5.3 Topographic Displacement

## **2.6 NUMERICAL EXAMPLES**

## **2.7 Inferences Based on the Relief Displacement Equation 41**

800 km / 500 miles



# Spaceborne Sensors

400 km / 250 miles

1.3 million ft to 2.5 million ft

400,000 m to 760,000 m

12 km / 40,000 ft



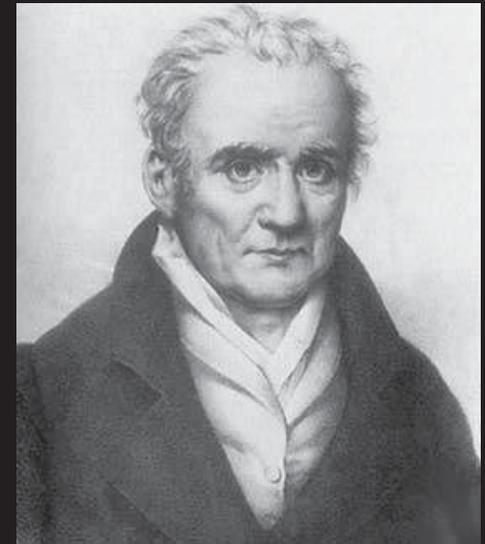
# Airborne Sensors

1 km / 3,000 ft



# Descriptive Geometry

- Gaspard **Monge** (1746-1818), the father of **descriptive geometry**, developed a graphical protocol which creates three-dimensional **virtual space** on a two-dimensional plane.
- Monge became a scientific and mathematical aide to Napoleon during his reign as general and emperor of France.



For more details see the lecture on  
ME114 : COMPUTER AIDED ENGINEERING DRAWING II  
<http://me114.cankaya.edu.tr/course.php?page=Lecture%20Notes>

# Aerial Photo (Image) vs Map

<b>Images</b>	<b>Maps</b>
<ol style="list-style-type: none"><li data-bbox="163 613 823 997">1. Central/ Perspective projection</li><li data-bbox="163 1062 873 1289">2. non-uniform scale</li><li data-bbox="163 1370 982 1458">3. actual features</li></ol>	<ol style="list-style-type: none"><li data-bbox="1064 613 1751 964">1. Orthogonal/ Orthographic projection</li><li data-bbox="1064 1029 1818 1117">2. uniform scale</li><li data-bbox="1064 1192 1570 1295">3. Symbols</li></ol>

# Aerial Photo (Image) vs Map

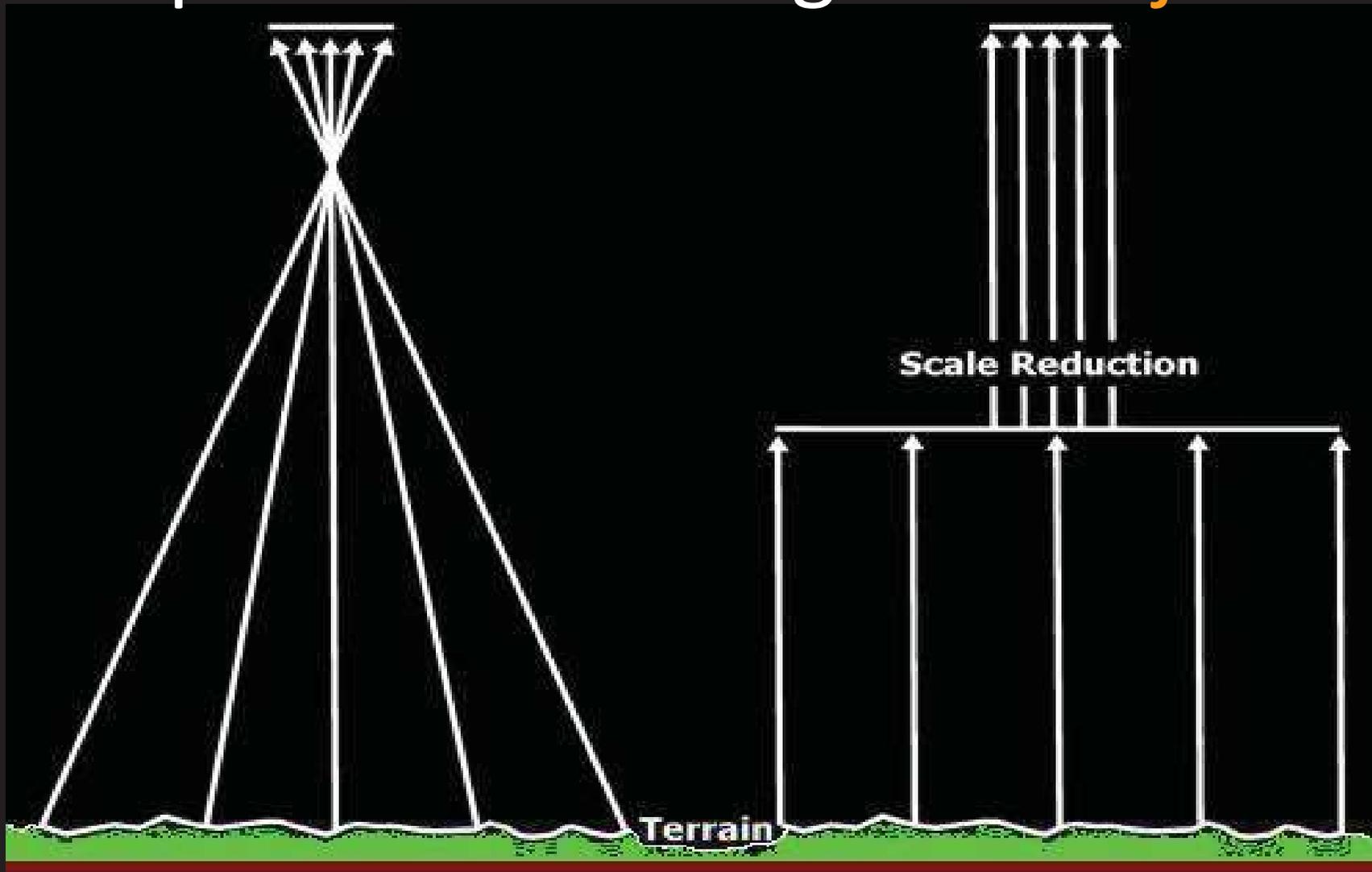
## Central Projection

- Projection of features in an aerial photograph
- Each point on a photo is imaged after reflected light passes through a single point (lens)
- Causes features with a height above a base elevation to radiate outward from the center

## Orthogonal Projection

- Each point is placed as if it is viewed from above
- Also called a map plane

# Perspective vs Orthogonal Projection



Central projection: **aerial photo**

Orthogonal projection: **map**

# Perspective vs Orthogonal Projection

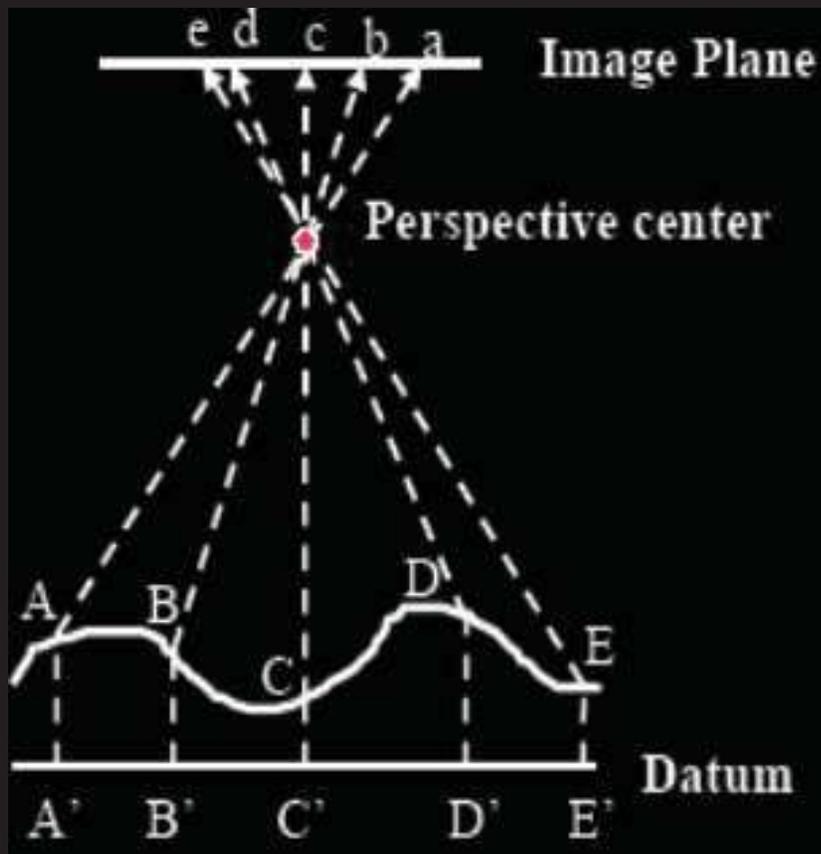
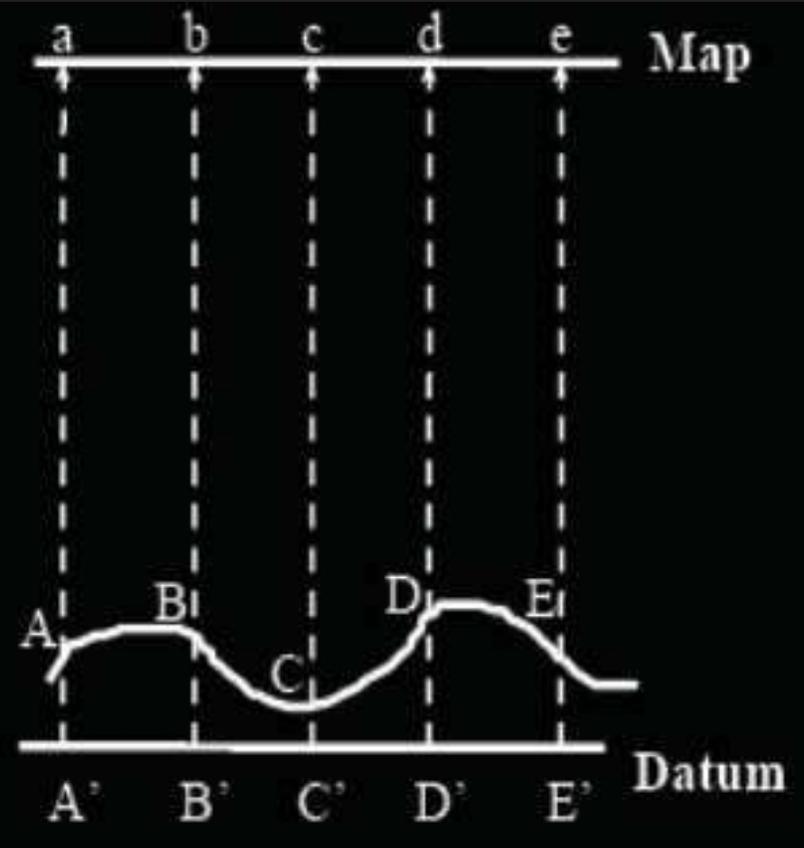
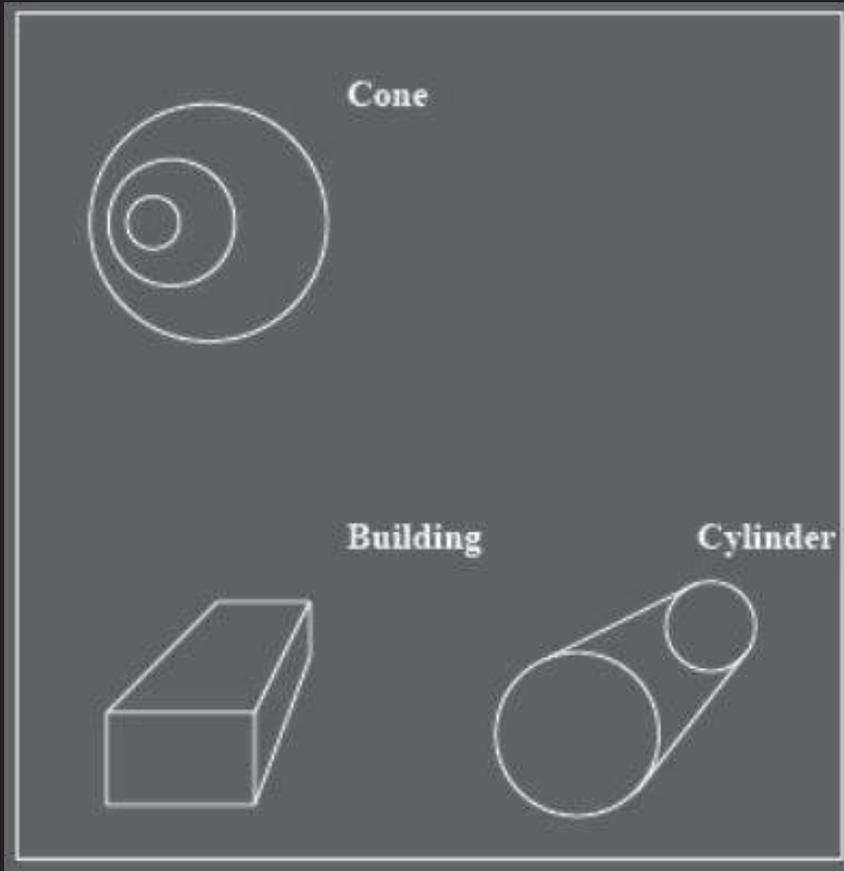


Image / aerial photo

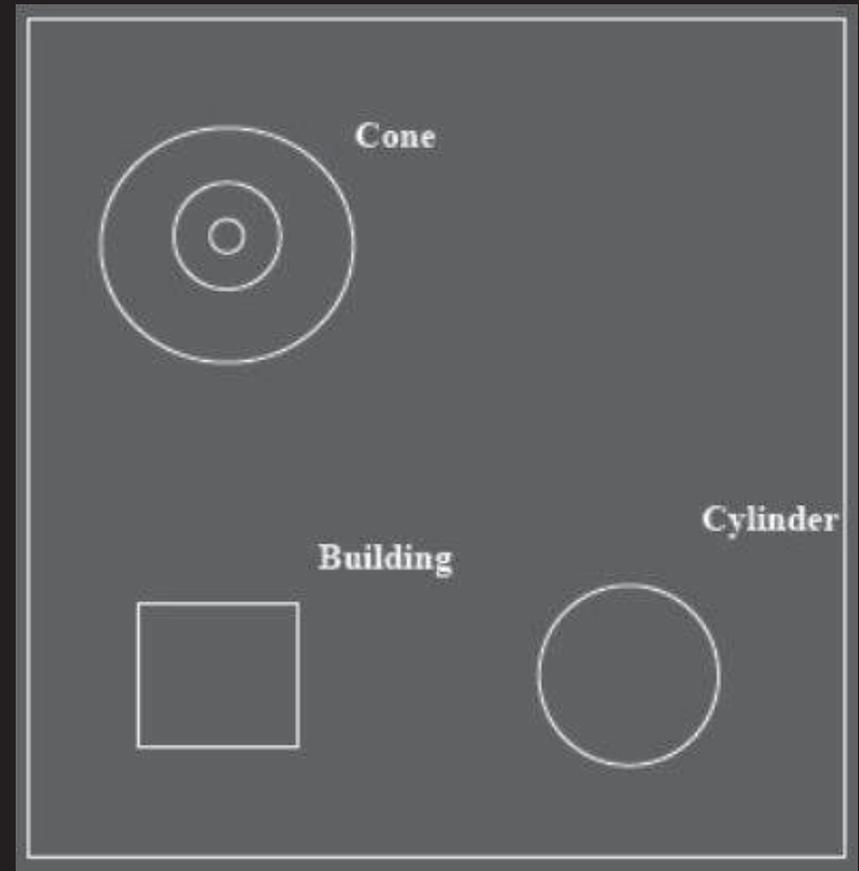


Map

# Perspective vs Orthogonal Projection

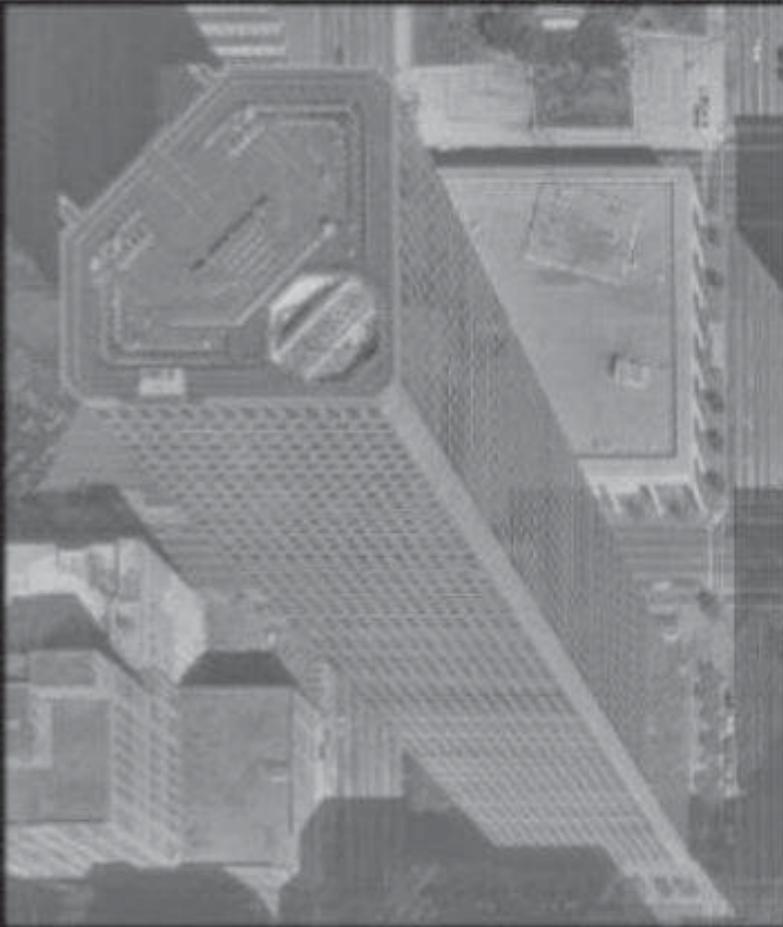


Perspective Projection

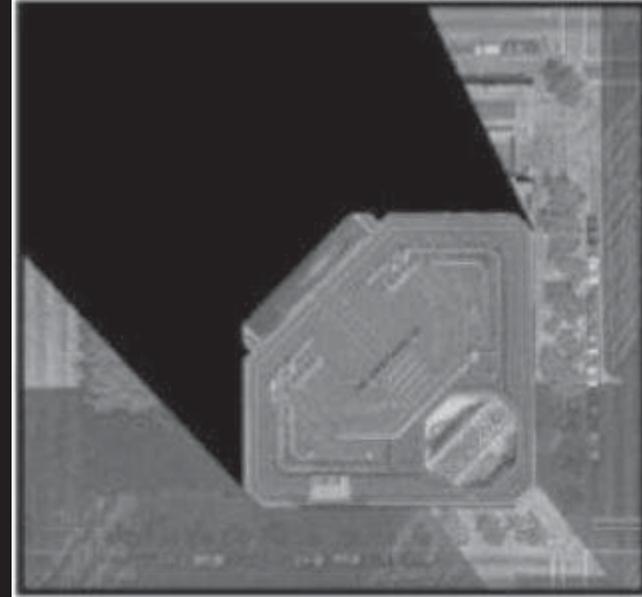


Orthogonal Projection

# Perspective vs Orthogonal Projection



Perspective Projection



Orthogonal Projection

# Perspective vs Orthogonal Projection



Perspective Projection



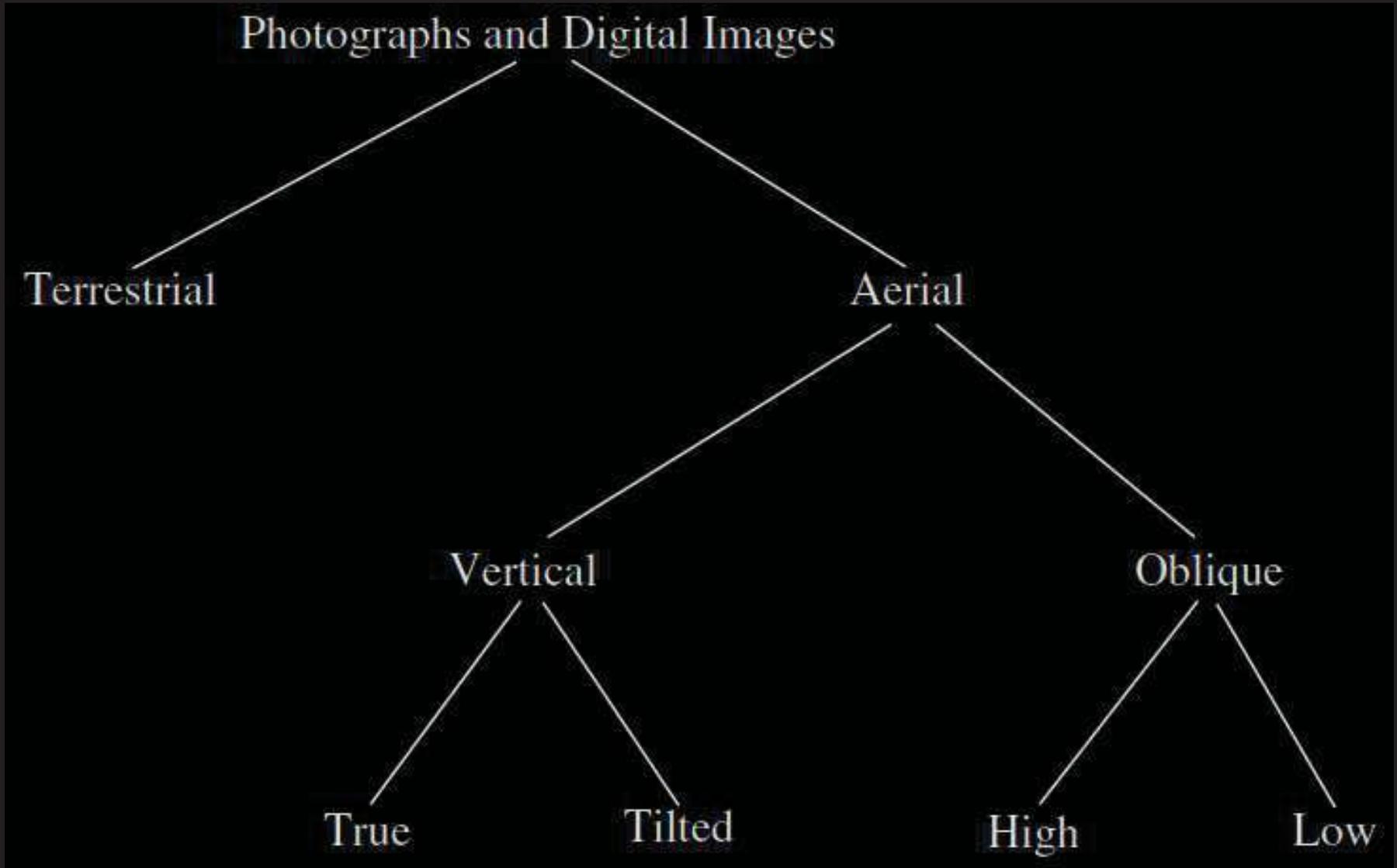
Orthogonal Projection

1-

# Classification/Types of aerial photographs

AP

# 1- Classification of AP



A classification of aerial photographs and digital images.

# 1- Classification of AP

## Terrestrial Photograph

10 Tips for  
Improving  
Your Wildlife  
Photography



# 1- Classification of AP

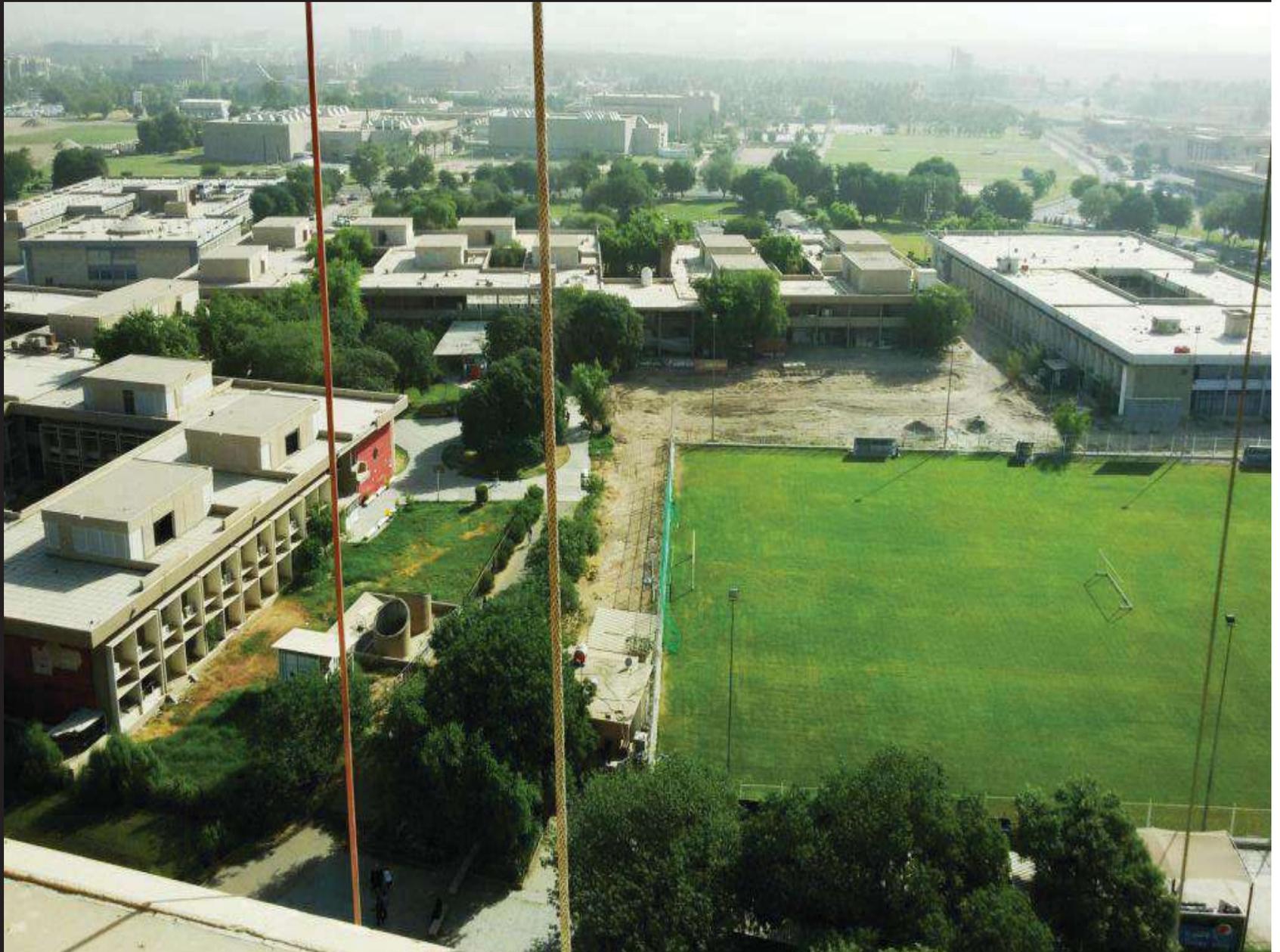
## True Vertical Photograph



<http://imagewerx.us/your-exclusive-source-for-true-vertical-aerial-photography/>

# 1- Classification of AP

Oblique



# 1- Classification of AP

**Tilted**

Raw



# 1- Classification of AP

**Tilted**

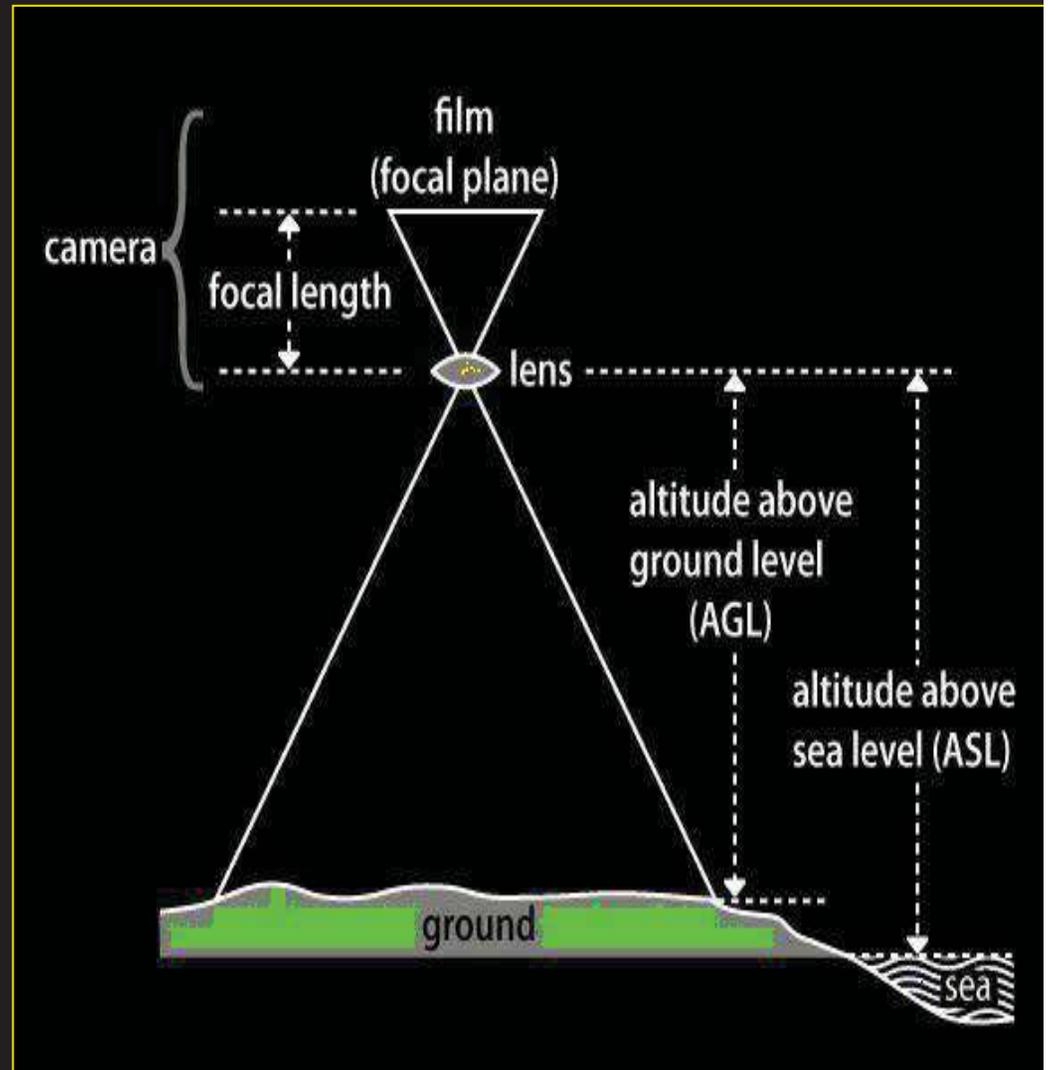
Rectified



# 1- Classification of AP according to *direction of exposure*

**1. Vertical photograph:**  
AP taken with the optical axis of the camera perpendicular to the horizontal plane.

Ground features appear in the photo in much the same way as the map of similar scale.



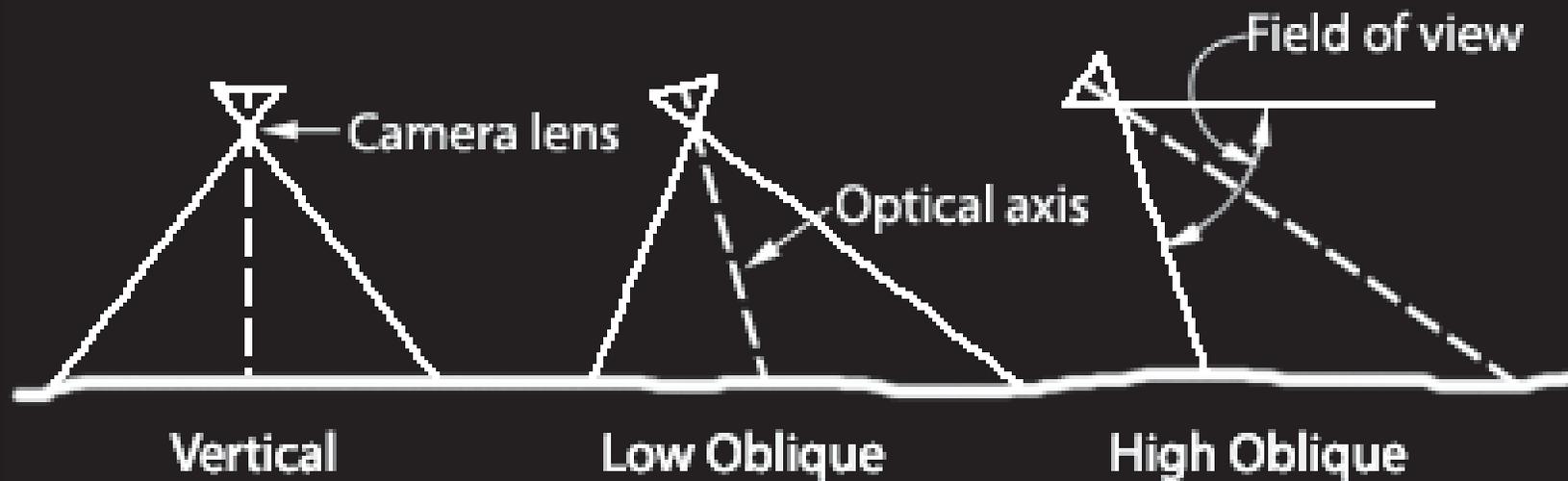
# 1- Classification of AP according to *direction of exposure*

**2. Oblique photograph:** AP taken with the camera axis tilted intentionally between the horizontal and vertical plane.

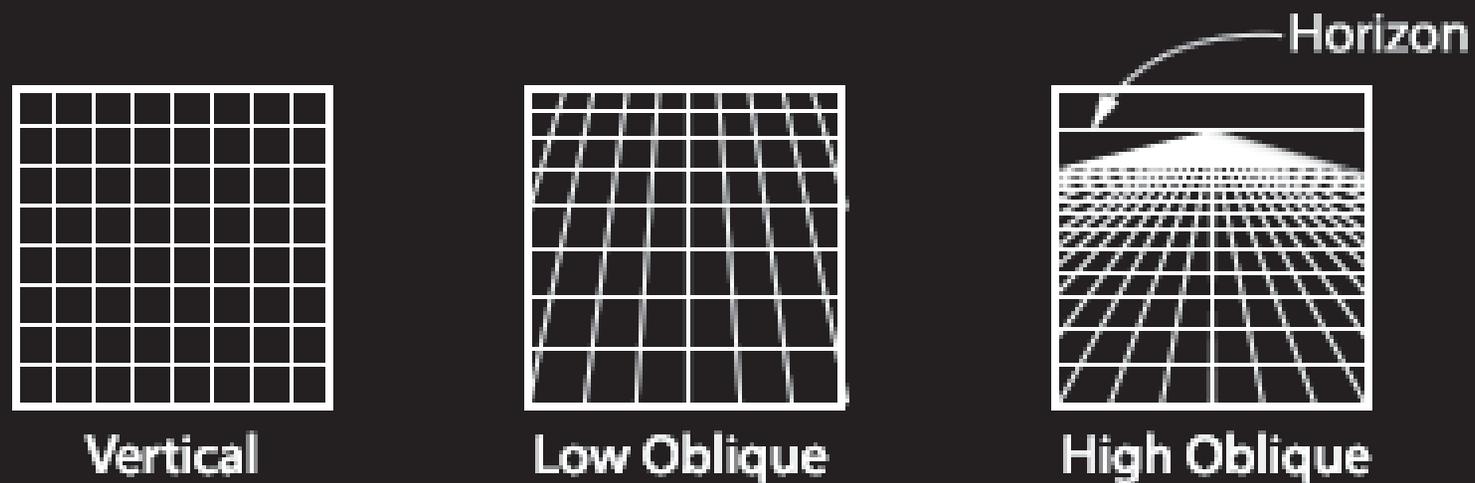
## ***Two types:***

a) **Low oblique:** The horizon does not show in the picture and the optical axis is generally **less than  $30^{\circ}$**  from the vertical.

b) **High oblique:** Horizon is seen in the AP and optical axis has an angle of  **$60^{\circ}$**  with the vertical.



Camera orientation for various types of aerial photographs



How a grid of section lines appears on various types of photos.

# 1- Classification of AP according to *direction of exposure*



Vertical AP



High oblique AP



Low oblique AP

# 1- Classification of AP

according to *Sensor & film*

**1. Panchromatic** (black and white) photography: is useful in recording the visible light in the range of 350-750milimicron of the EMR. This film produces 'normal' picture i.e. dark object appears dark and light object appears light.

**2. Infrared** (black and white) photography: The infrared film is sensitive to visible light as well as near infrared radiation (350-900 mm).

**3. Color** photography: This film also called true color film registers all the colors that are visible to human eyes (400-700mm).

# 1- Classification of AP

according to *Sensor & film  
emulation*

**4. False color** photography: Three layered film is sensitive to green, red and near-infrared radiation instead of usual blue, green and red radiation...Used in differencing manmade and natural object, healthy and diseased trees, between deciduous and evergreen trees.

**5 Multiband or multispectral** photography: Photographs the same area simultaneously with several films using various filters.

# 1- Classification of AP

according to *Sensor & film  
emulation*



**panchromatic**



**true color**



**false-color infrared**

With an airborne digital camera, images can be captured simultaneously in grayscale (also called panchromatic), true color (RGB), and false-color infrared (CIR).

SOURCE: Fugro EarthData.

# 1- Classification of AP according to *Scale*

1. *Very large scale* > larger than 1:10,000  
(for detailed studies e.g. logging planning, damage survey).

2. *Large scale* > 1:10,000-1:20,000

3. *Medium scale* > 1:20,000-1:40,000

(e.g. for inventory and forest cover mapping, plantation site selection)

# 1- Classification of AP according to *Scale*

4. Small scale > 1:40,000-1:70,000

5. Very small scale > 1:70,000-

1:100,000 (for nationwide survey,

reconnaissance survey)

# 1- Classification of AP according to *focal length ( f )*

1. Narrow-angle ( $f = 12$  inches)
2. Normal-angle ( $f = 8.25$  inches)
3. Wide-angle ( $f = 6$  inches)
4. Super-wide-angle ( $f = 3.5$  inches)

Classification	Focal Length	Angle of Coverage
Narrow angle	12 in. = 304.8 mm	Less than 60°
Normal angle	8¼ in. = 209.5 mm	60° to 75 °
Wide angle	6 in. = 152.4 mm	75° to 100°
Super-wide angle	3½ in. = 88.9 mm	More than 100°

**Figure 2.4.** Relationship between focal length and the angle of coverage. As the focal length increases, the angle of coverage decreases.



*Thank you*

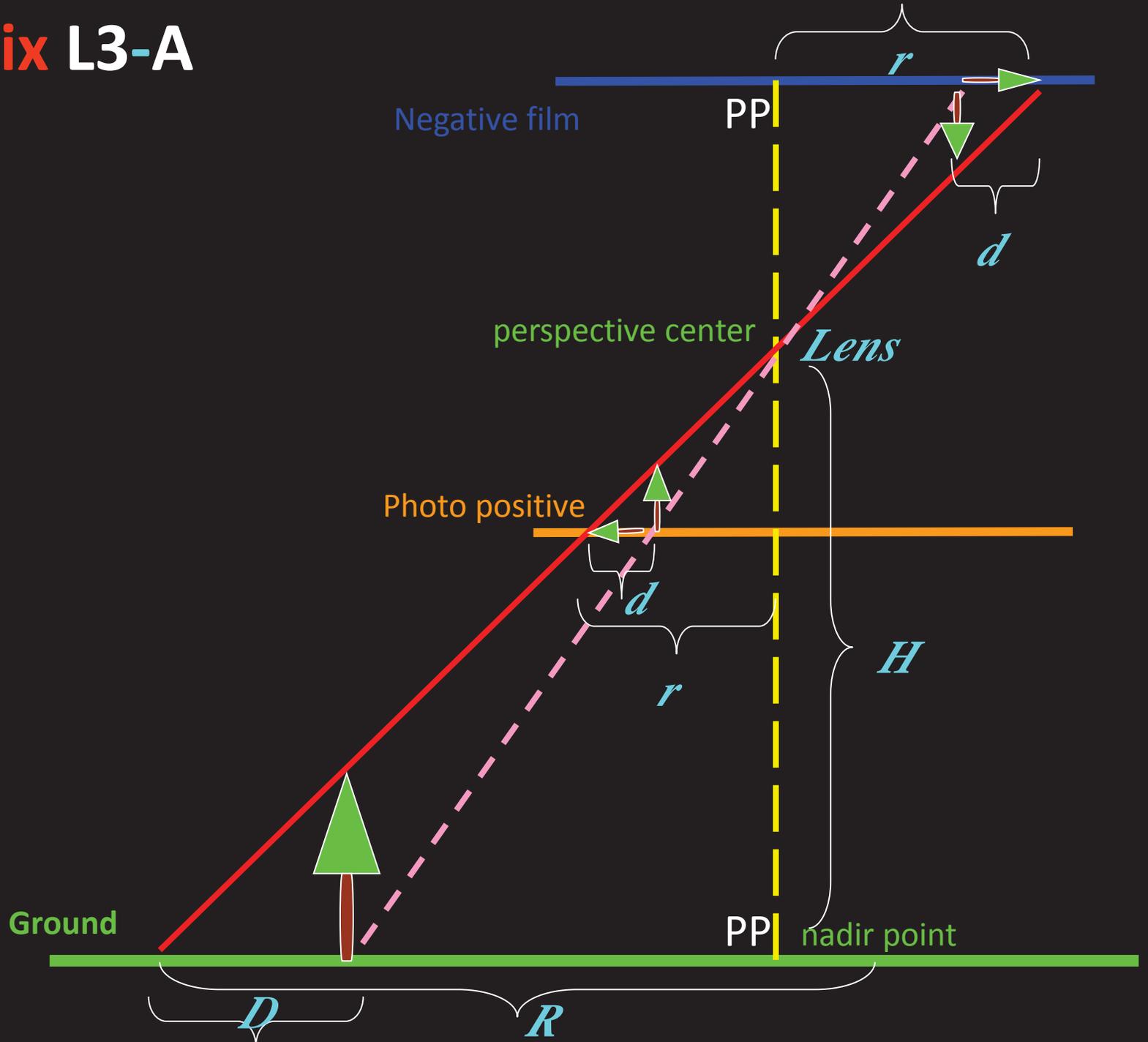
Any Questions ?



**END**

**of Lecture**

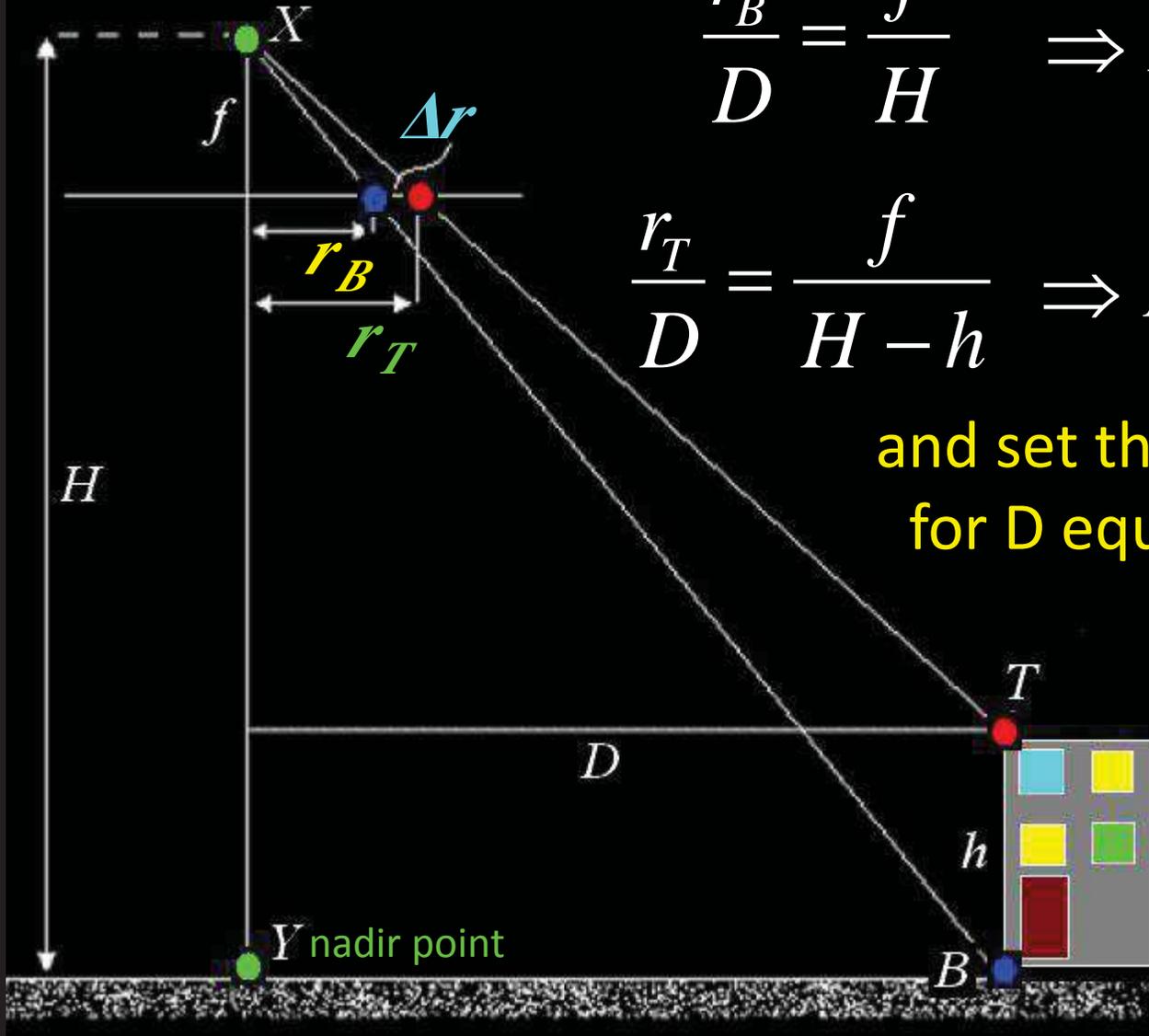
# Appendix L3-A



# Appendix L3-A

We may write two expressions for distance  $D$  in this figure, in terms of radial image distances

perspective center



$$\frac{r_B}{D} = \frac{f}{H} \Rightarrow D = \frac{H \times r_B}{f}$$

$$\frac{r_T}{D} = \frac{f}{H - h} \Rightarrow D = \frac{r_T \times (H - h)}{f}$$

and set the two expressions for  $D$  equal to each other,

## Appendix L3-A

$$\frac{H \times r_B}{f} = \frac{r_T \times (H - h)}{f}$$

$$\Rightarrow H \times r_T - h \times r_T - H \times r_B = 0$$

$$\Rightarrow H \times (r_T - r_B) = h \times r_T$$

$$\Rightarrow \Delta r = d = \frac{h \times r}{H}$$

## Appendix L3-B

# Some Key Terms in Aerial Photography

(compiled by Hugh Millward from  
various sources)

- **Air-base (AB)** (a.k.a. **Camera Base**): Ground distance between centers (PPs) of adjacent photos along a flight line.
- **Angle of Coverage**: the angle of the cone of rays passing the front of a camera lens. Normal angle =  $60^\circ - 75^\circ$ , Wide-angle = over  $75^\circ$
- **Average Photo-base (PB)**: For adjacent air photos, the average of the distance between the PP and CPP on each photo
- **Camera-base (CB)** (a.k.a. **Air-base**): Ground distance between centres (PPs) of adjacent photos along a flight line
- **Conjugate Principal Point (CPP)** (a.k.a. **Corresponding P.P.**): The location of a Principal Point from one photo on an adjacent photo along the flight line
- **Control Point**: A reference point precisely located on both the ground and the photo (ground control point) on both a map and the photo (map control point), or on two adjacent photos (photo control point)
- **Controlled Mosaic**: A series of overlapping air photos that have been rectified and aligned with ground control points, to allow planimetrically-correct distance measurements
- **Crab**: Rotation of the camera (and aircraft) relative to the flightline
- **Drift**: Lateral shift of the aircraft from the planned flightline
- **Eye-base (EB)**: Interpupillary distance, normally  $6.4 (\pm 0.4)$  cm.
- **Fiducial Marks**: Marks built into aerial cameras which appear on the sides or in the corners of the photo (or both), and which are used to determine the precise location of the principal point.

- **Focal length:** Distance from the optical centre of the lens to the focal plane, when the camera is focussed at infinity.
- **Forward Overlap:** (a.k.a. **Endlap**): The amount of overlap between successive photos in a flight line to allow for stereo viewing (usually 60 – 70%)
- **High Oblique photo:** An air photo which shows the horizon line (usually having high vertical tilt, of 60° or more)
- **Isocentre:** The point on an air photo which lies halfway between the Principal Point and the Nadir (Tilt-displacement radiates from this)
- **Low Oblique Photo:** An air photo tilted from the vertical, but not enough for the horizon to be visible (usually having vertical tilt of 3° - 60°)
- **Mosaic:** A series of overlapping air photos
- **Nadir:** That point on the ground vertically beneath the camera lens (or aircraft), or the point on the photo which corresponds to it. (Topographic Displacement radiates from this)
- **Orthophoto:** A vertical air photo which has been rectified to remove parallax
- **Parallax:** The apparent displacement of the position of an object, with respect to a reference point, caused by a shift in the point of observation.
- **Photo-base (PB):** On a single air photo, the distance between the photo's principal point and the CPP of an adjacent photo (see Average PB)

- **Principal Point (PP):** The geometric centre of an aerial photograph, located at the intersection of lines drawn between the fiducial marks (i.e., at the intersection of the x and y axes). (Lens distortion radiates from this)
- **Radial Line Triangulation (RLT):** The production of planimetrically-correct (i.e. uniform scale) maps from two or more adjacent vertical air photos, using the techniques of resection and intersection.
- **Rectification:** The process of converting a vertical air photo to remove displacements caused by tilt or topography (i.e., to remove parallax).
- **Sidelap (a.k.a. Lateral overlap):** The amount of overlap between air photos in adjacent flight lines (usually 20 – 30%)
- **Stereogram:** A stereopair or stereo-triplet mounted for proper stereovision (conjugate points  $5.7 \pm 0.3$  cm apart)
- **Tilt:** Rotation of the camera away from the vertical, about the x- or y-axis
- **Tilt Displacement:** Changes in position caused by scale variations related to the tilt of the camera, about either the x-or y-axis
- **Topographic Displacement (a.k.a. Relief Displacement, Radial Displacement, or Planimetric Shift due to Elevation):** Changes in position caused by scale variations related to differences in elevation or height.
- **Uncontrolled Mosaic:** A series of overlapping air photos which have not been aligned to ground control points
- **Vertical Air Photo:** An air photo with less than  $3^\circ$  of vertical tilt
- **X-axis:** For a single photo, the line through the photo showing direction of flight at the centre of the photo (i.e., nose-to-tail axis)
- **Y-axis:** the line at right-angles to the x-axis (i.e., wingtip-to-wingtip axis)

# Appendix 3-1

How is the Principal Point determined from aerial photographs ?

Principal Point is determined by the photographs Fiducial Marks

What aspects of photographic geometry cause differences between Nadir and the Principal Point?

- Topographic displacement affects often increase from Nadir
- Tilt displacement affects increase away from the Isocenter of the photograph

Why are most aerial photographs taken from a tilted angle opposed to a vertical position?

What two aspects of aerial photography are used to define the scale of a photograph, and how are they calculated?

Scale is determined by the

1- focal length of a lens/

2- photograph height.

Focal Length = 4cm , Flying Height = 100000cm ,  $4/100000 = \text{Scale } 1/25,000$

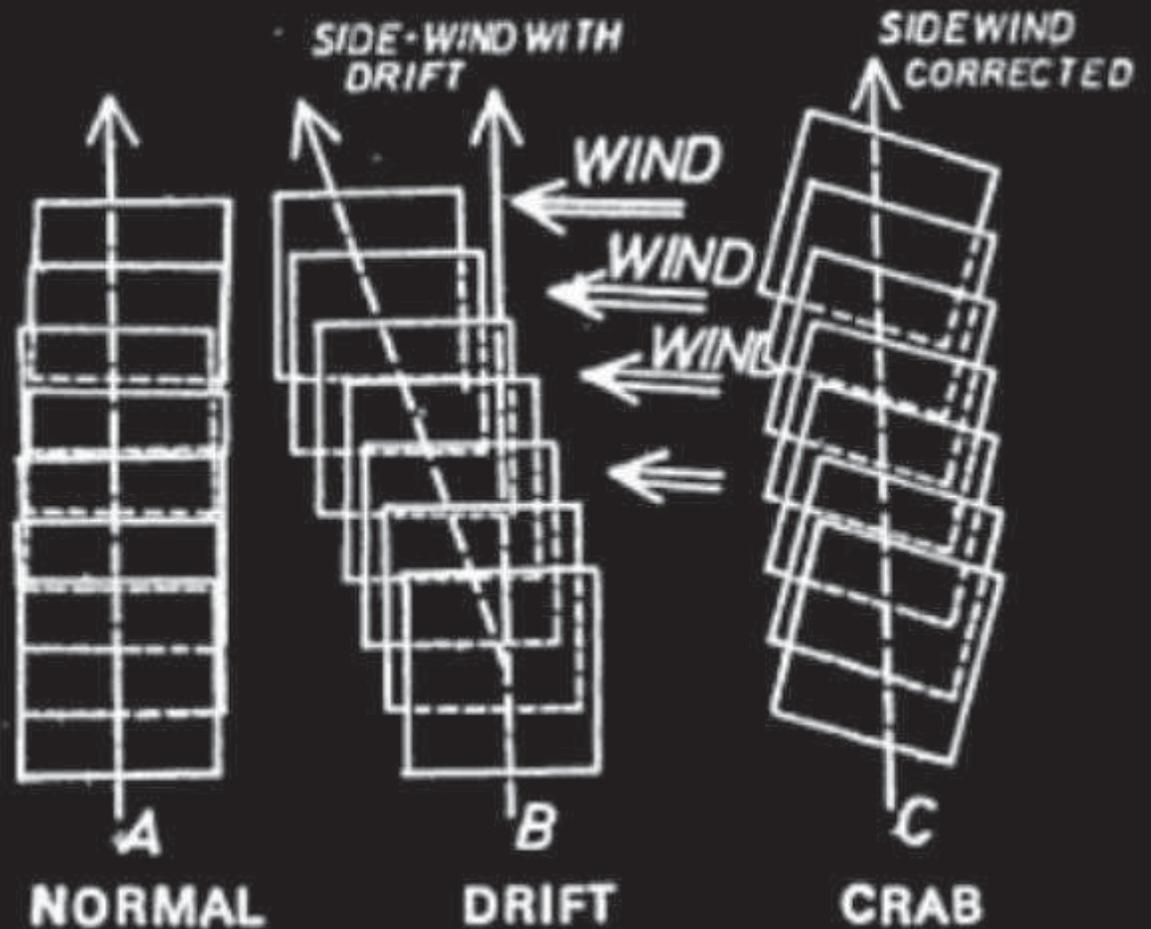
# Appendix 3-3

## Crab

Opposite line of photographs are not parallel to flight line is known as crab of photograph.

## Drift

When aircraft is swayed away from its preplanned flight line then it is known as drift.





بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

# *Aerial Photography & Photogrammetry*

## Lecture 3-2

# Geometry of a Vertical Aerial Photograph

*Lecturer: Faisel Ghazi Mohammed*

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[faiselgm73@gmail.com](mailto:faiselgm73@gmail.com)

*2017-2019*

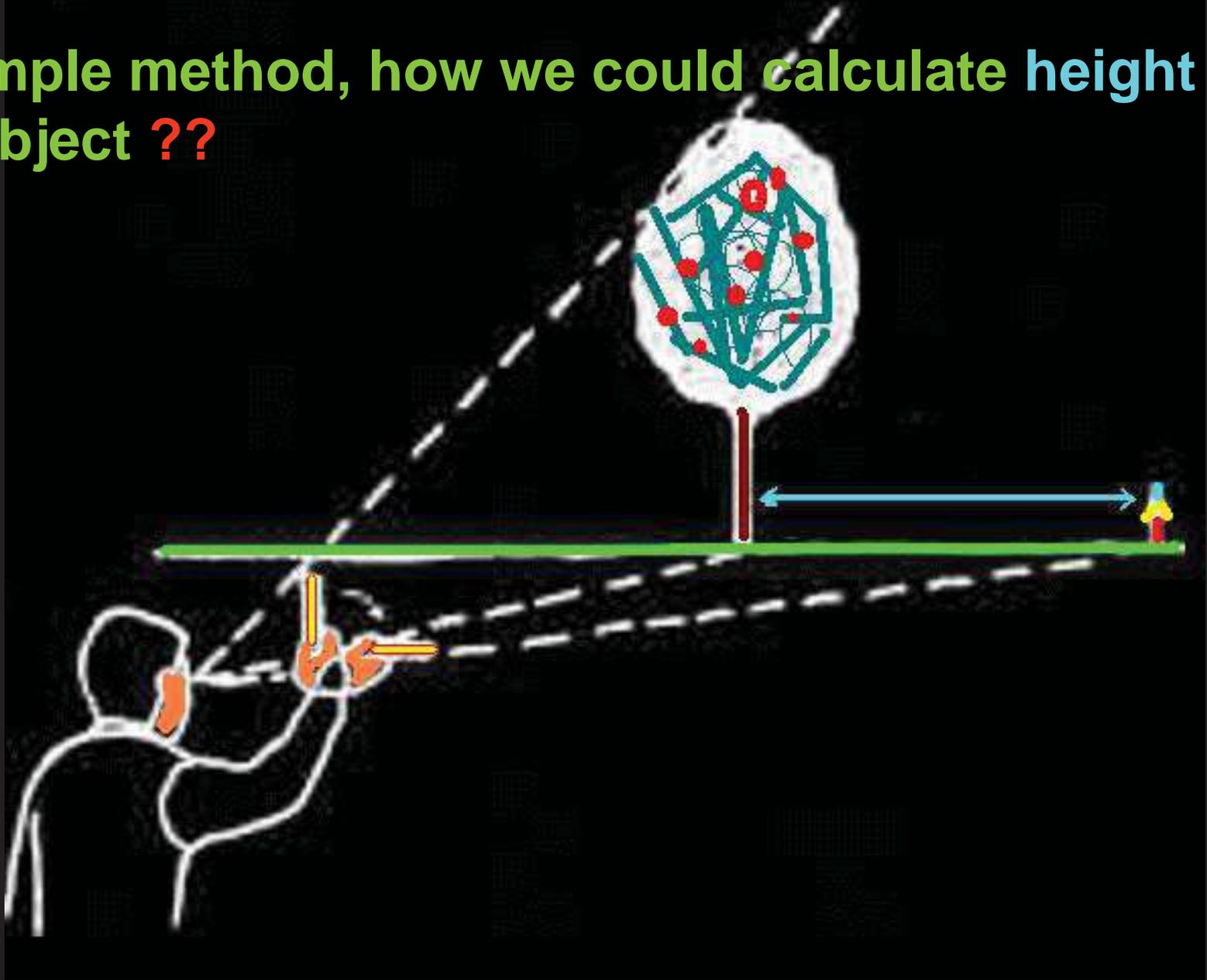
*All rights reserved*

2-

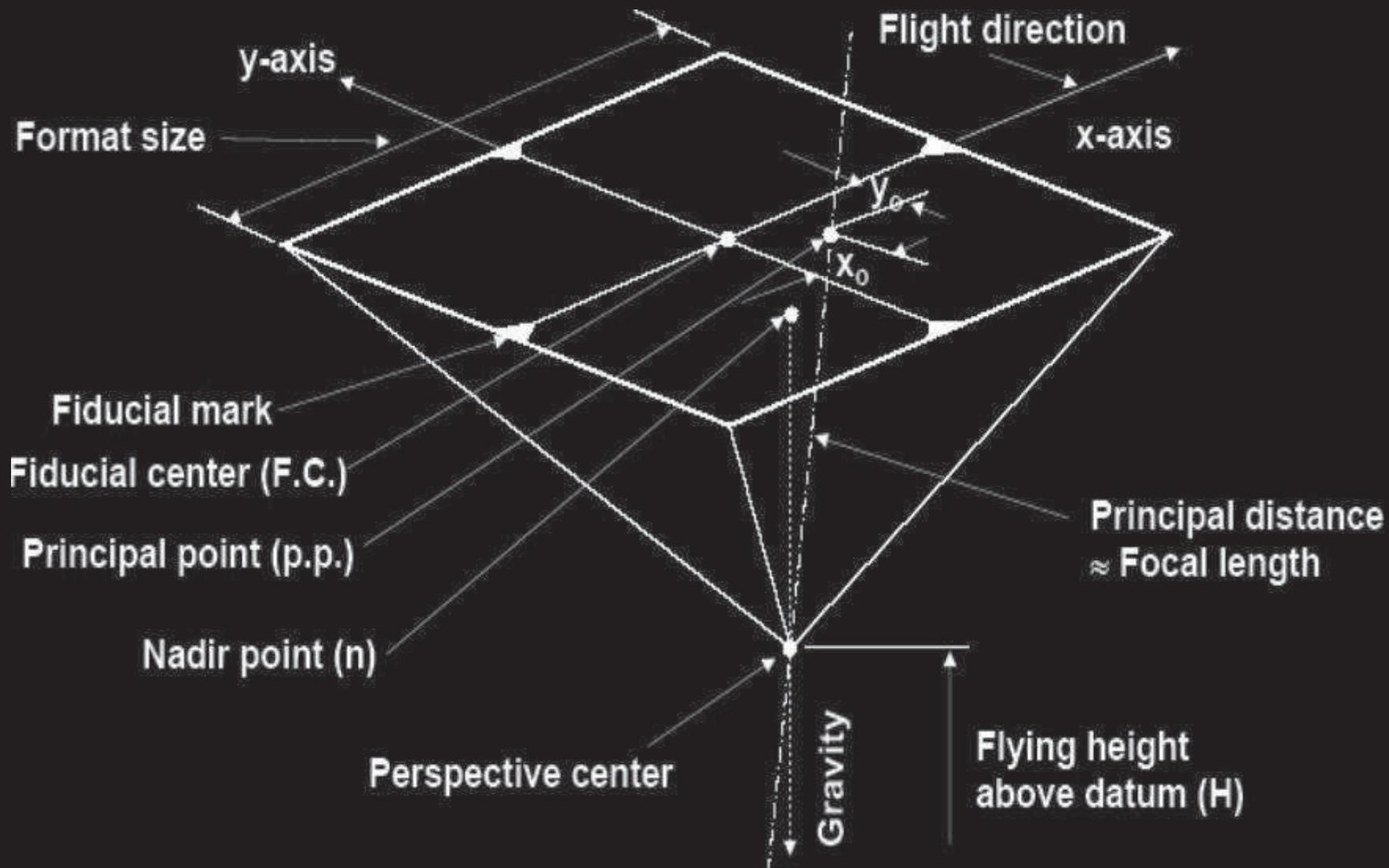
# The Coordinate Axes

## 2- The Coordinate Axes

In a Simple method, how we could calculate height of an object ??



## 2- The Coordinate Axes



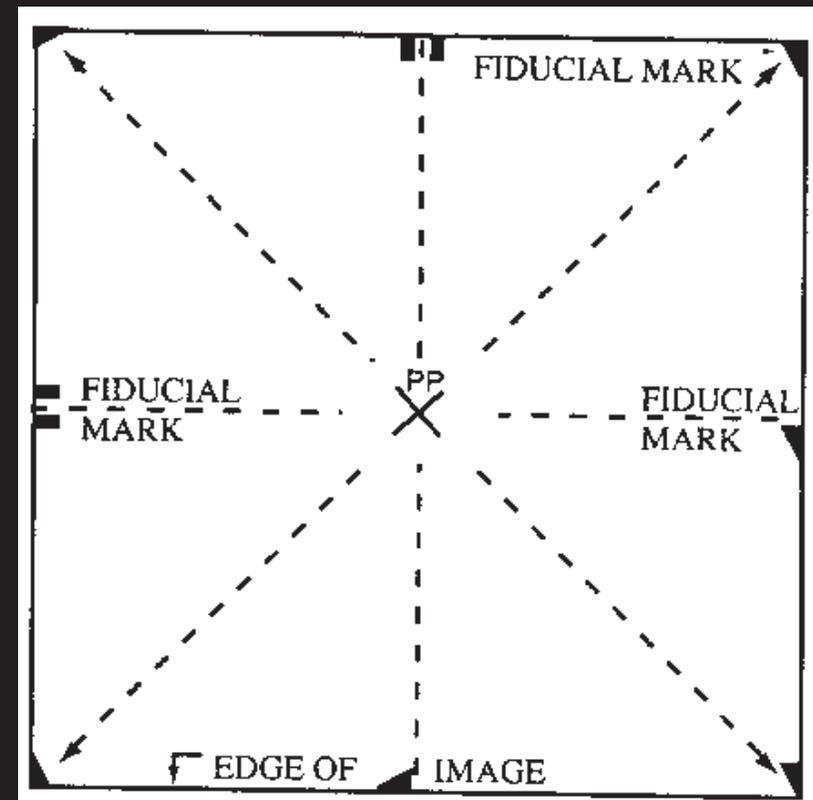
## 2- The Coordinate Axes

### Fiducial marks

Fiducial marks are optically projected fine crosses, dots, half arrows, or other geometric figures located either on the sides of the photo.

Use:

Fiducial marks are reference marks that define the coordinate axes and the geometric center of a single AP.



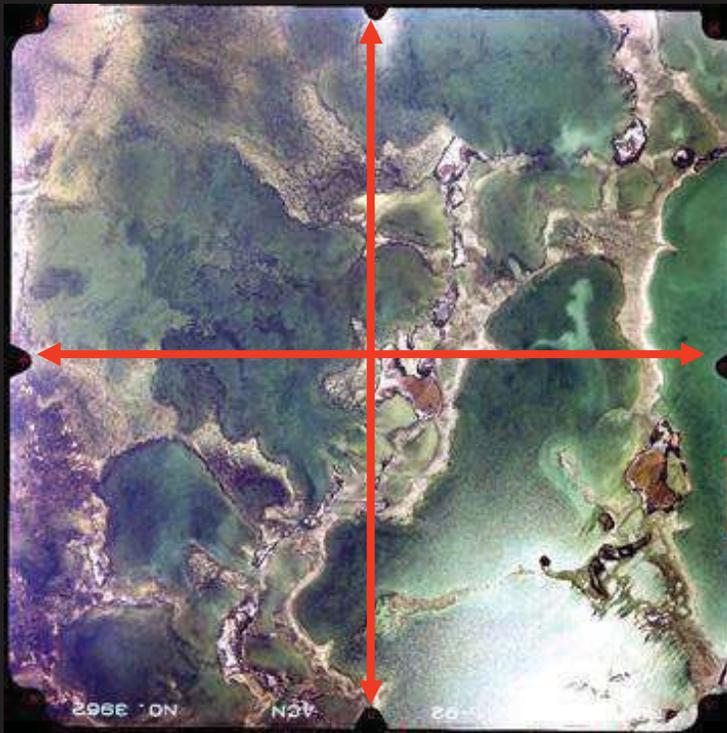
## 2- The Coordinate Axes

Fiducial  
marks



## 2- The Coordinate Axes

Fiducial  
axes



## 2- The Coordinate Axes

Principal  
point

Marginal  
information

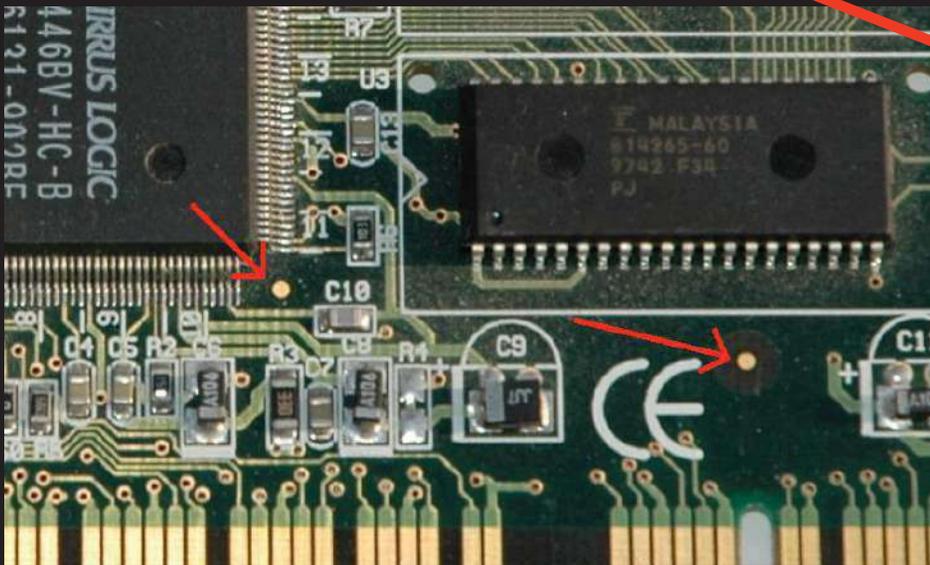




Fiducial mark



Rulers make good fiducial markers

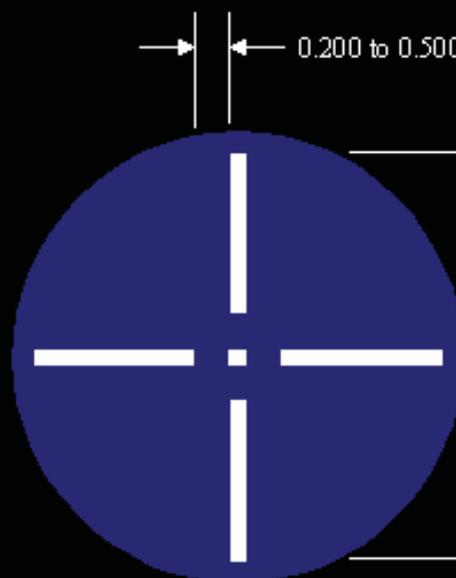


## 2- The Coordinate Axes

Examples of  
Acceptable  
Forms of  
Fiducial  
Marks

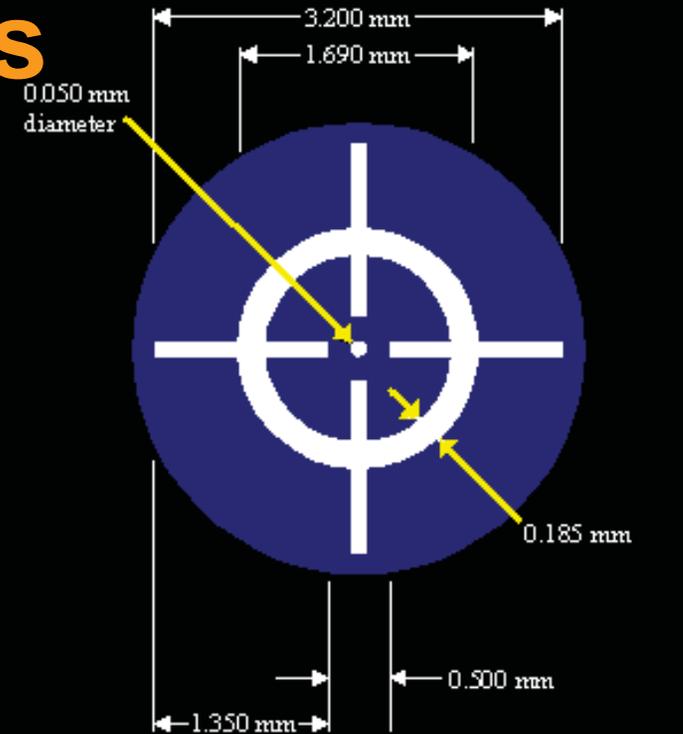


0.050 to 0.120 mm



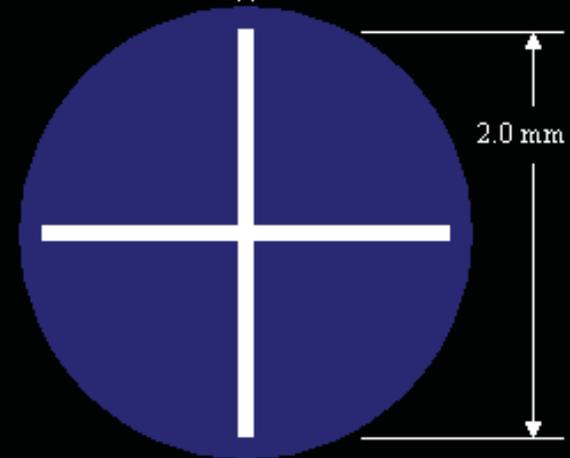
0.200 to 0.500 mm

2.0 to  
4.5 mm



0.050 mm  
diameter

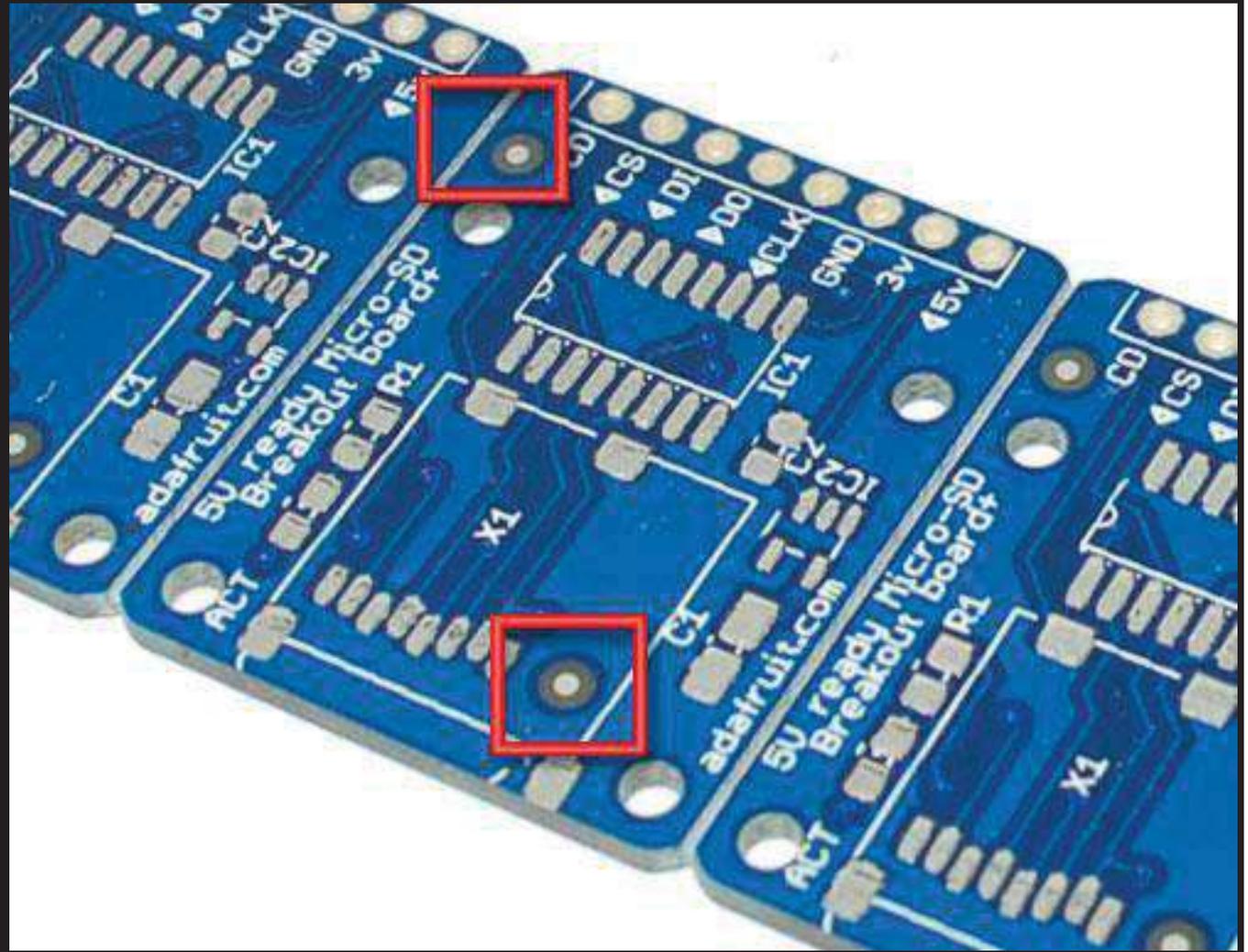
0.050 to 0.100 mm



2.0 mm

Fiducials are little target registration marks that are printed on **PCBs**, they are placed on the top copper layer (and bottom if you're doing 2-layers) and allow the vision system of the pick and place to recognize where the PCB is at.

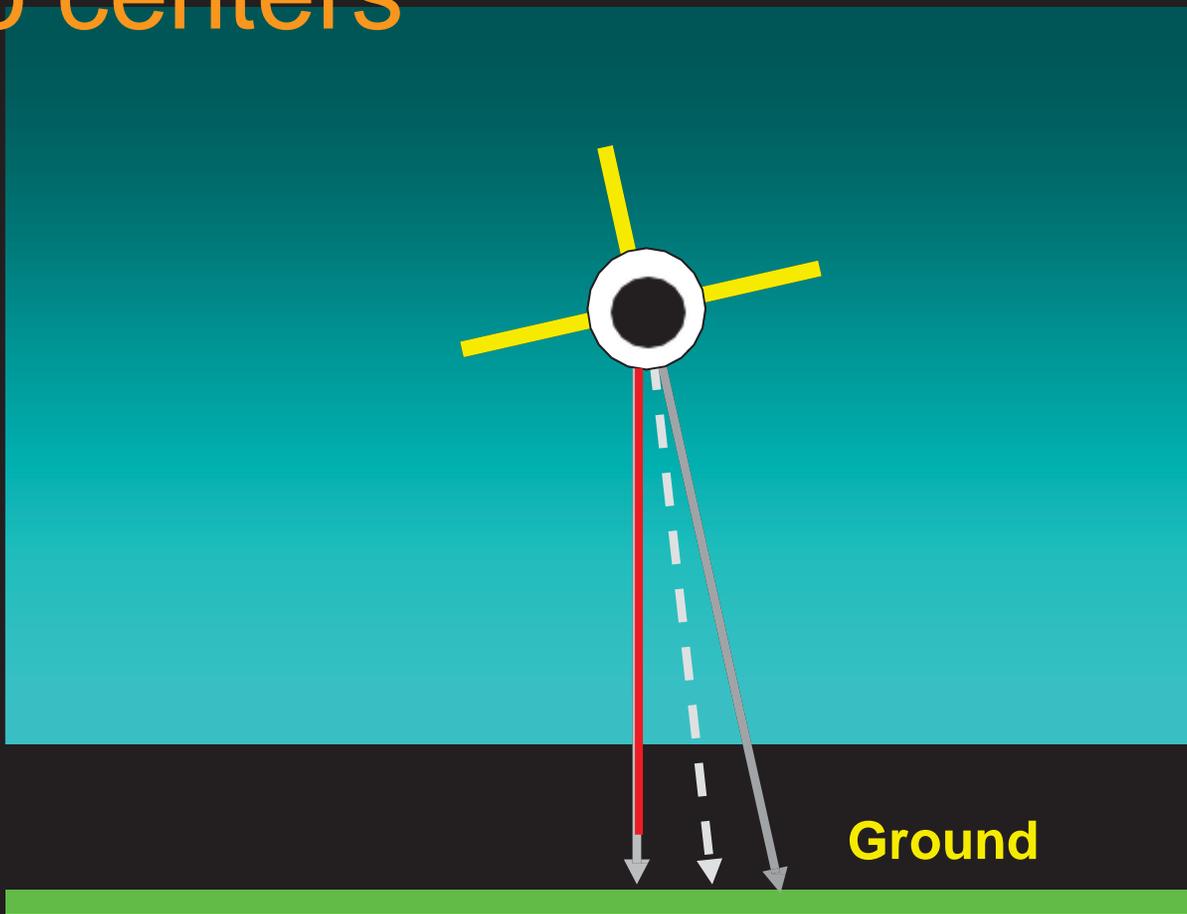
They are not placed on the mask or silk because they are not as precisely aligned to the parts as the copper itself.



3-

Three photo  
centers

# 3- Three photo centers



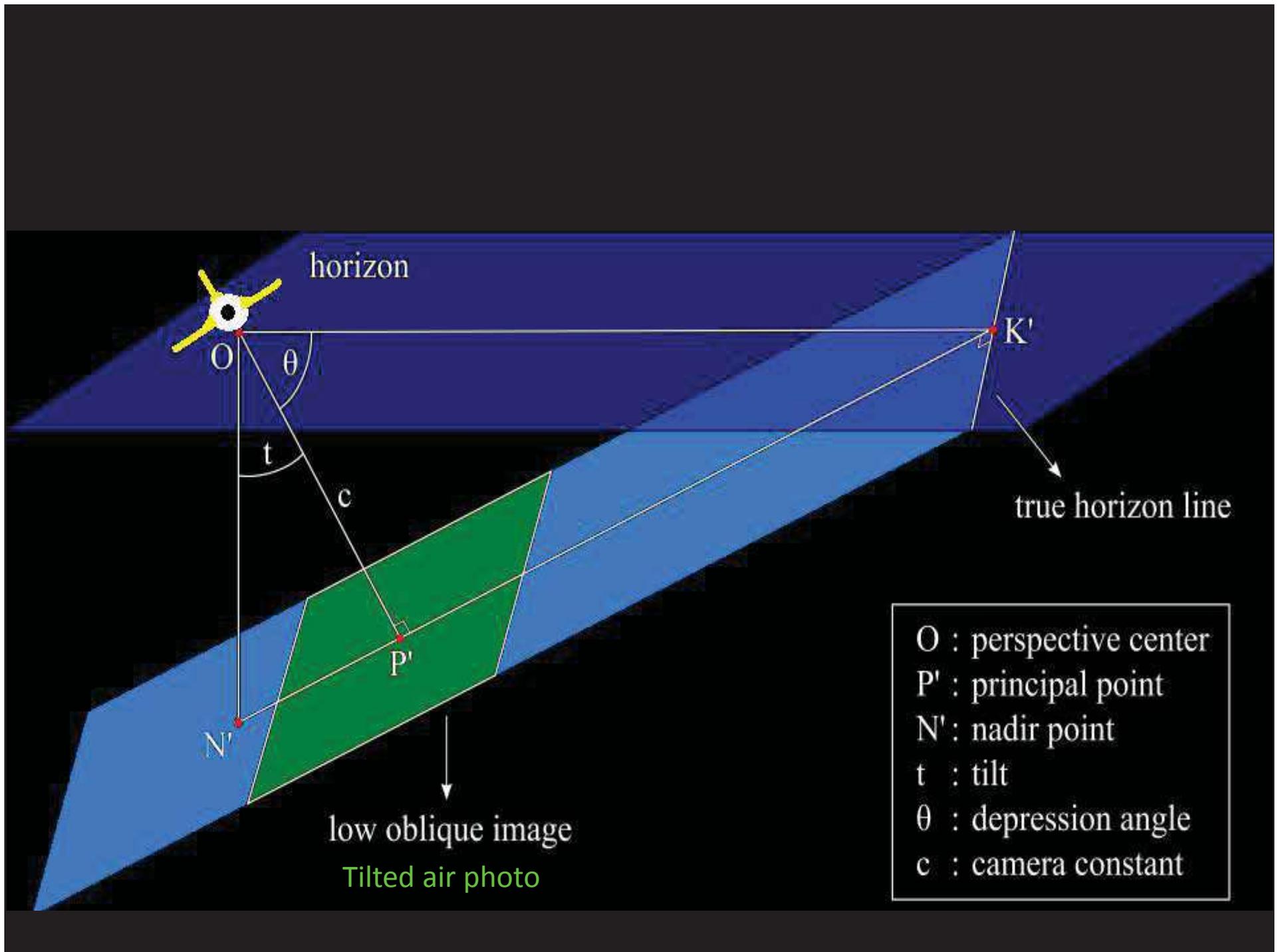
Nadir  
Isocenter  
PPoint

## 3- Three photo centers

1. **Principal Point** – geometric center of the photograph; intersection of the line normal to the image plane through the PC
2. **Nadir** – point vertically below the camera at the time the photo was taken; intersection of the plumb line through the PC with the image plane
3. **Isocenter** – point halfway between the principal point and nadir; point intersected by the bisector of the angle between plumb line and optical axis

# Principal Line

- Line of maximum tilt
- Line connecting the principal point, isocenter and nadir
- All lines perpendicular to this line are lines of **zero inclination** or zero phototilt :  
→ this means that all points along a perpendicular line have uniform scale





*Thank you*

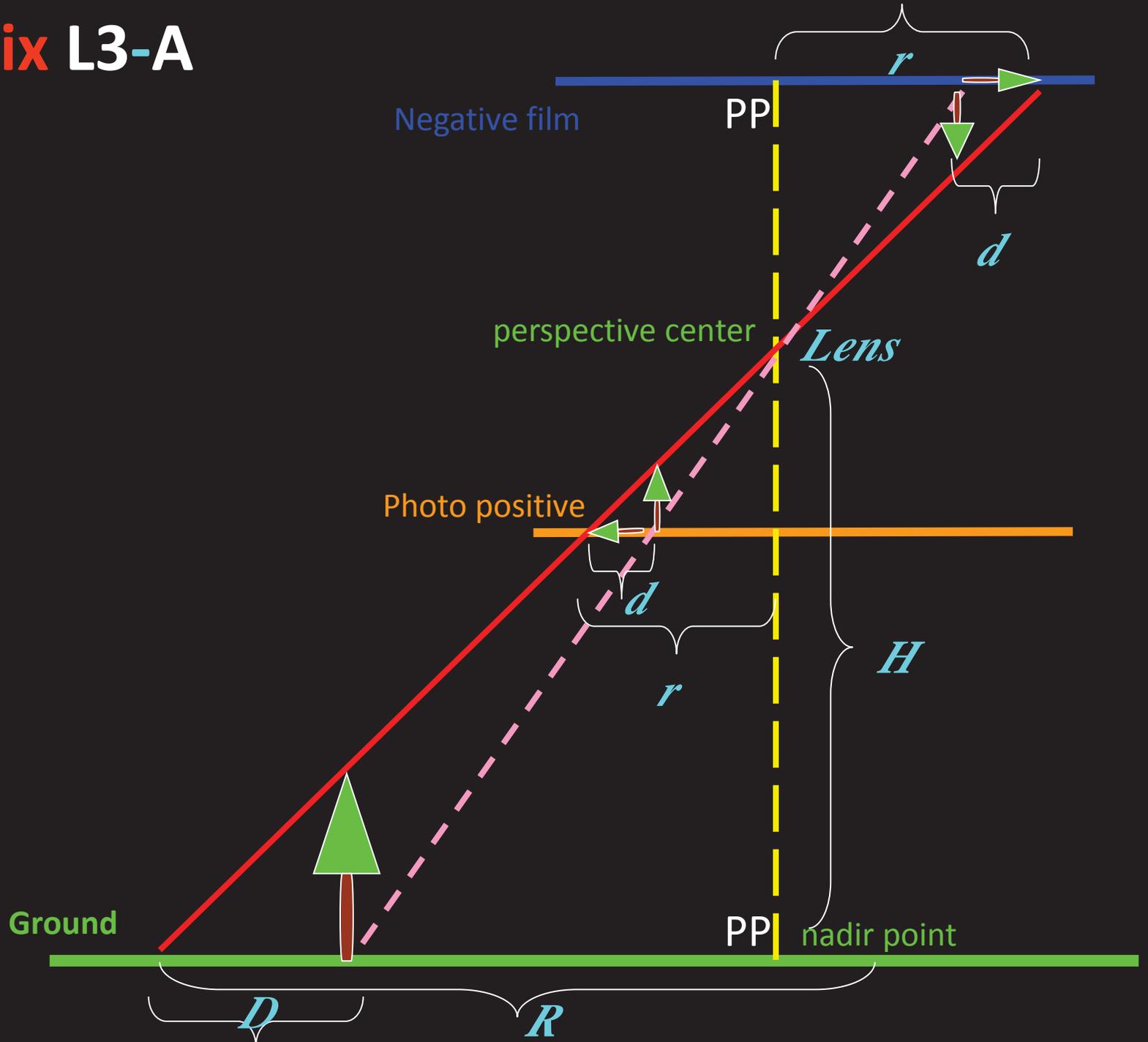
Any Questions ?



**END**

**of Lecture**

# Appendix L3-A





## Appendix L3-A

$$\frac{H \times r_B}{f} = \frac{r_T \times (H - h)}{f}$$

$$\Rightarrow H \times r_T - h \times r_T - H \times r_B = 0$$

$$\Rightarrow H \times (r_T - r_B) = h \times r_T$$

$$\Rightarrow \Delta r = d = \frac{h \times r}{H}$$

## Appendix L3-B

# Some Key Terms in Aerial Photography

(compiled by Hugh Millward from  
various sources)

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- **Uncontrolled Mosaic:** A series of overlapping air photos which have not been aligned to ground control points
- **Vertical Air Photo:** An air photo with less than  $3^\circ$  of vertical tilt
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- **Y-axis:** the line at right-angles to the x-axis (i.e., wingtip-to-wingtip axis)

# Appendix 3-1

How is the Principal Point determined from aerial photographs ?

Principal Point is determined by the photographs Fiducial Marks

What aspects of photographic geometry cause differences between Nadir and the Principal Point?

- Topographic displacement affects often increase from Nadir
- Tilt displacement affects increase away from the Isocenter of the photograph

Why are most aerial photographs taken from a tilted angle opposed to a vertical position?

What two aspects of aerial photography are used to define the scale of a photograph, and how are they calculated?

Scale is determined by the

1- focal length of a lens/

2- photograph height.

Focal Length = 4cm , Flying Height = 100000cm ,  $4/100000 = \text{Scale } 1/25,000$

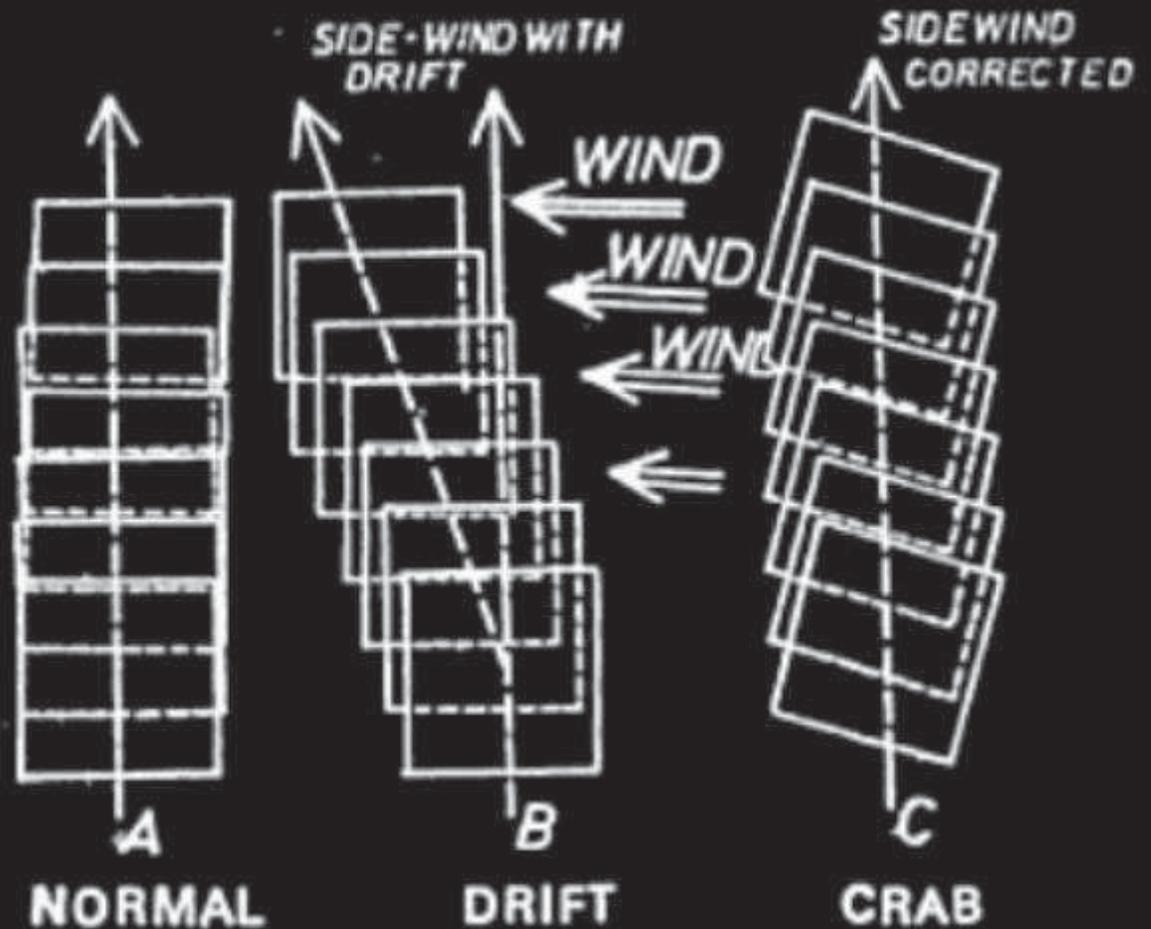
# Appendix 3-3

## Crab

Opposite line of photographs are not parallel to flight line is known as crab of photograph.

## Drift

When aircraft is swayed away from its preplanned flight line then it is known as drift.





بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

# *Aerial Photography & Photogrammetry*

## Lecture 3-3

# Geometry of a Vertical Aerial Photograph

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# 4

## Distortion and Displacement

4-1 Lens Distortion

4-2 Tilt Displacement

4-3 Topographic Displacement

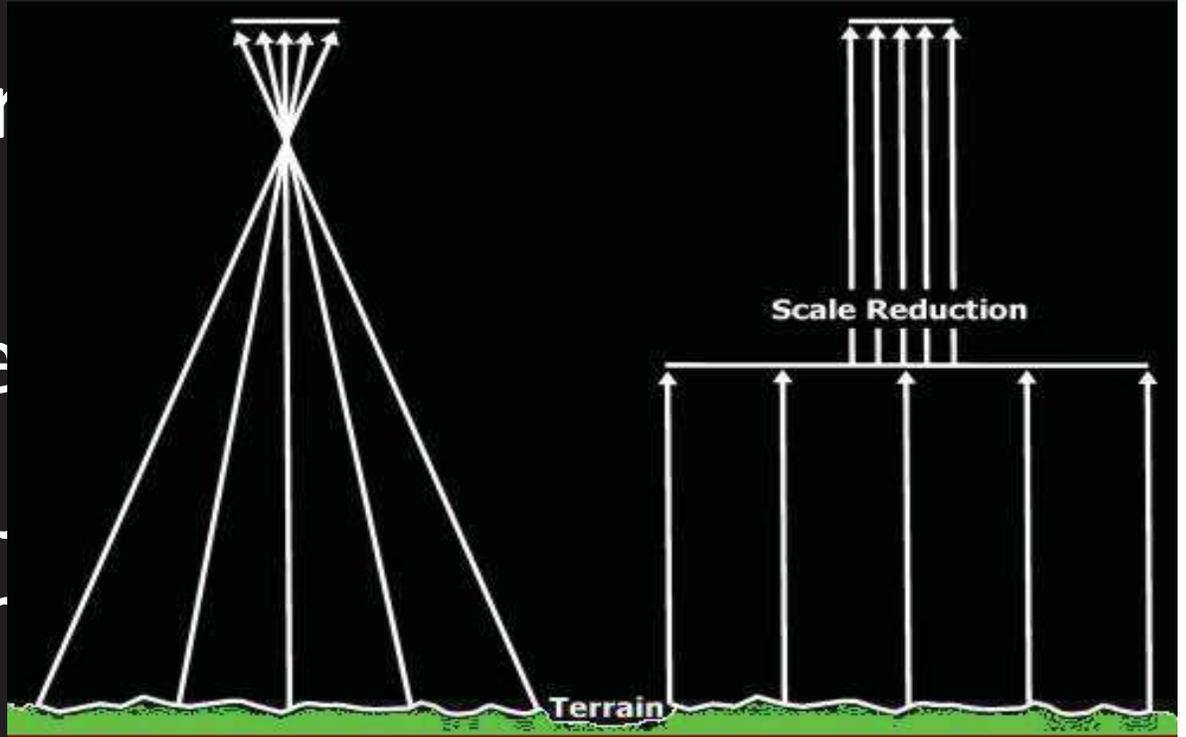
## 4- Distortion and Displacement

- **Distortion** is any shift in the position of an image on a photograph that **alters** the perspective characteristics of the image.

**Displacement** is any shift in the position of an image on a photograph that does **not alter** the perspective characteristics of the photograph. **Makes stereo viewing possible. Also, allows us to measure heights and make topographic maps**

## 4- Distortion and Displacement

- A vertical aerial photograph is a distorted product of an optical projection.
- A photo is the optical product of an aerial camera or central projection (Figure 2.8 in text – page 50).
- Unlike a map on stable base material, an aerial photo is **subject to** distortion and displacement.



## 4- Distortion and Displacement

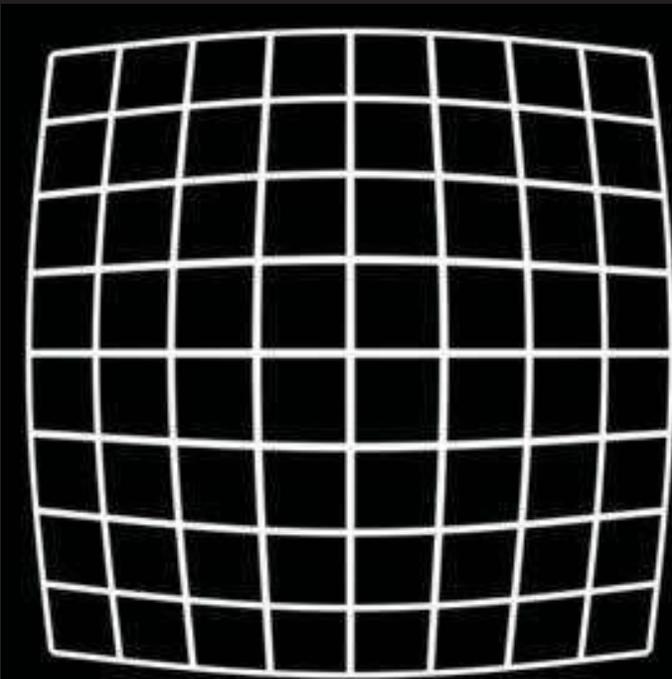
Distortions	Displacement
<ul style="list-style-type: none"><li>1. Film and paper shrinkage</li><li>2. Atmospheric distortions</li><li>3. Image motion</li><li>4. Lens distortion</li></ul> <p>The effects (1,2) are usually negligible in most cases except for precise mapping projects</p>	<ul style="list-style-type: none"><li>1. curvature of the Earth</li><li>2. tilt</li><li>3. topographic relief and height of features</li></ul> <p>The effect (1) are usually negligible in most cases except for precise mapping projects</p>

# 4- Distortion and Displacement

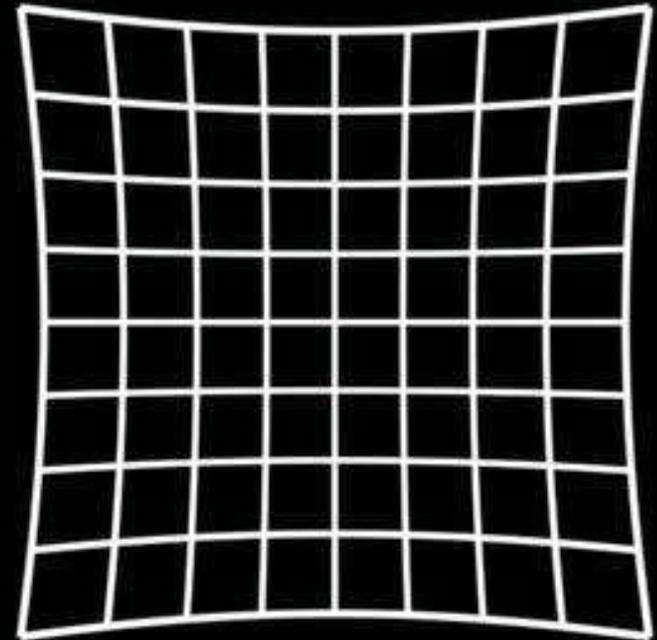
## Lens Distortion

1. Small effects due to the **flaws in the optical** components (lens) of camera systems leading to distortions
2. typically **more serious at the edges** of photo
3. **Radial** from the principal point
4. Makes objects **appear either closer** to, or farther from the principal point than they actually are
5. May be corrected using **calibration curves**
6. Examples: car windows/windshields, carnival mirrors

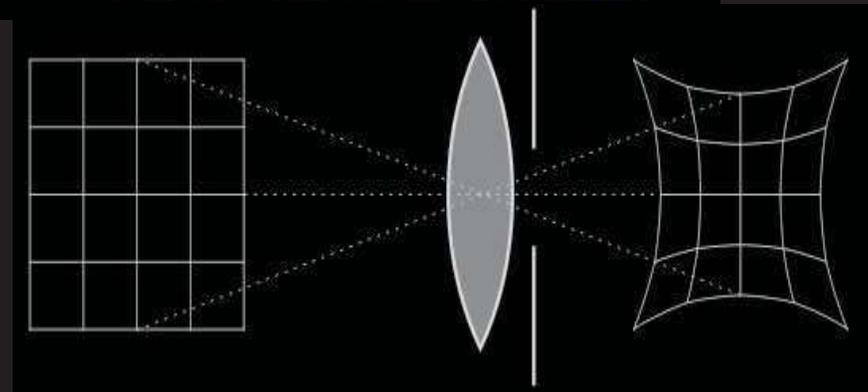
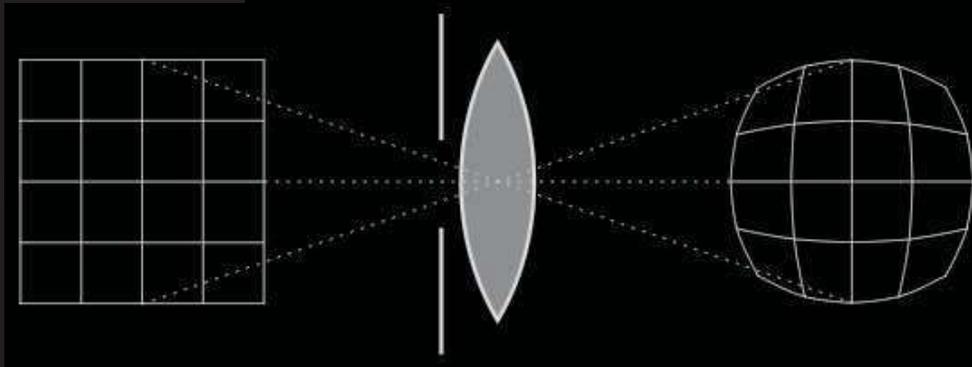
# 4- Distortion and Displacement



Barrel Distortion



Pincushion Distortion



Lens distortion graphic

# 4- Distortion and Displacement

## Lens Distortion



# 4- Distortion and Displacement

## Lens Distortion

Zoomed IN



Zoomed OUT



135mm

70mm

50mm

24mm

12mm

## 4- Distortion and Displacement



Lens distortion – Barrel effect

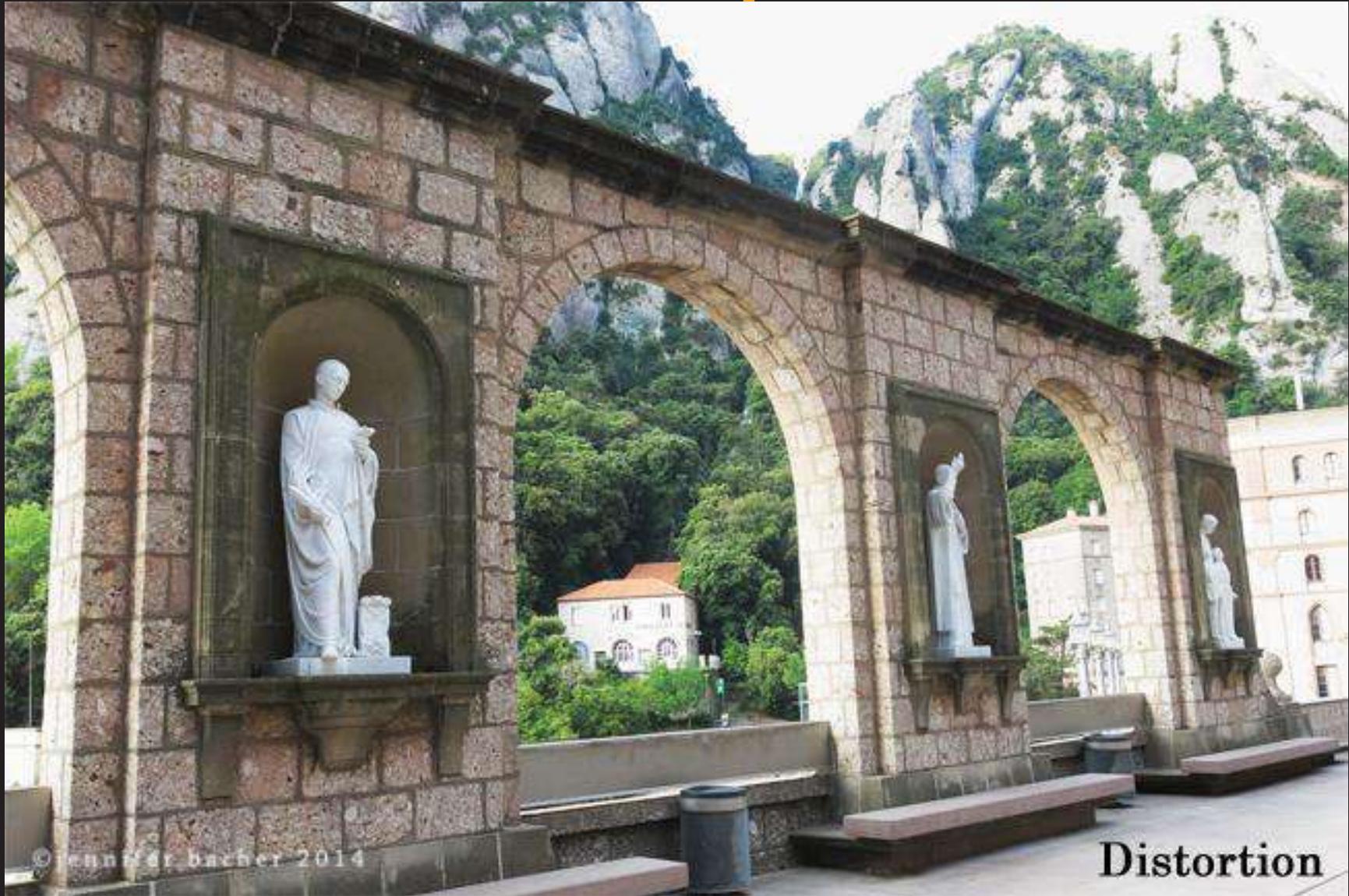
# 4- Distortion and Displacement



# 4- Distortion and Displacement

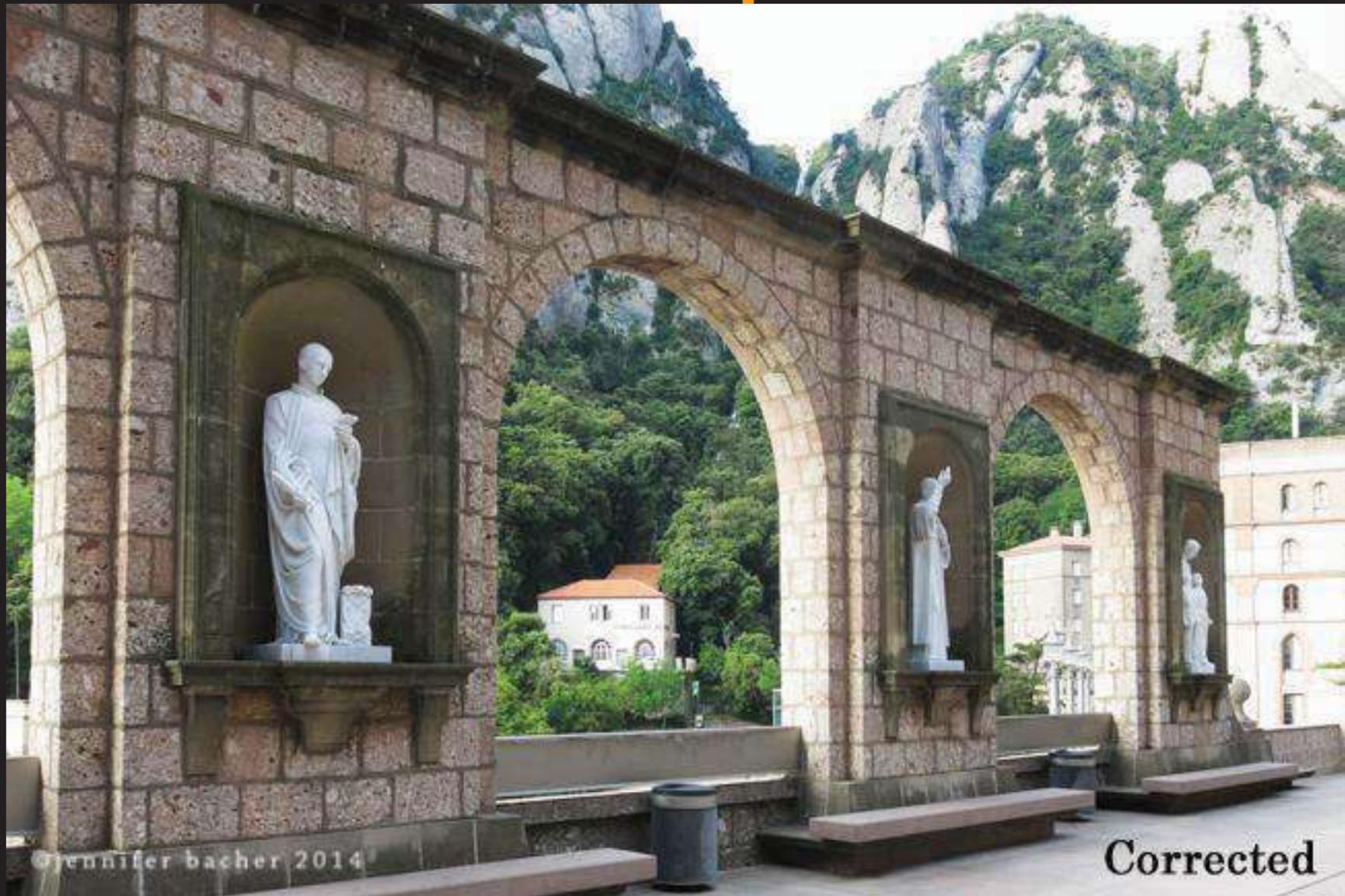


# 4- Distortion and Displacement



Distortion

# 4- Distortion and Displacement



Corrected

© Jennifer Bacher 2014

# 4- Distortion and Displacement

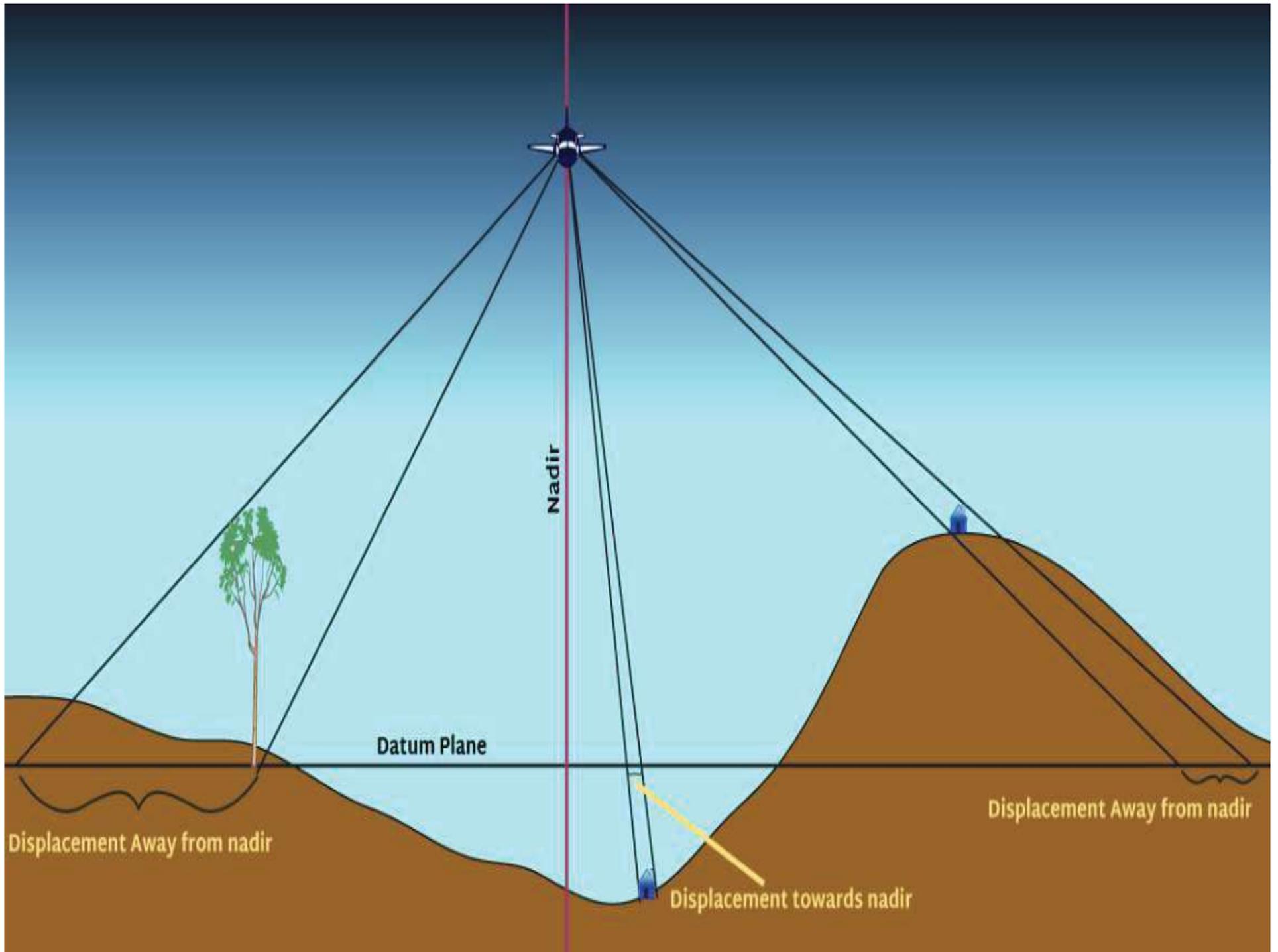
## Lens Distortion



## 4- Distortion and Displacement

### Displacement

- Shift in the location of an object in a photo, which does not change the perspective characteristics of the photo
  - Fiducial distance between an object's image and its true plan position, caused by change in elevation



# 4- Distortion and Displacement

## Types of Displacement

1. **Tilt Displacement** – radial from the isocenter
2. **Relief Displacement** – radial from the nadir
3. **Curvature of the Earth** – negligible effect  
(except for precise mapping projects)

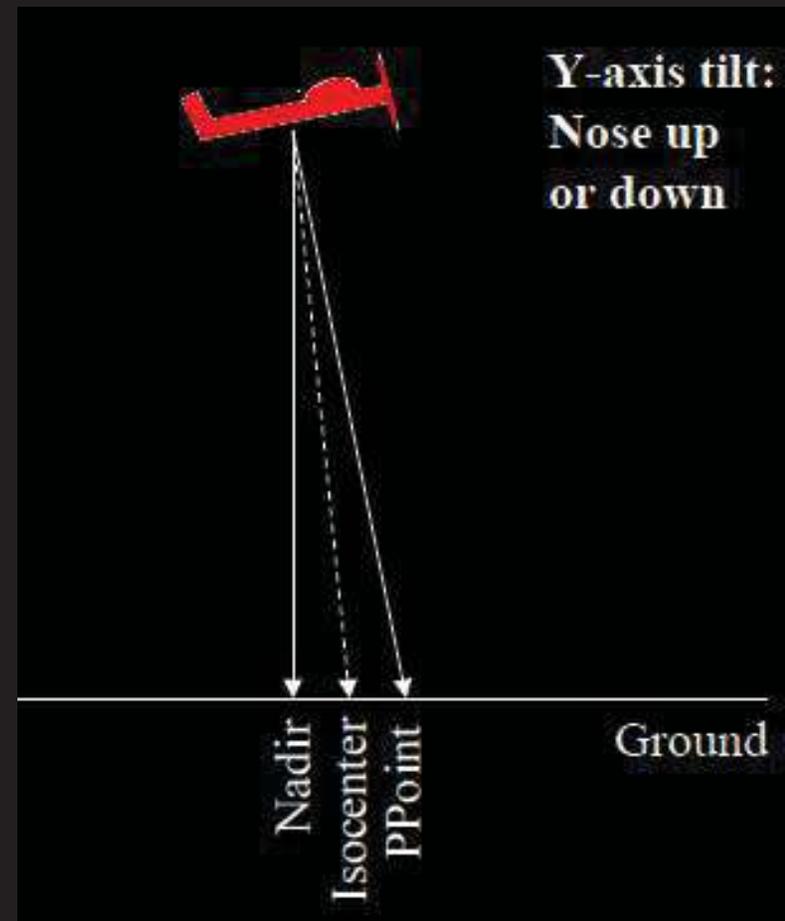
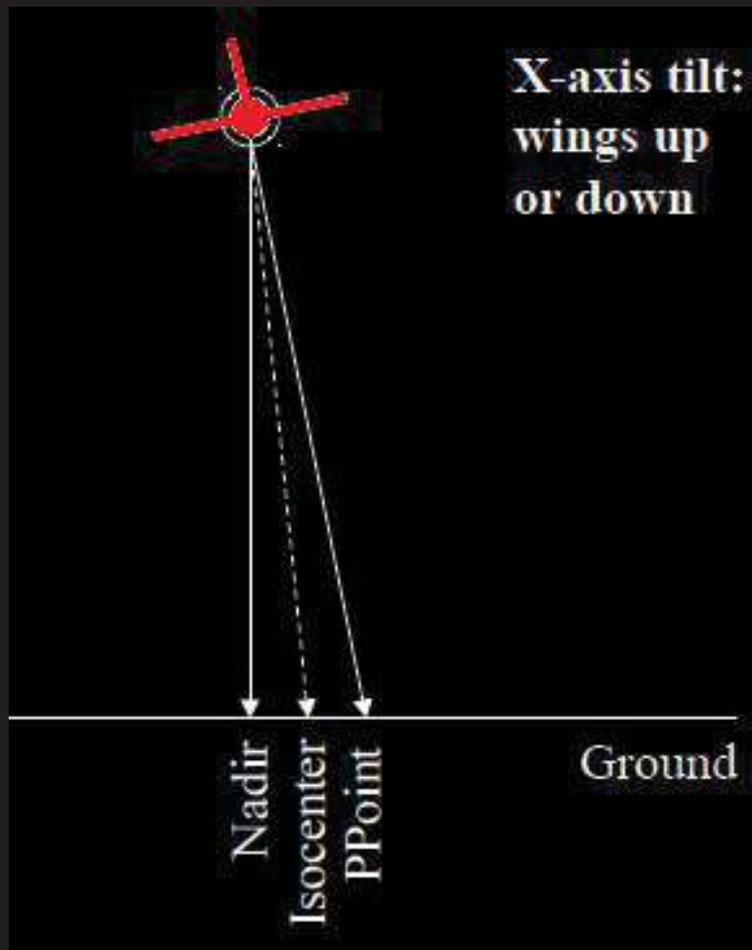
# 4- Distortion and Displacement

## 1- Tilt Displacement

- Radiates from the **isocenter** of a photograph.
- Caused by the **aircraft not** being perfectly **horizontal** at the time of exposure of the film.
- If the **amount of tilt** is known, photographs can be **rectified (expensive)**.
- If we can determine the **direction of the tilt**, in terms of “upper side” of the tilt and the “lower side” of the tilt, we can determine how **landscape features** are being **displaced**.

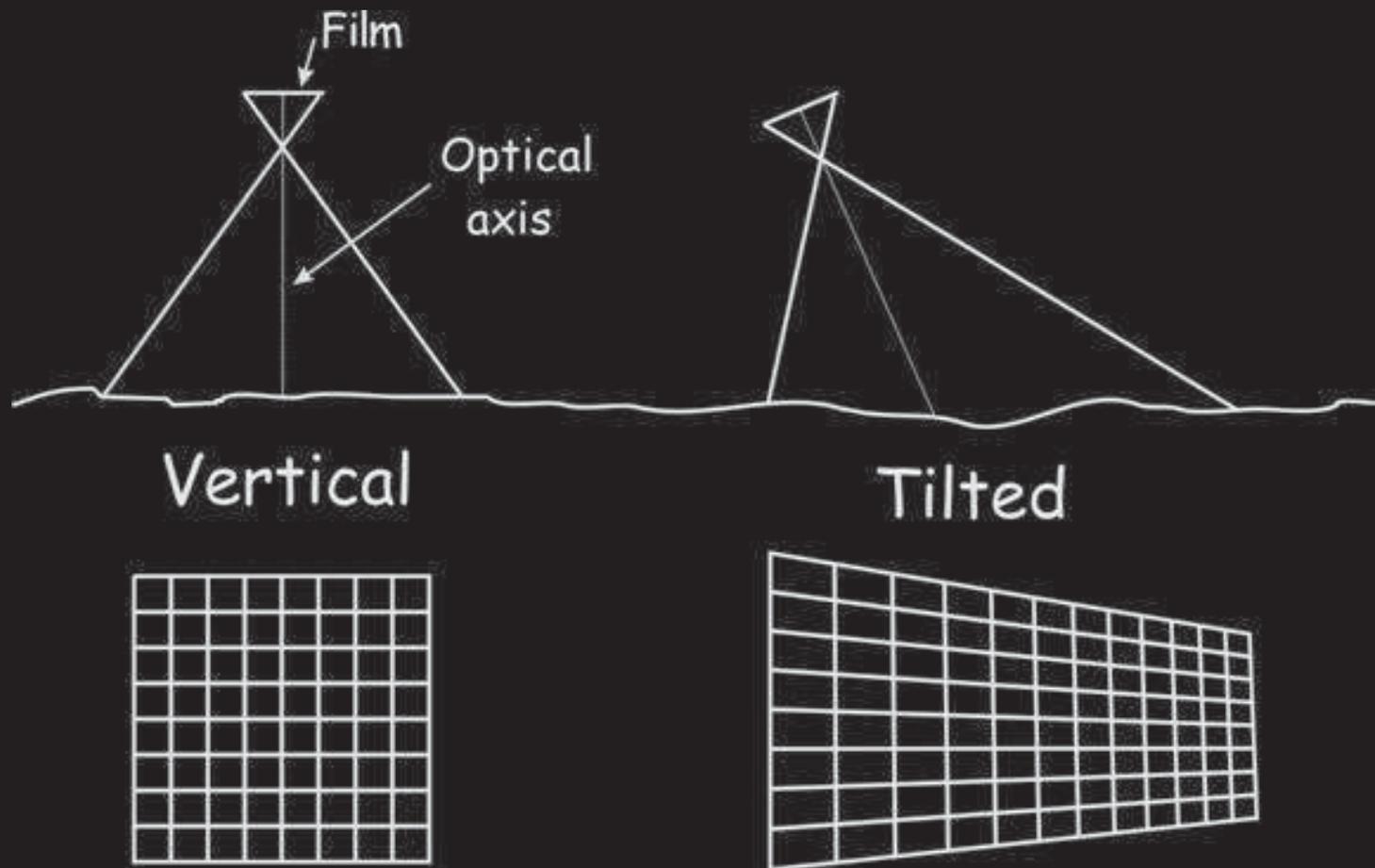
# 4- Distortion and Displacement

## 1- Tilt Displacement



# 4- Distortion and Displacement

## 1- Tilt Displacement



# Terrain

- **Terrain** or **relief** (also **topographical relief**) involves the vertical and horizontal dimensions of land surface.
- The term **bathymetry** is used to describe underwater relief, while hypsometry studies terrain relative to sea level.
- The Latin word *terra* (the root of *terrain*) means "earth"

# Terrain

Relief  
map

of Sierra  
Nevada



# Relief

- Relief (or *local relief*) refers specifically to the quantitative measurement of vertical elevation change in a landscape.
- It is the difference between maximum and minimum elevations within a given area, usually of limited extent.
- The relief of a landscape can change with the size of the area over which it is measured, making the definition of the scale over which it is measured very important. Because it is related to the slope of surfaces within the area of interest and to the gradient of any streams present, the relief of a landscape is a useful metric in the study of the Earth's surface.

# Relief

A shaded and colored image (i.e. terrain is enhanced) of varied terrain from the [Shuttle Radar Topography Mission](#). This shows [elevation model](#) of New Zealand's [Alpine Fault](#) running about 500 km (300 mi) long. The [escarpment](#) is flanked by a vast chain of hills between the [fault](#) and the [mountains](#) of [New Zealand's Southern Alps](#). Northeast is towards the top.

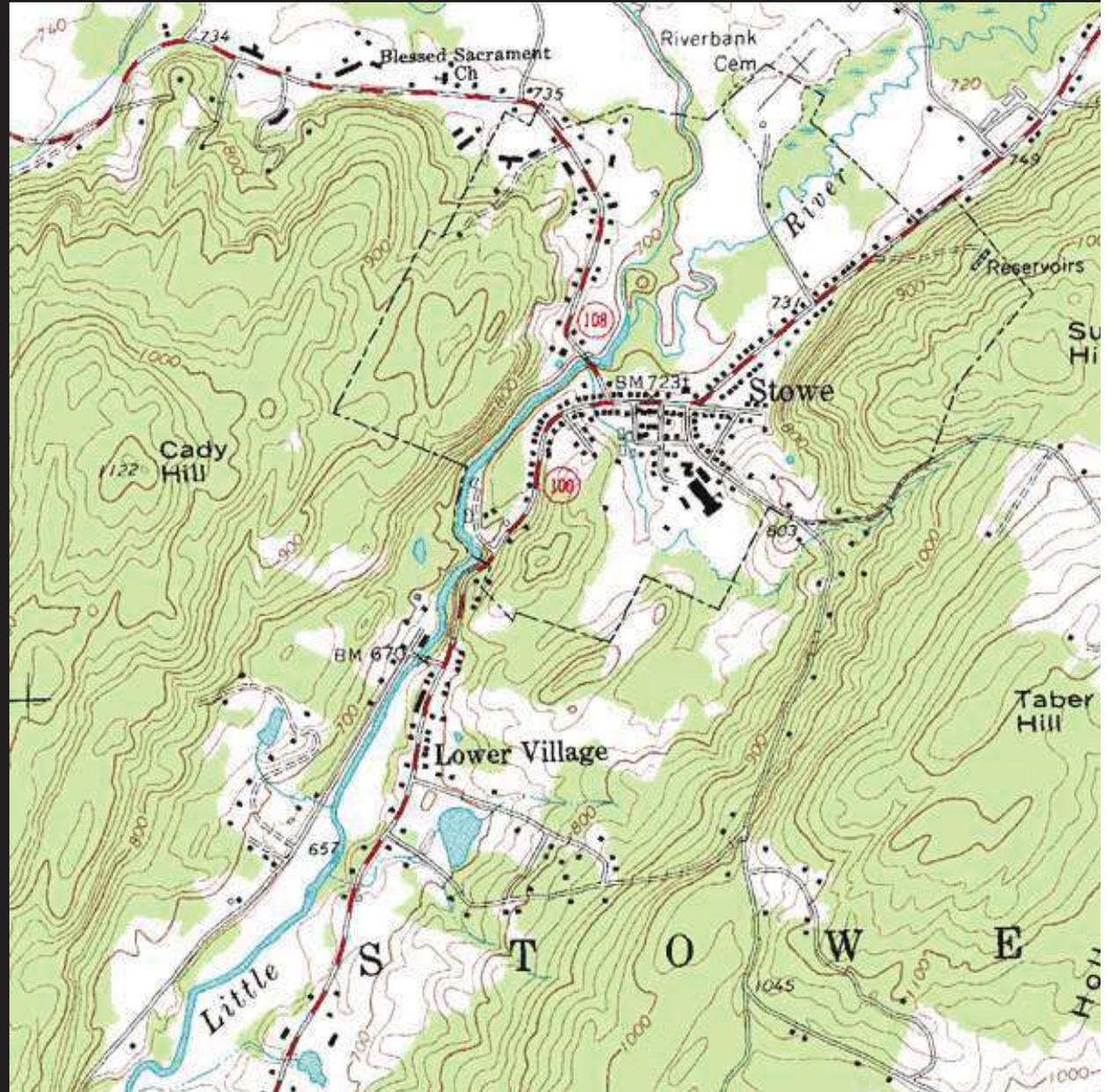


# Topography

- **Topography** is the study of the shape and features of the surface of the Earth and other observable astronomical objects including planets, moons, and asteroids.
- The topography of an area could refer to the surface shapes and features themselves, or a description (especially their depiction in maps).

# Topography

A topographic map with contour intervals





*Thank you*

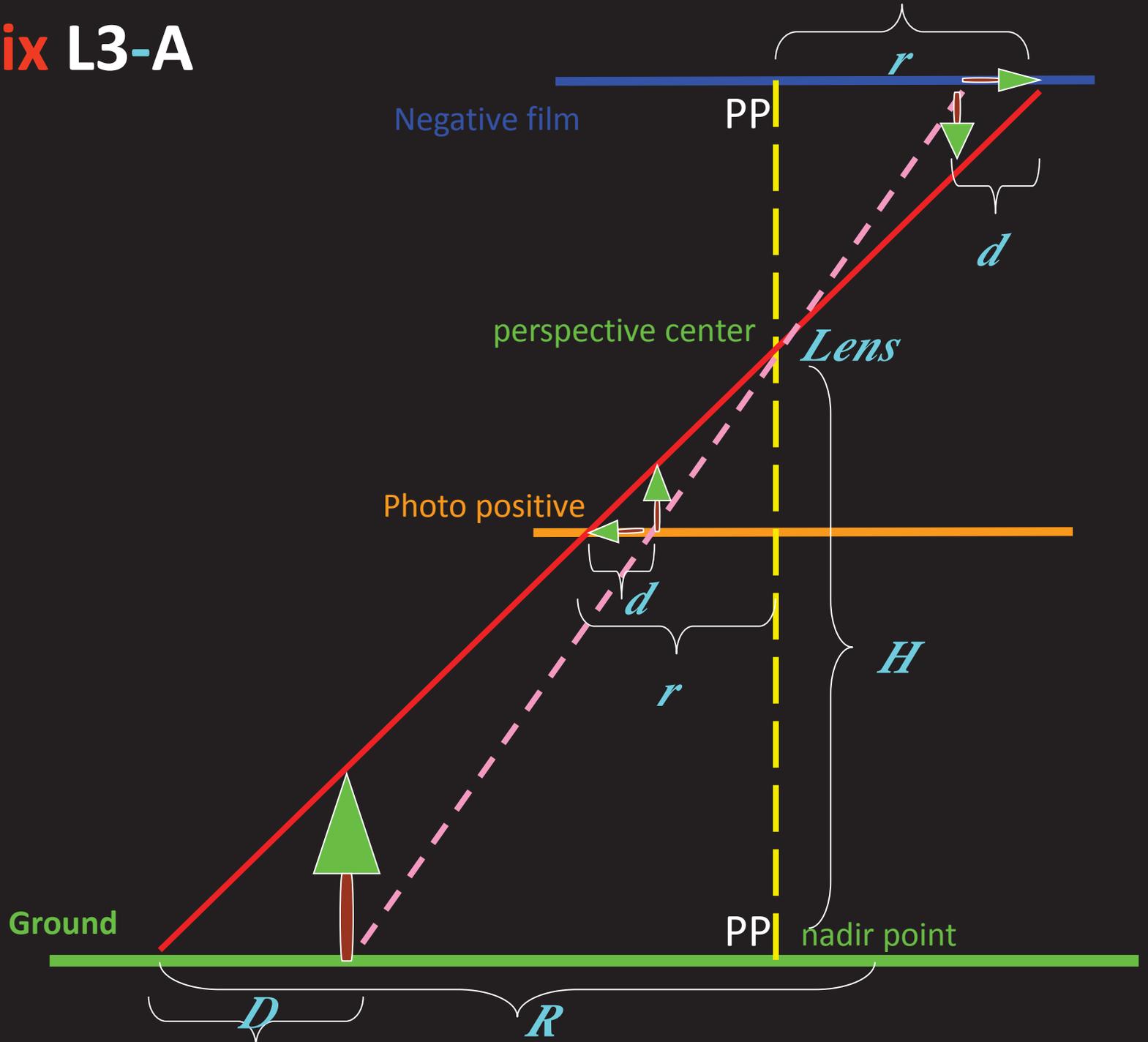
Any Questions ?



**END**

**of Lecture**

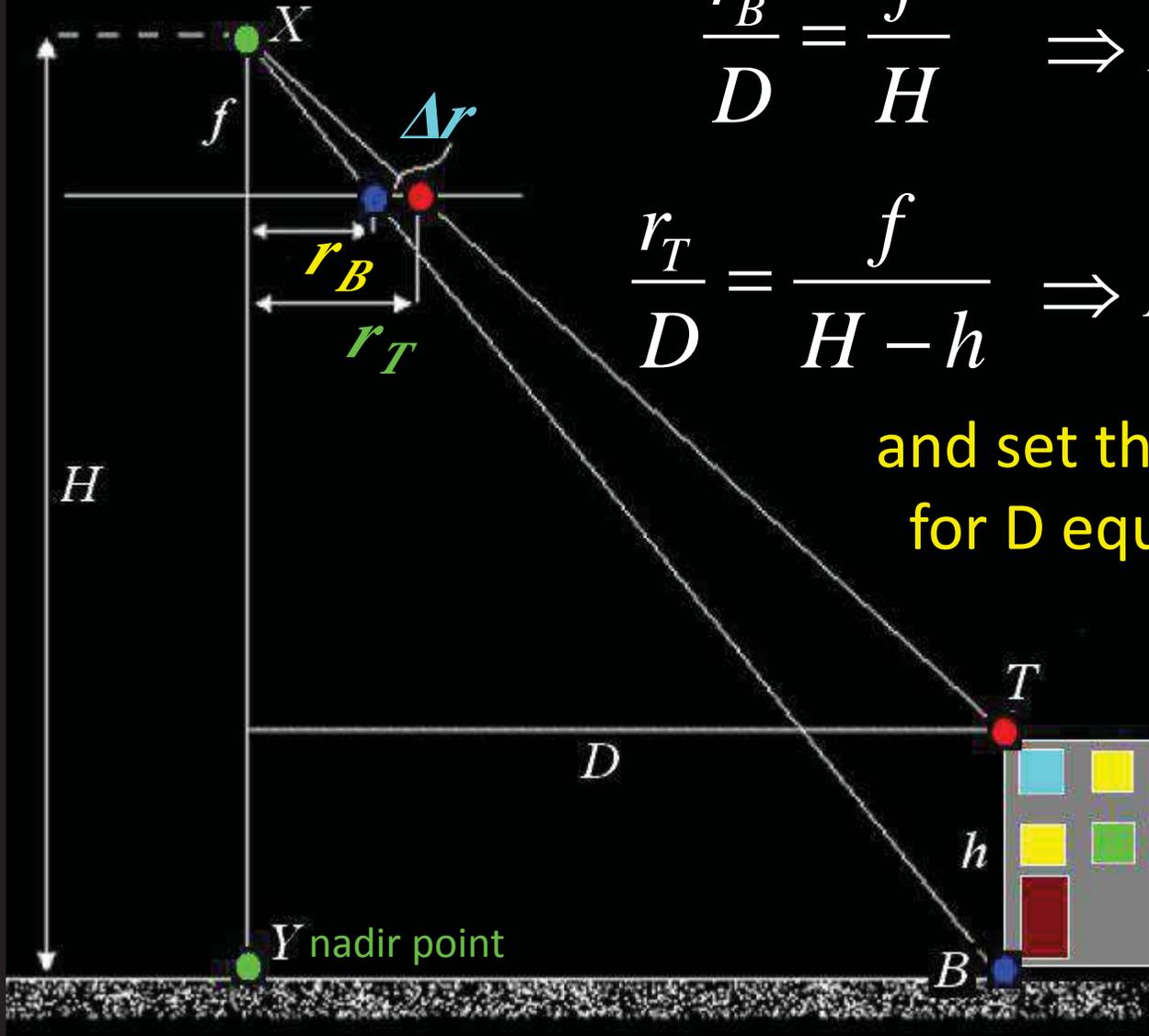
# Appendix L3-A



# Appendix L3-A

We may write two expressions for distance  $D$  in this figure, in terms of radial image distances

perspective center



$$\frac{r_B}{D} = \frac{f}{H} \Rightarrow D = \frac{H \times r_B}{f}$$

$$\frac{r_T}{D} = \frac{f}{H - h} \Rightarrow D = \frac{r_T \times (H - h)}{f}$$

and set the two expressions for  $D$  equal to each other,

## Appendix L3-A

$$\frac{H \times r_B}{f} = \frac{r_T \times (H - h)}{f}$$

$$\Rightarrow H \times r_T - h \times r_T - H \times r_B = 0$$

$$\Rightarrow H \times (r_T - r_B) = h \times r_T$$

$$\Rightarrow \Delta r = d = \frac{h \times r}{H}$$

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How is the Principal Point determined from aerial photographs ?

Principal Point is determined by the photographs Fiducial Marks

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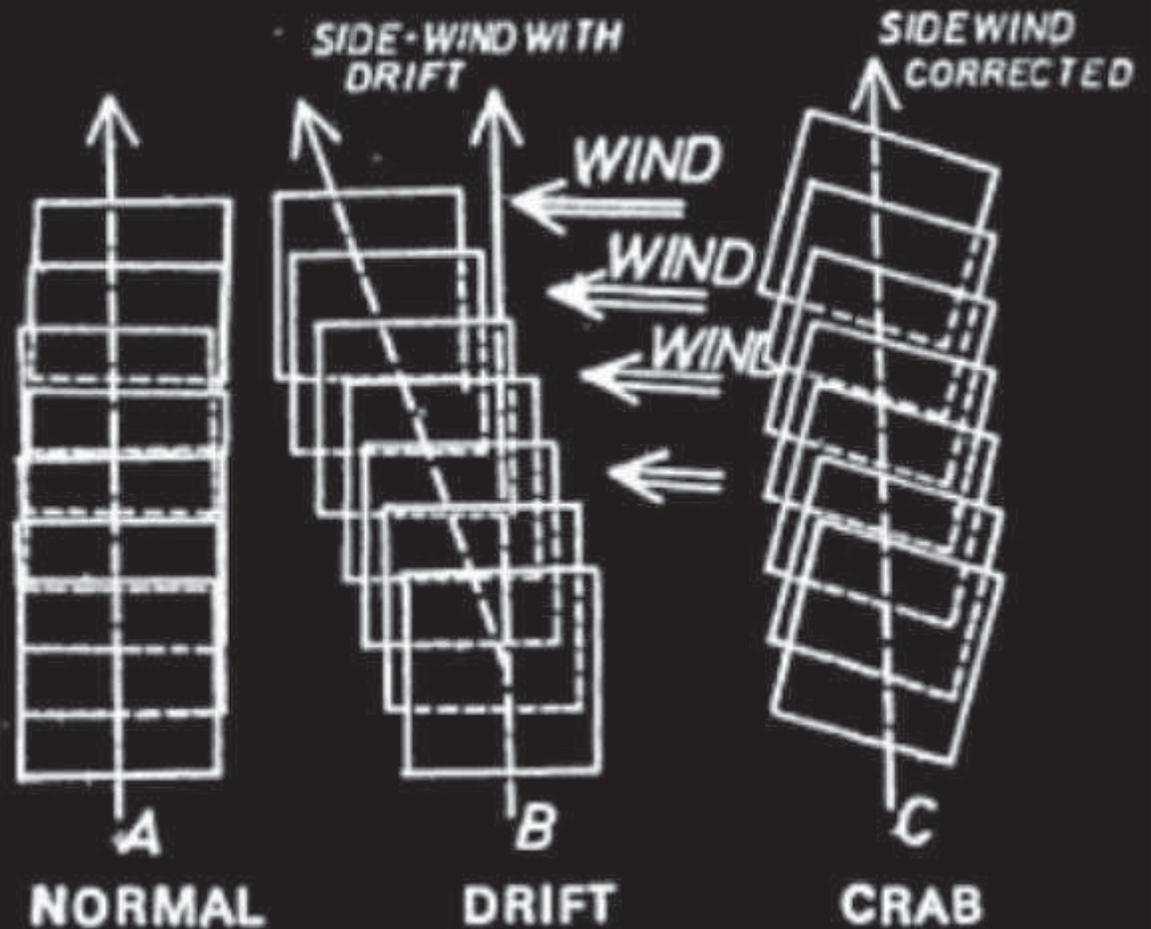
# Appendix 3-3

## Crab

Opposite line of photographs are not parallel to flight line is known as crab of photograph.

## Drift

When aircraft is swayed away from its preplanned flight line then it is known as drift.





بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

# *Aerial Photography & Photogrammetry*

## Lecture 3-4

# Geometry of a Vertical Aerial Photograph

*Lecturer: Faisel Ghazi Mohammed*

Email: [faisel@scbaghdad.edu.iq](mailto:faisel@scbaghdad.edu.iq)  
[faiselgm73@gmail.com](mailto:faiselgm73@gmail.com)

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# 4- Distortion and Displacement

## 2- Topographic/relief Displacement

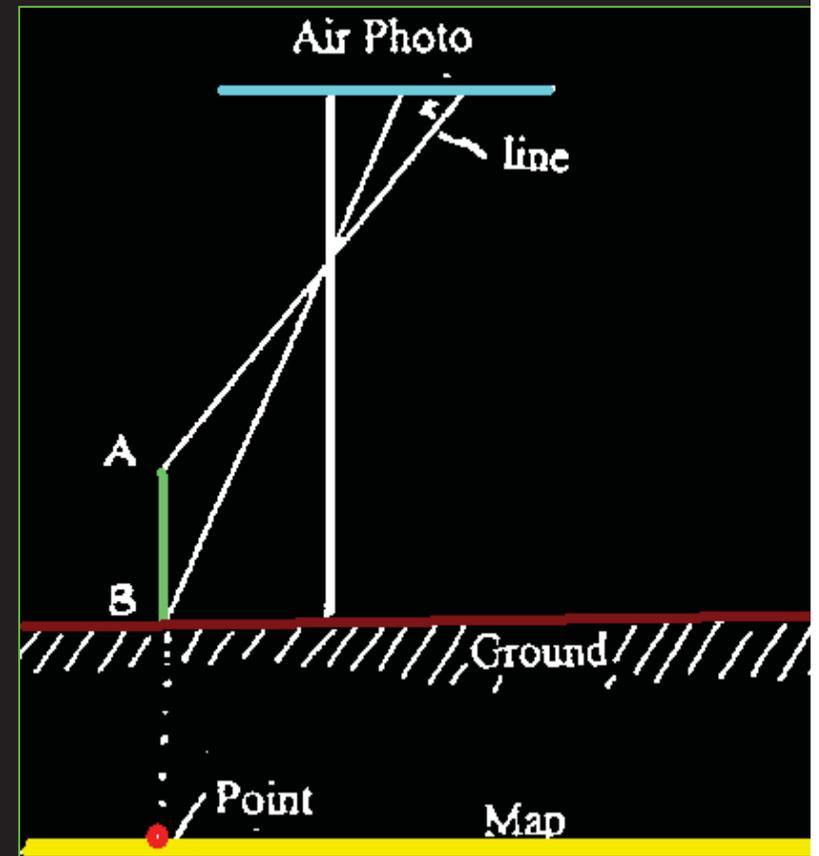
- Objects will tend to lean outward, i.e. be radially displaced.
- The greater the object is from the **principal point**, the greater the radial displacement.
- **Example:** cooling towers towards the edge of photo show greater radial displacement.



# 4- Distortion and Displacement

## 2- Relief Displacement

- If the terrain is completely flat and horizontal, the vertical photograph is same as map.
- If difference in height occur, then relief displacement becomes appearance.
- Pole **AB** appears as a point on map and as line on aerial photo.
- So aerial photo cannot be used as a map.



# 4- Distortion and Displacement

## 2- Relief Displacement

### How to prepare maps from aerial photographs?

- To overcome relief displacement, which is shown above, **two photographs must** taken of the same area at different positions.
- These photographs are overlapped and viewed stereo viewing devices and using this principle a map can be prepared.

# 4- Distortion and Displacement

## 2- Relief Displacement

Where:

- $d$  = Photo displacement, in inches or millimeters at the same scale as the datum
- $r$  = Radial distance on the photo from the nadir to the displaced point, in inches or millimeters
- $h$  = Height of the object (or vertical distance between two elevations), in feet or meters ( $h$  can be either + or -)
- $H$  =  $A - E$  = Flying height above the datum (nadir or base of the object)
- $A$  = Altitude of aircraft above sea level
- $E$  = Elevation of the datum

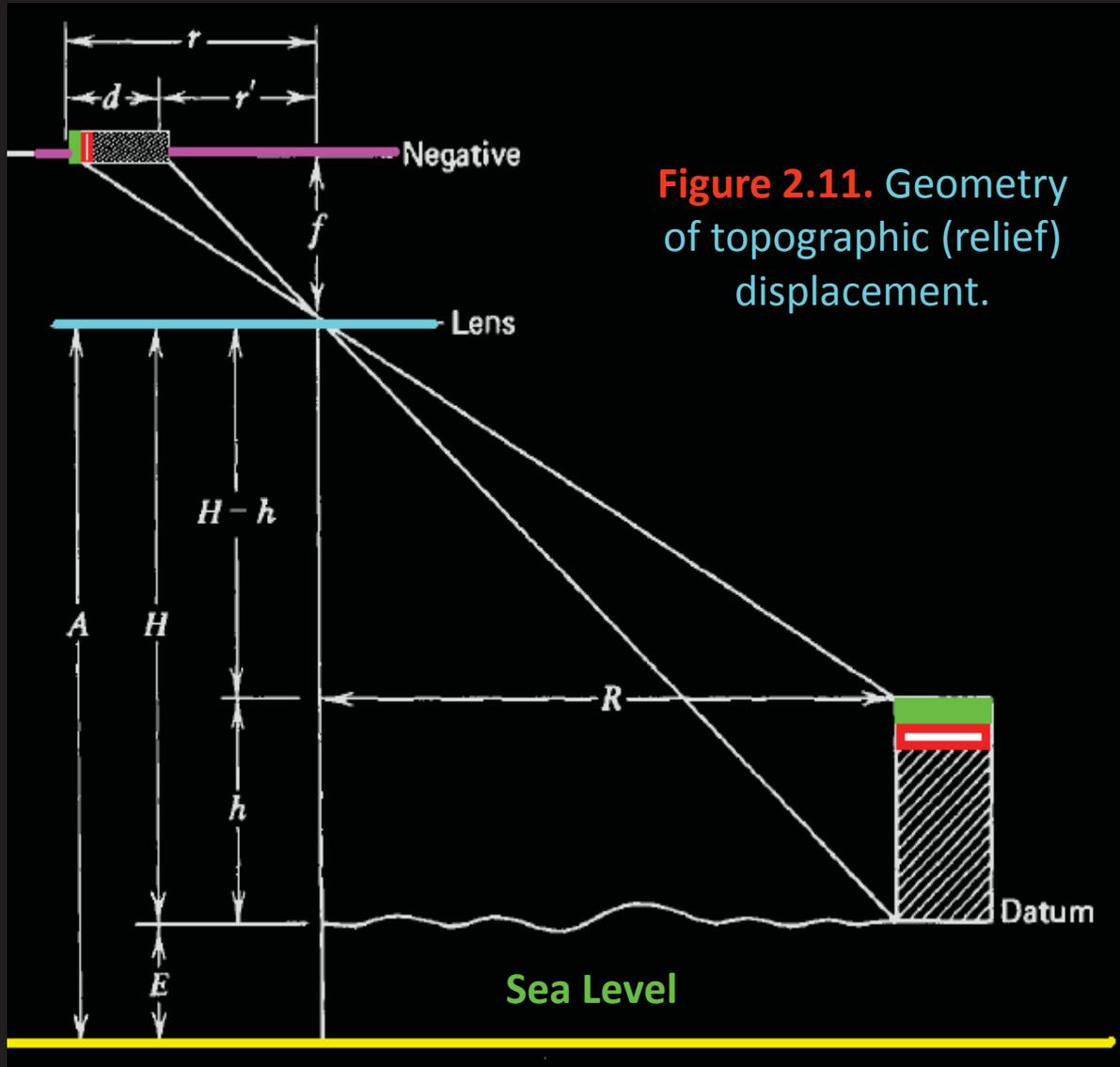


Figure 2.11. Geometry of topographic (relief) displacement.

# 4- Distortion and Displacement

## 2- Relief Displacement

Derive the equation for relief displacement

- We can derive an equation for computing the amount of topographic displacement from Figure 2.11 by using geometric relationships of similar triangles.
- Because the relief displacement,  $d$  ( $\Delta r$ ) is equal to  $r$  minus  $r'$ , we want to consider the following relationships:

$$\frac{f}{r} = \frac{H - h}{R} \quad \text{therefore} \quad r = \frac{f(R)}{H - h}$$

$$\frac{f}{r'} = \frac{H}{R} \quad \text{therefore} \quad r' = \frac{f(R)}{H}$$





## 4- Distortion and Displacement

### 2- Relief Displacement

#### Exercise:

Derive the equation for relief displacement

$$\Delta r = \frac{h \times r}{H}$$

Check your final solution steps within the one in :

**Appendix L3-A**

# 4- Distortion and Displacement

## 2- Relief Displacement

- If all object units are the same (e.g., meters) and all image units are the same (e.g., millimeters), the units of the equation will be consistent.
- Hence, *if we know the flying height*, we can calculate the height of any object in a photograph!
- The height of an object can be estimated from a single aerial photograph provided we have one auxiliary piece of information, the flying height. See **example 2**

# 4- Distortion and Displacement

## 2- Relief Displacement

### Example 1

- A 1:15000 aerial photograph was taken using a wide-angle camera. A point on the photograph was identified and its measured distance from the center is 5.4 centimeters.
- If the corresponding point on the ground is elevated from the datum by 60 meters,
- Determine the displacement due to relief and the correct radial distance of the point from the center of the photo.

# 4- Distortion and Displacement

## 2- Relief Displacement

### Solution

$$scale = \frac{\text{camera focal length}}{\text{flying height above terrain}}$$

$$f = 6 \text{ inches} \quad (\text{see figure 2.4 in the text})$$

$$\Delta r = \frac{r \times h}{H}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$r = 5.4 \text{ cm}, h = 6000 \text{ cm}$$

$$\frac{1}{15000} = \frac{6}{H} \rightarrow H = 90000 \text{ (inches)} \times 2.54 = 228600 \text{ cm}$$

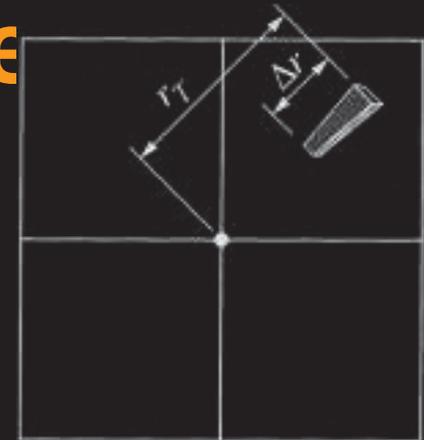
$$\Delta r = \frac{5.4 \times 6000}{228600} = 0.1417322 \text{ cm}$$

This means that the *correct* or *map position* of the point is **0.1 cm closer** to the nadir than actually shown on the photo (photo displacement is away from the nadir)

## 4- Distortion and Displacement

### 2- Relief Displacement

#### Example 2



The flying height above the base of the building shown in the following figure is 500 *m* for a vertical photograph *H*. When measuring the image, the relief displacement of the building ( $\Delta r$ ) is 4 mm and the radial distance from the principal point to the top of the object  $r = 75$  mm... What is the height of the building?

## 4- Distortion and Displacement

### 2- Relief Displacement

#### Solution

$$\Delta r = \frac{h \times r}{H} \quad \Rightarrow \quad h = \frac{\Delta r \times H}{r}$$

$$= 500m \times \frac{4mm}{75mm}$$

The building is in reality **26.7 m** high.

## 4- Distortion and Displacement

### Relief Displacement

#### Quiz 1

The top and bottom of a utility pole in an image are 129.8 mm and 125.2 mm, respectively, from the principal point of a vertical photograph. What is the height of the pole if the flying height above the base of the pole is 875m?

# 4- Distortion and Displacement

## Relief Displacement

### Quiz 1- Answer

$$\Delta r = \frac{h \times r}{H} \quad \Rightarrow \quad h = \frac{\Delta r \times H}{r}$$

## 4- Distortion and Displacement

### Relief Displacement

Please see the  
**2.6 NUMERICAL EXAMPLE**  
and the next one  
in text book  
placed in page 40

# *Homework's*

( all )

Questions and Problem

in Page 43

are required

# 4- Distortion and Displacement

## Inferences Based on the Relief Displacement Equation

1. Topographic displacement varies **directly** with the height of the object. A 1,000-foot mountain would be displaced twice as far as a 500-foot mountain.
2. Topographic displacement varies **directly** with the radial distance from the nadir to the object. A particular elevation 4 inches from the nadir will have twice the displacement as the same elevation 2 inches from the nadir.
3. There is no topographic displacement at the **nadir**. If  $r$  is zero, so is  $d$ .

# 4- Distortion and Displacement

## Inferences Based on the Relief

### Displacement Equation

4. Assuming the datum elevation to be at the nadir, points *above* the nadir are displaced radially *away* from the nadir and points *below* the nadir are displaced radially *toward* the nadir.
5. Finally, topographic displacement varies **inversely** with the flying height above the base of the object. Therefore, there is very little topographic displacement on photographs taken from high altitudes, such as orbiting space stations.



*Thank you*

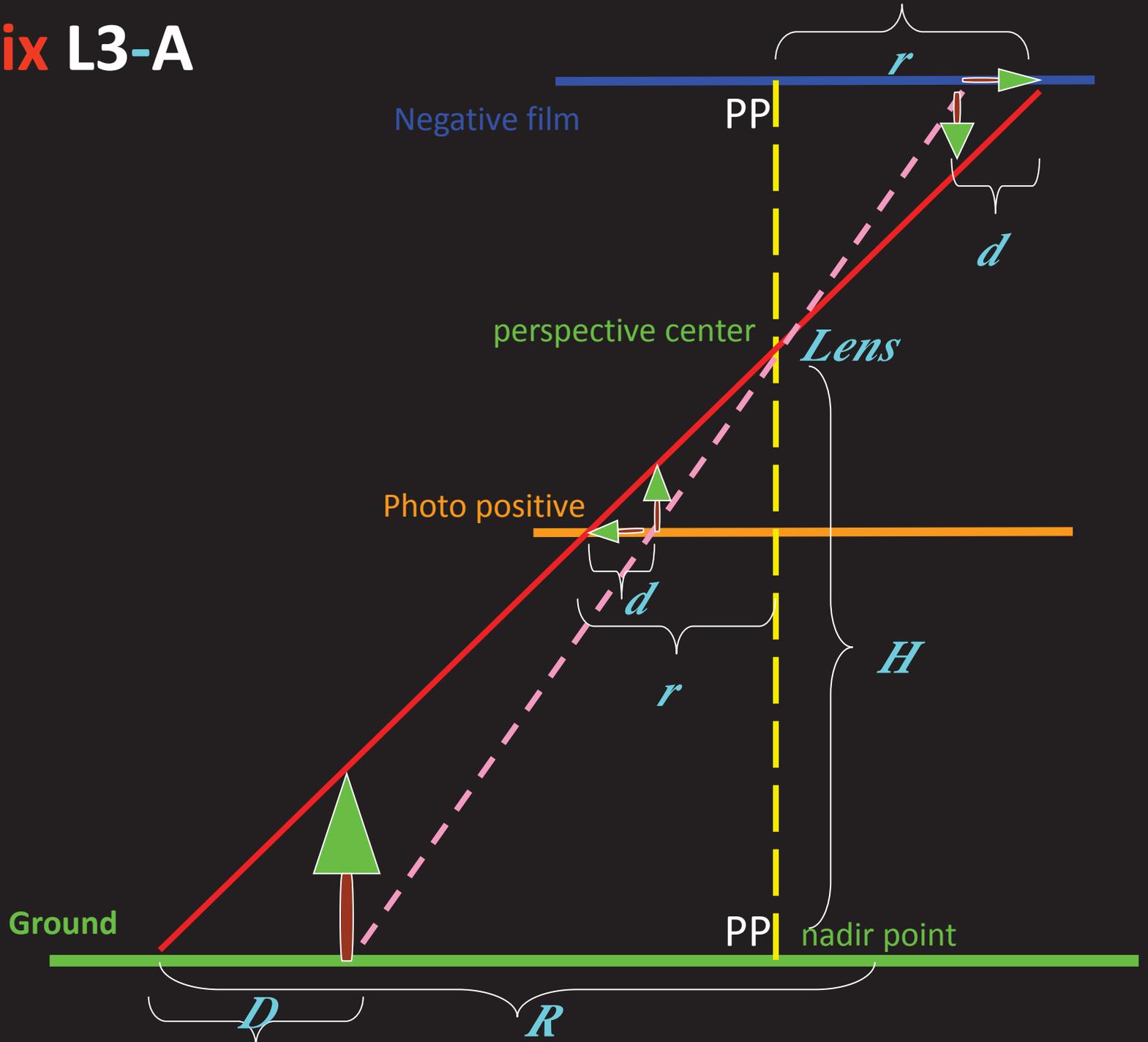
Any Questions ?



**END**

**of Lecture**

# Appendix L3-A





## Appendix L3-A

$$\frac{H \times r_B}{f} = \frac{r_T \times (H - h)}{f}$$

$$\Rightarrow H \times r_T - h \times r_T - H \times r_B = 0$$

$$\Rightarrow H \times (r_T - r_B) = h \times r_T$$

$$\Rightarrow \Delta r = d = \frac{h \times r}{H}$$

## Appendix L3-B

# Some Key Terms in Aerial Photography

(compiled by Hugh Millward from  
various sources)

- **Air-base (AB)** (a.k.a. **Camera Base**): Ground distance between centers (PPs) of adjacent photos along a flight line.
- **Angle of Coverage**: the angle of the cone of rays passing the front of a camera lens. Normal angle =  $60^\circ - 75^\circ$ , Wide-angle = over  $75^\circ$
- **Average Photo-base (PB)**: For adjacent air photos, the average of the distance between the PP and CPP on each photo
- **Camera-base (CB)** (a.k.a. **Air-base**): Ground distance between centres (PPs) of adjacent photos along a flight line
- **Conjugate Principal Point (CPP)** (a.k.a. **Corresponding P.P.**): The location of a Principal Point from one photo on an adjacent photo along the flight line
- **Control Point**: A reference point precisely located on both the ground and the photo (ground control point) on both a map and the photo (map control point), or on two adjacent photos (photo control point)
- **Controlled Mosaic**: A series of overlapping air photos that have been rectified and aligned with ground control points, to allow planimetrically-correct distance measurements
- **Crab**: Rotation of the camera (and aircraft) relative to the flightline
- **Drift**: Lateral shift of the aircraft from the planned flightline
- **Eye-base (EB)**: Interpupillary distance, normally  $6.4 (\pm 0.4)$  cm.
- **Fiducial Marks**: Marks built into aerial cameras which appear on the sides or in the corners of the photo (or both), and which are used to determine the precise location of the principal point.

- **Focal length:** Distance from the optical centre of the lens to the focal plane, when the camera is focussed at infinity.
- **Forward Overlap:** (a.k.a. **Endlap**): The amount of overlap between successive photos in a flight line to allow for stereo viewing (usually 60 – 70%)
- **High Oblique photo:** An air photo which shows the horizon line (usually having high vertical tilt, of 60° or more)
- **Isocentre:** The point on an air photo which lies halfway between the Principal Point and the Nadir (Tilt-displacement radiates from this)
- **Low Oblique Photo:** An air photo tilted from the vertical, but not enough for the horizon to be visible (usually having vertical tilt of 3° - 60°)
- **Mosaic:** A series of overlapping air photos
- **Nadir:** That point on the ground vertically beneath the camera lens (or aircraft), or the point on the photo which corresponds to it. (Topographic Displacement radiates from this)
- **Orthophoto:** A vertical air photo which has been rectified to remove parallax
- **Parallax:** The apparent displacement of the position of an object, with respect to a reference point, caused by a shift in the point of observation.
- **Photo-base (PB):** On a single air photo, the distance between the photo's principal point and the CPP of an adjacent photo (see Average PB)

- **Principal Point (PP):** The geometric centre of an aerial photograph, located at the intersection of lines drawn between the fiducial marks (i.e., at the intersection of the x and y axes). (Lens distortion radiates from this)
- **Radial Line Triangulation (RLT):** The production of planimetrically-correct (i.e. uniform scale) maps from two or more adjacent vertical air photos, using the techniques of resection and intersection.
- **Rectification:** The process of converting a vertical air photo to remove displacements caused by tilt or topography (i.e., to remove parallax).
- **Sidelap (a.k.a. Lateral overlap):** The amount of overlap between air photos in adjacent flight lines (usually 20 – 30%)
- **Stereogram:** A stereopair or stereo-triplet mounted for proper stereovision (conjugate points  $5.7 \pm 0.3$  cm apart)
- **Tilt:** Rotation of the camera away from the vertical, about the x- or y-axis
- **Tilt Displacement:** Changes in position caused by scale variations related to the tilt of the camera, about either the x-or y-axis
- **Topographic Displacement (a.k.a. Relief Displacement, Radial Displacement, or Planimetric Shift due to Elevation):** Changes in position caused by scale variations related to differences in elevation or height.
- **Uncontrolled Mosaic:** A series of overlapping air photos which have not been aligned to ground control points
- **Vertical Air Photo:** An air photo with less than  $3^\circ$  of vertical tilt
- **X-axis:** For a single photo, the line through the photo showing direction of flight at the centre of the photo (i.e., nose-to-tail axis)
- **Y-axis:** the line at right-angles to the x-axis (i.e., wingtip-to-wingtip axis)

# Appendix 3-1

How is the Principal Point determined from aerial photographs ?

Principal Point is determined by the photographs Fiducial Marks

What aspects of photographic geometry cause differences between Nadir and the Principal Point?

- Topographic displacement affects often increase from Nadir
- Tilt displacement affects increase away from the Isocenter of the photograph

Why are most aerial photographs taken from a tilted angle opposed to a vertical position?

What two aspects of aerial photography are used to define the scale of a photograph, and how are they calculated?

Scale is determined by the

1- focal length of a lens/

2- photograph height.

Focal Length = 4cm , Flying Height = 100000cm ,  $4/100000 = \text{Scale } 1/25,000$

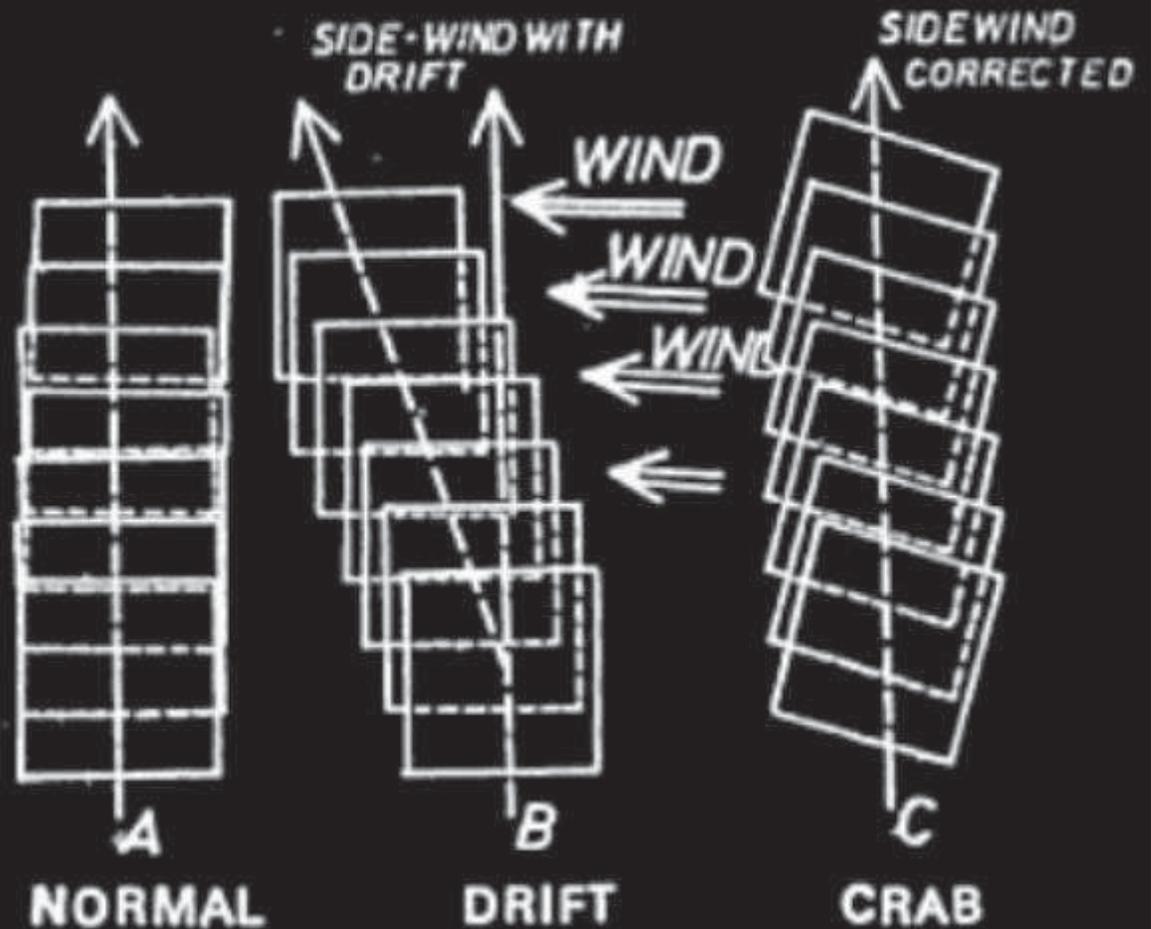
# Appendix 3-3

## Crab

Opposite line of photographs are not parallel to flight line is known as crab of photograph.

## Drift

When aircraft is swayed away from its preplanned flight line then it is known as drift.





بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

## Lecture 3

# Advanced Image Processing

# Image Segmentation

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**2022-2021**

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1. Thresholding
2. Edge detection,
3. Edge linking
4. Region based segmentation  
(Region growing )
5. Region splitting and Merging

# Elements of Image Analysis

## Preprocess

Image acquisition, restoration, and enhancement



## Intermediate process

Image segmentation and feature extraction

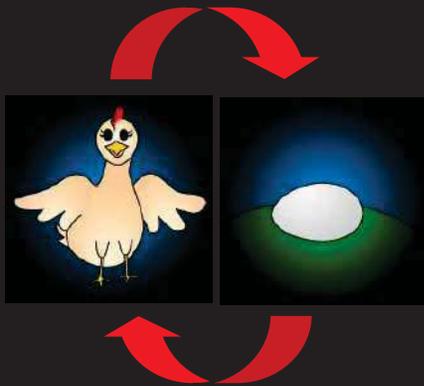


## High level process

Image interpretation and recognition

# Elements of Image Analysis

## Chicken & Egg



- To perform better segmentation, we need an *object* model. This model includes methods to describe higher-level properties such as object shape
- To measure object shape, we need a good segmentation



# 1- Image Segmentation

- The purpose of image segmentation is to partition an image into *meaningful* regions with respect to a particular application
- The segmentation is based on measurements taken from the image and might be *grey level, colour, texture, depth* or *motion*

# What is segmentation?



boundaries



labels



pseudocolors



mean colors

(different ways of displaying the output)

# 1- Image Segmentation

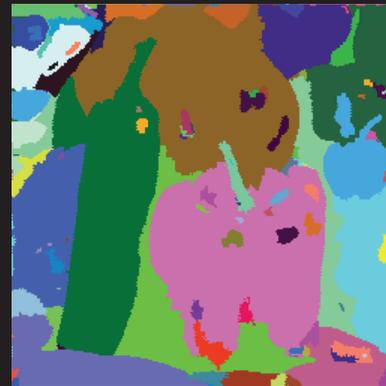
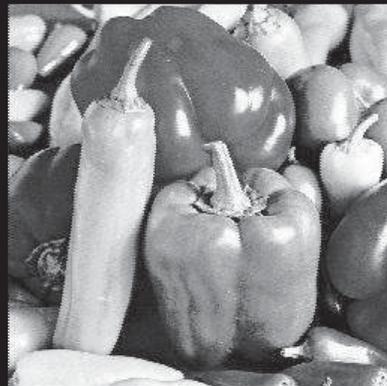
Usually image segmentation is an initial and vital step in a series of processes aimed at overall image understanding

Applications of image segmentation include

1. Identifying objects in a scene for object-based measurements such as **size** and **shape**
2. Identifying objects in a moving scene for *object-based video compression (MPEG4)*
3. Identifying objects which are at different distances from a sensor using depth measurements from a laser range finder enabling **path planning** for a **mobile** robots

# Image Segmentation-based on greyscale

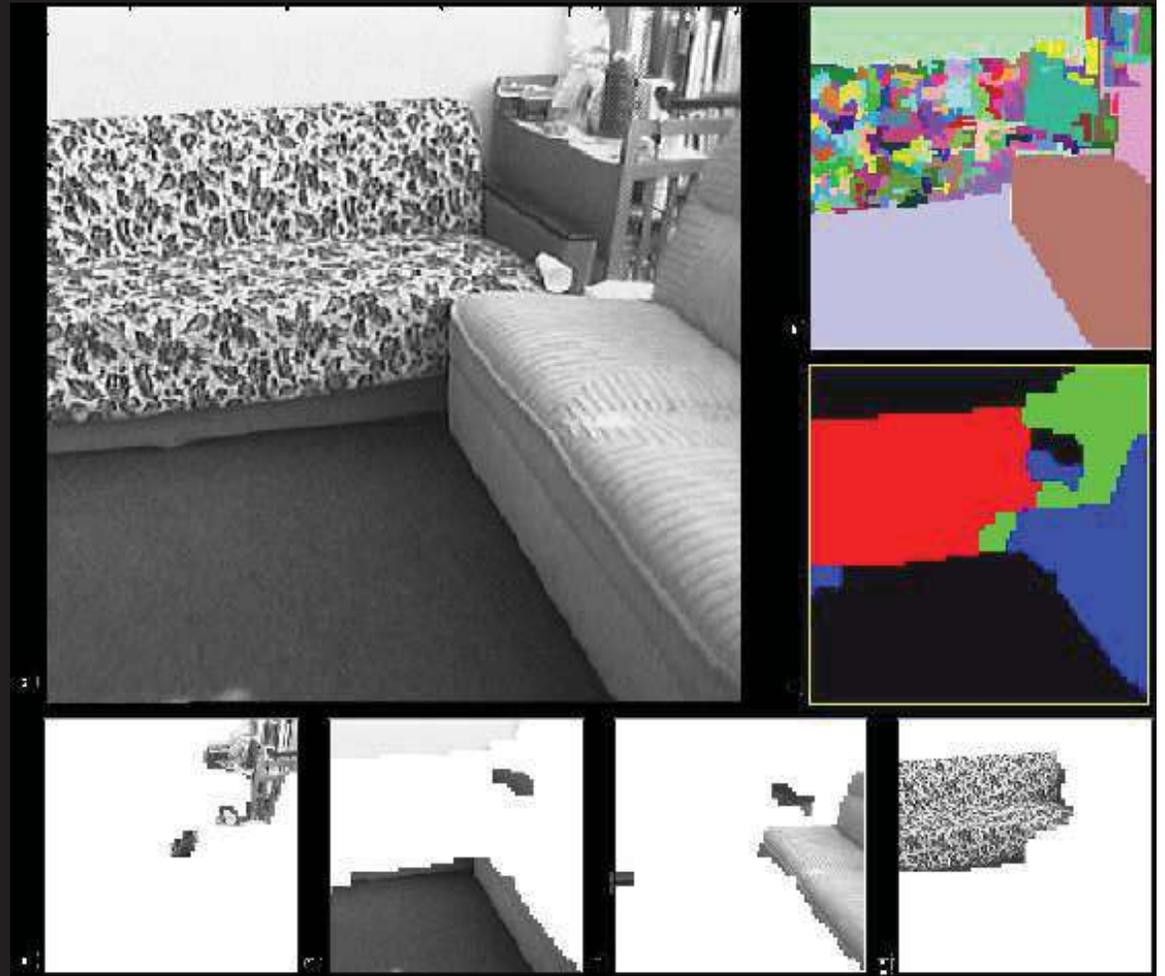
Very simple 'model' of greyscale leads to inaccuracies in object labelling



# Image Segmentation-based on texture

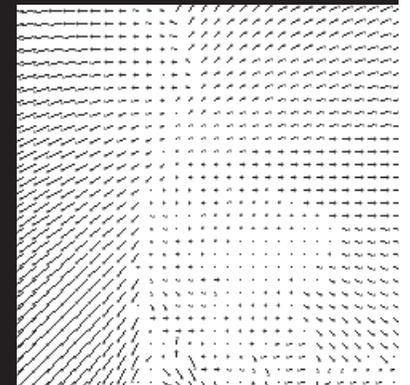
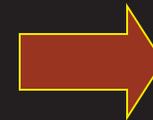
Segmentation

Enables  
object  
surfaces with  
varying  
patterns of  
grey to be  
segmented

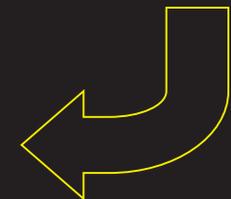
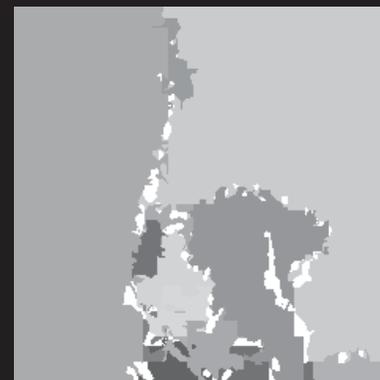


# Image Segmentation-based on motion

The main difficulty of motion segmentation is that an intermediate step is required to (either implicitly or explicitly) **estimate** an *optical flow field*



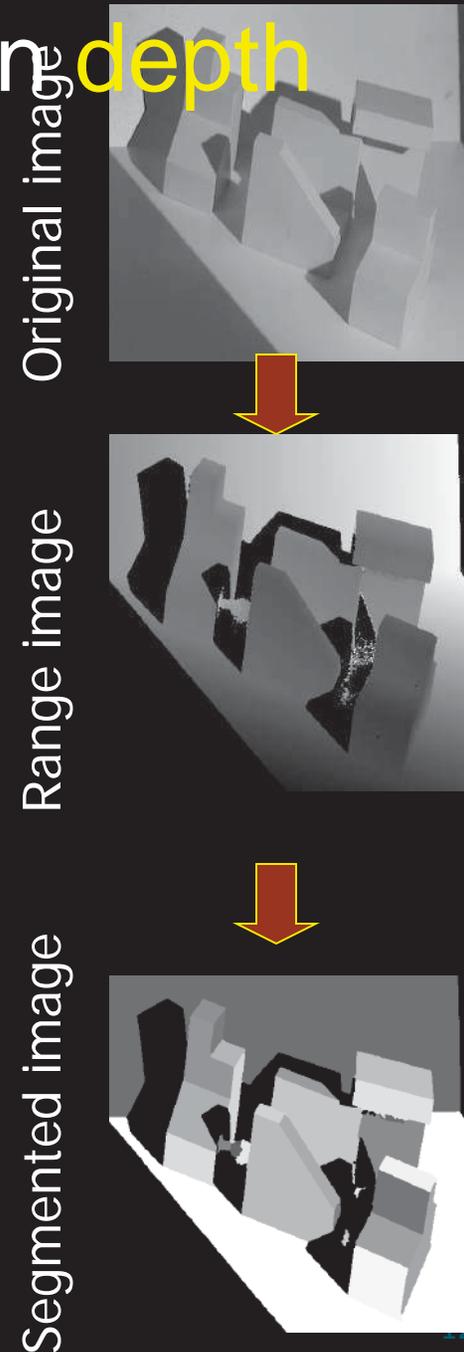
The segmentation must be **based on** this estimate and not, in general, the true flow



# Image Segmentation- based on depth

This example shows a range image, obtained with a laser range finder

A segmentation based on the range (the object distance from the sensor) is useful in guiding mobile robots



# Image Segmentation : Methods

## Pixel-based (oriented) Thresholding

- **GrayLevel** Thresholding
- GrayLevel **Histogram-based** Thresholding
- Basic **Adaptive** Thresholding
- **Optimal** Global and Adaptive Thresholding
- GrayLevel **Clustering**
- Relaxation **Labelling**
- The Expectation/Maximization (**EM**) algorithm
- **Color** Images Segmentation
  - Color Segmentation in **HSI** Color Space
  - Color Segmentation in **RGB** Vector Space
- **Multispectral** Images Segmentation
- Self Organizing Map for **Color** Image Segmentation

# Image Segmentation : Methods

## Region-based

- Region growing
- Split and merge

## Edge-based

## Model-based

## Physics-based

## Graph-based

## Lecture 3-a

# Image Segmentation

# Thresholding

based

# Contents

Today we will continue to look at the problem of segmentation, this time though in terms of thresholding

In particular we will look at:

1. What is Thresholding?
2. Simple Thresholding
3. Adaptive Thresholding

# Graylevel Thresholding

- Thresholding is usually the **first step** in any segmentation approach
- We have talked about **simple single** value thresholding already
- **Single** value thresholding can be given mathematically as follows:

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > T \quad (\text{object point}) \\ 0 & \text{if } f(x, y) \leq T \quad (\text{background point}) \end{cases}$$

**$T$**  : global thresholding

# Graylevel Thresholding

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > T \quad (\text{object point}) \\ 0 & \text{if } f(x, y) \leq T \quad (\text{background point}) \end{cases}$$

$T$  : global thresholding

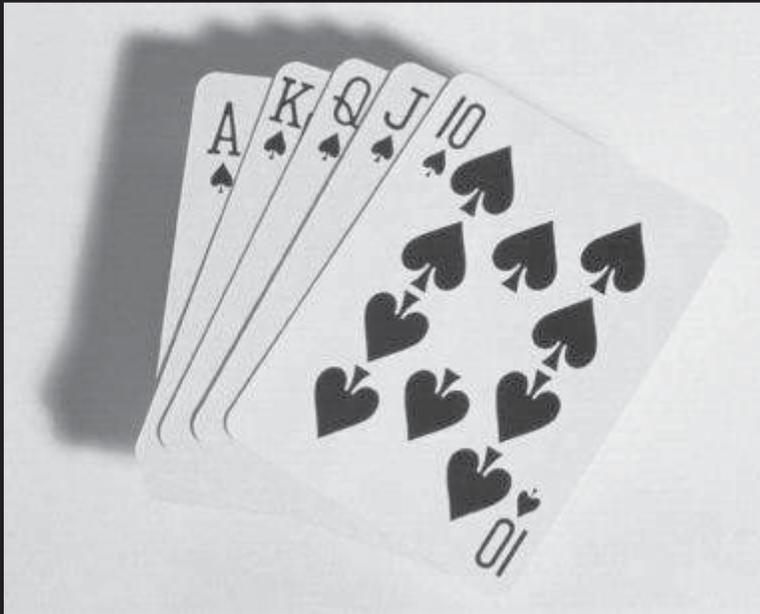
Multiple thresholding

$$g(x, y) = \begin{cases} a & \text{if } f(x, y) > T_2 \\ b & \text{if } T_1 < f(x, y) \leq T_2 \\ c & \text{if } f(x, y) \leq T_1 \end{cases}$$

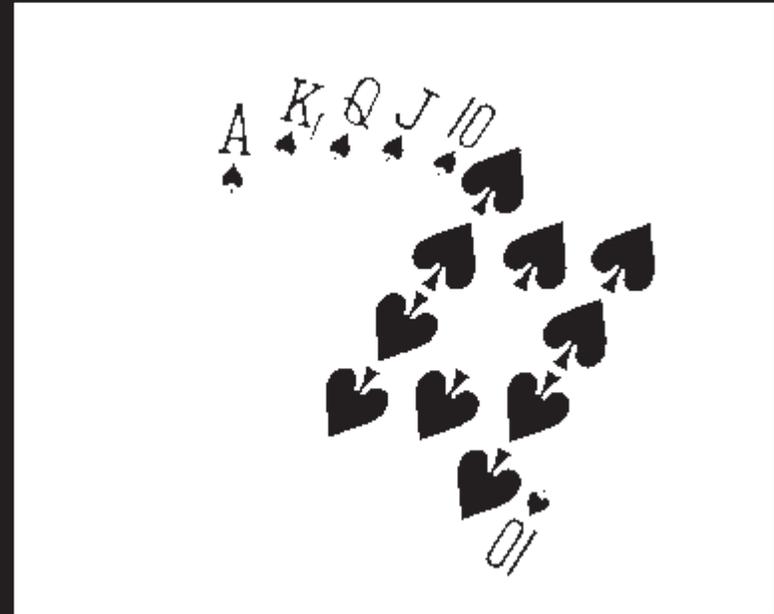
# Graylevel Thresholding

## Thresholding Example

Imagine a poker playing robot that needs to visually interpret the cards in its hand



Original Image

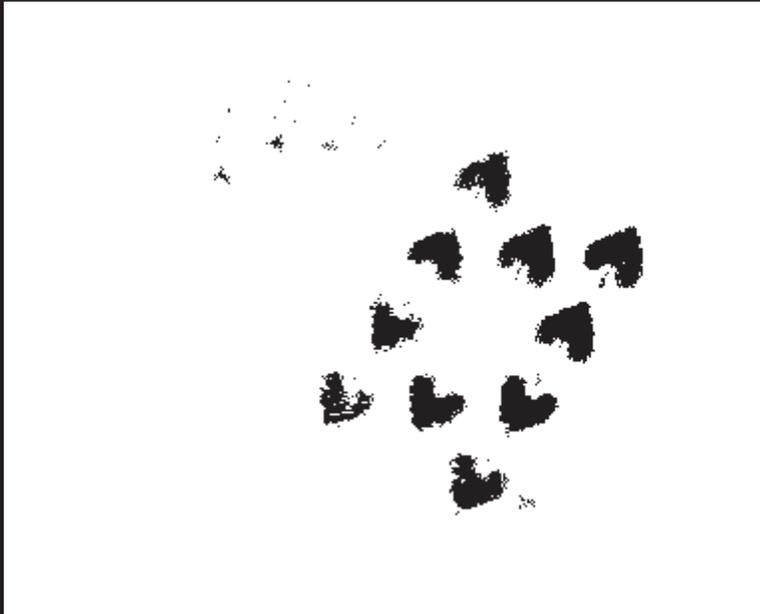


Thresholded Image

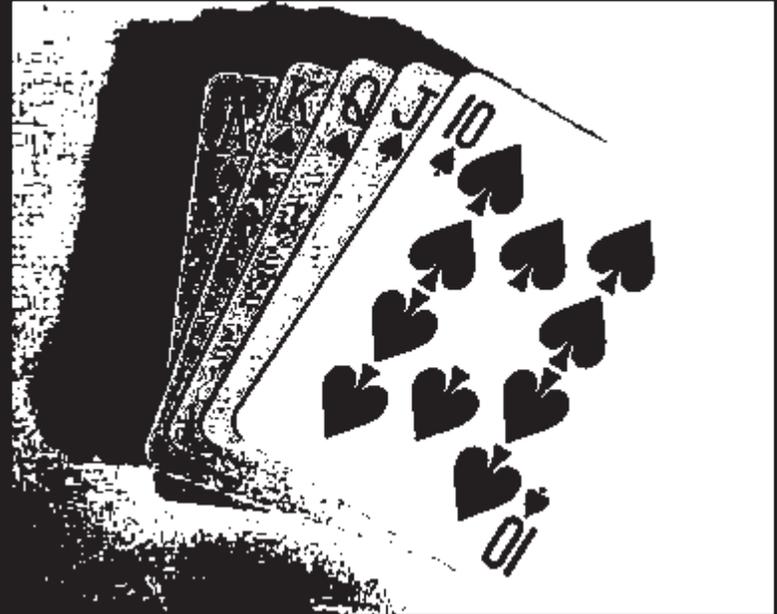
# Graylevel Thresholding

## But Be Careful

If you get the threshold wrong the results can be disastrous



Threshold Too Low



Threshold Too High

# Graylevel Histogram-based Thresholding

## Automatic Threshold Level Selection (Basic Global Thresholding)

- Based on the histogram of an image
- Partition the image histogram using a single global threshold
- The success of this technique very strongly depends on how well the histogram can be partitioned
- Effective for bimodal histogram

# Automatic Threshold Level Selection (Basic Global Thresholding)

*Algorithm:* effective for **bimodal** histogram

The basic global threshold,  $T$ , is calculated as follows:

1. Select an initial estimate for  $T$  (typically the average grey level in the image)
2. Segment the image using  $T$  to produce two groups of pixels:  $G_1$  consisting of pixels with grey levels  $>T$  and  $G_2$  consisting of pixels with grey levels  $\leq T$
3. Compute the average grey levels of pixels in  $G_1$  to give  $\mu_1$  and  $G_2$  to give  $\mu_2$
4. Compute a new threshold value: 
$$T = \frac{\mu_1 + \mu_2}{2}$$
5. Repeat steps 2 – 4 until the difference in  $T$  in successive iterations is less than a predefined limit  $T_\infty$ .

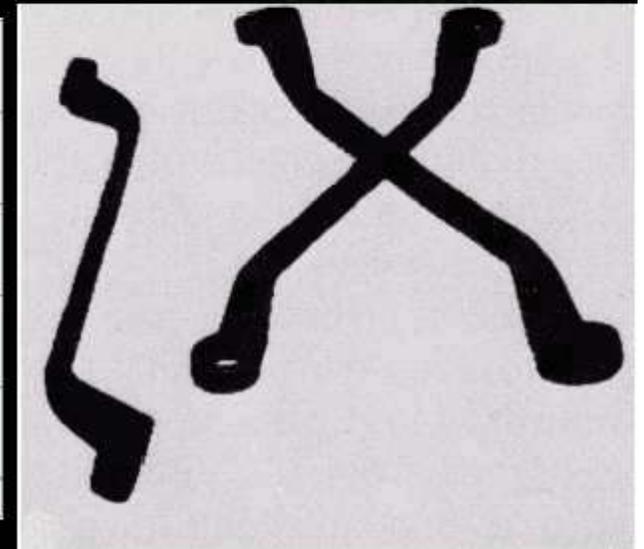
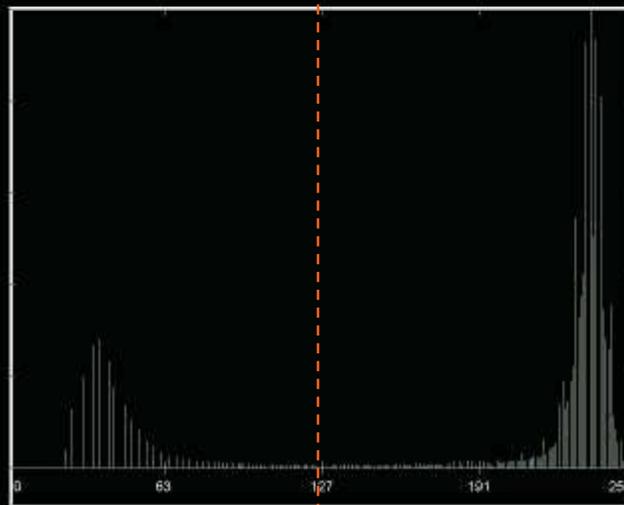
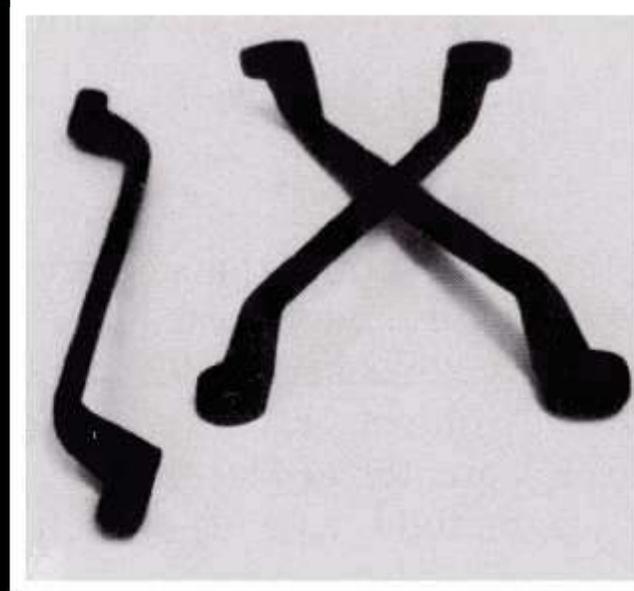
This algorithm works **very well** for finding thresholds when the **histogram** is **suitable**

# Graylevel Histogram-based Thresholding

Automatic Threshold Level Selection (Basic Global Thresholding)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

## Thresholding Example 1

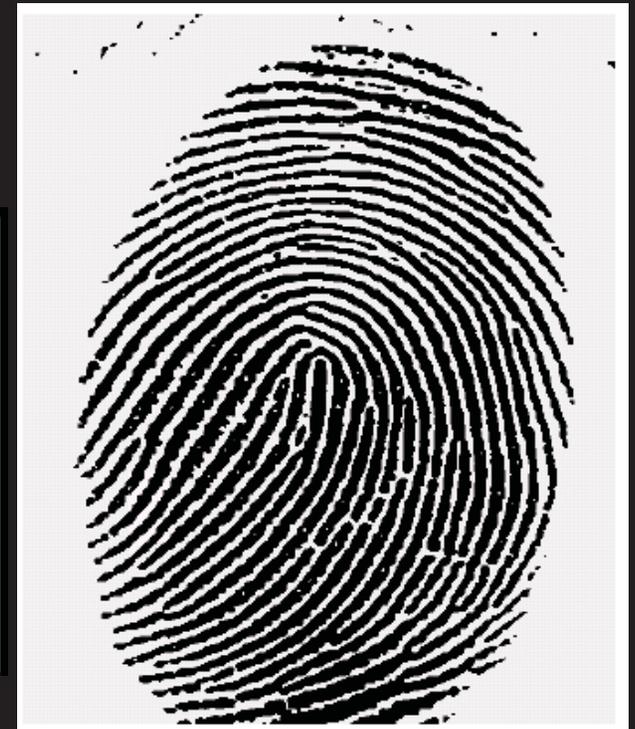
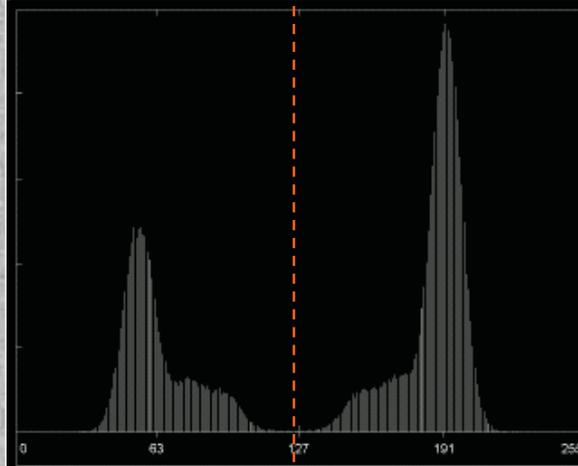


# Graylevel Histogram-based Thresholding

Automatic Threshold Level Selection (Basic Global Thresholding)

## Thresholding Example 2

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

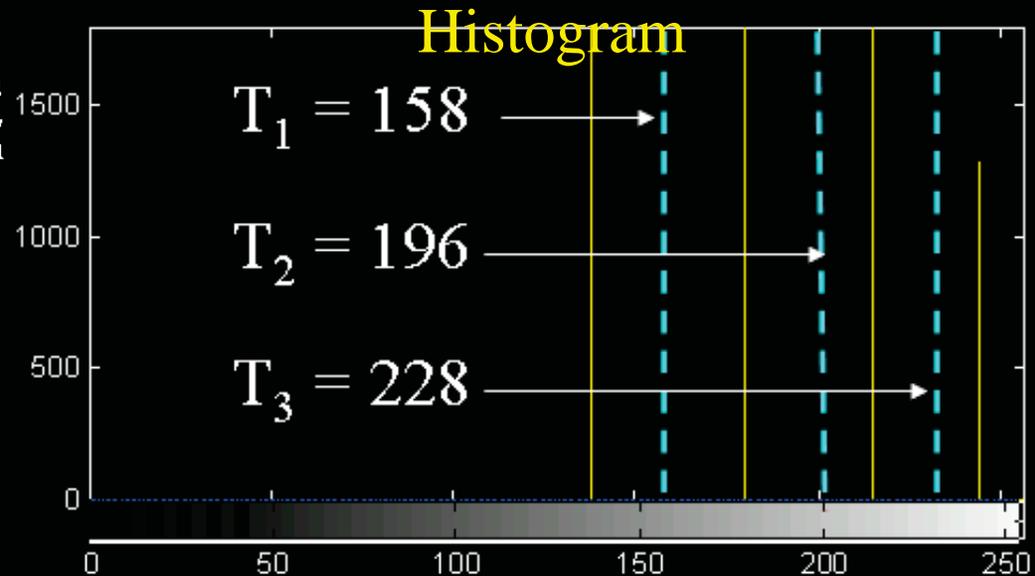
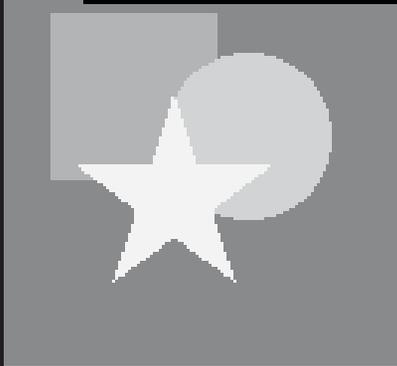


# Graylevel Histogram-based Thresholding

## Automatic Threshold Level Selection (Basic Global Thresholding)

Multiple thresholding

$$g(x, y) = \begin{cases} a & \text{if } f(x, y) > T_2 \\ b & \text{if } T_1 < f(x, y) \leq T_2 \\ c & \text{if } f(x, y) \leq T_1 \end{cases}$$



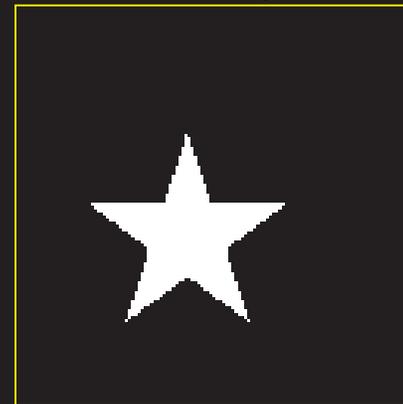
$T_1 < P < T_2$



$T_2 < P < T_3$



$P > T_3$

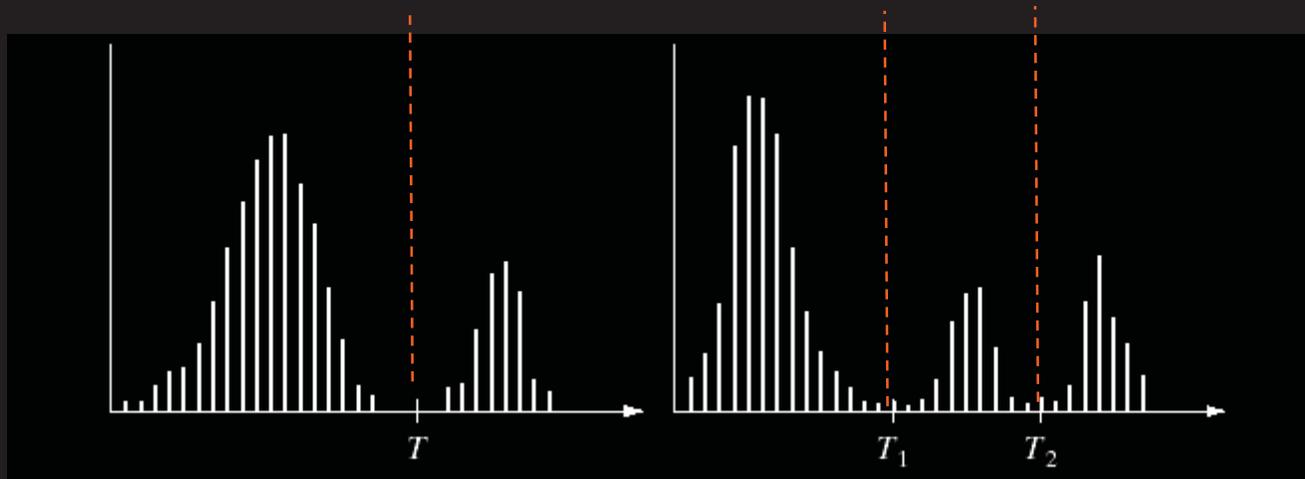


# Graylevel Histogram-based Thresholding

## Automatic Threshold Level Selection (Basic Global Thresholding)

### Problems With Single Value Thresholding

- **Single** value thresholding only works for **bimodal** histograms
- Images with other kinds of histograms need more than a single threshold

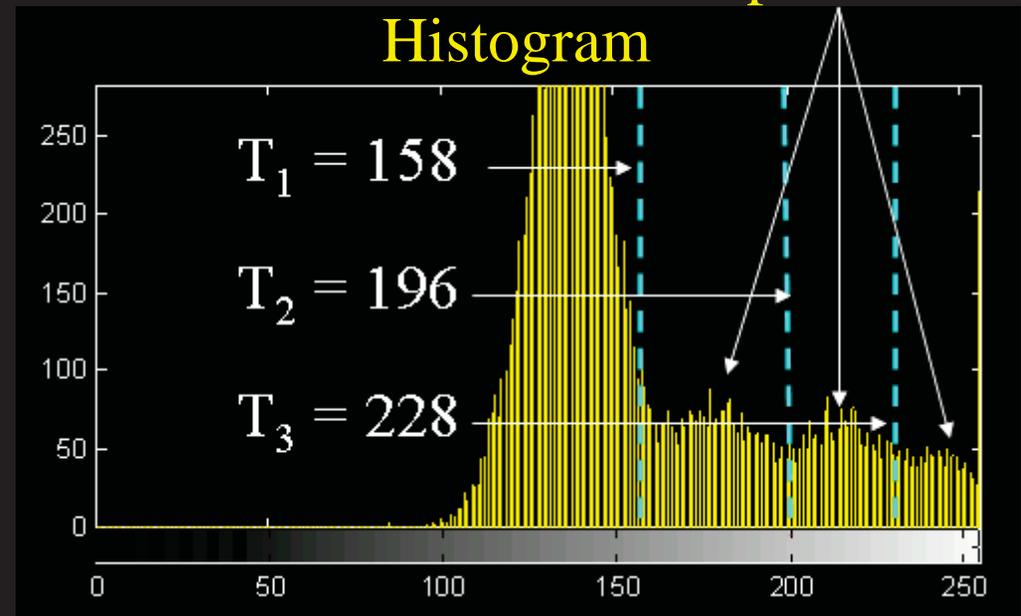
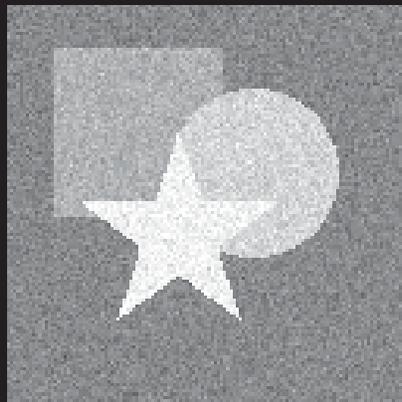


# Graylevel Histogram-based Thresholding

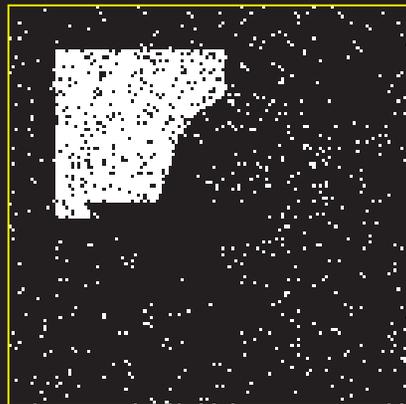
Automatic Threshold Level Selection (Basic Global Thresholding)  
peak

## Noise Problem

Image degraded by  
Gaussian noise  
( $\sigma=12$ )



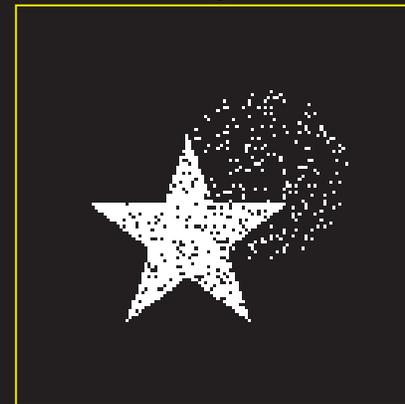
$$T_1 < P < T_2$$



$$T_2 < P < T_3$$



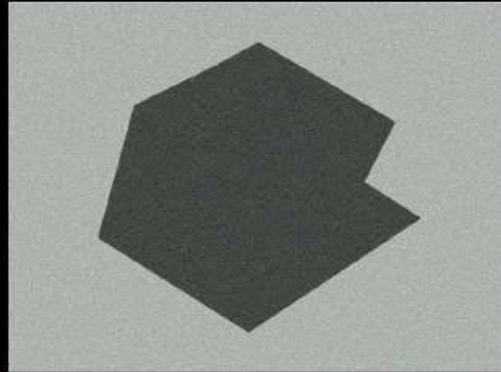
$$P > T_3$$



# Graylevel Histogram-based Thresholding

## Nonuniform Illumination Problem

Reflectance  
Function  $r(x,y)$

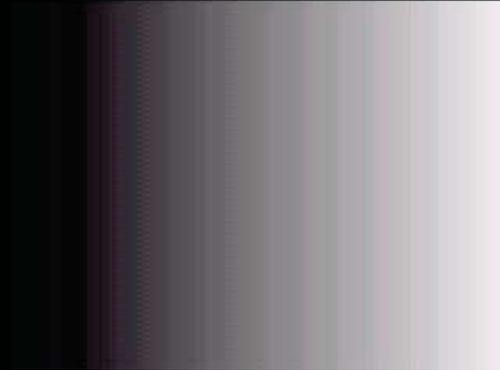
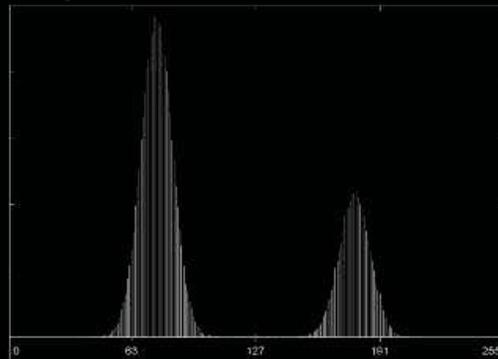


An image can be expressed as

$$f(x, y) = i(x, y)r(x, y)$$

$i(x,y)$  = illumination component  
 $r(x,y)$  = reflectance component

Histogram



Illumination  
Function  $i(x,y)$

$r(x,y)$

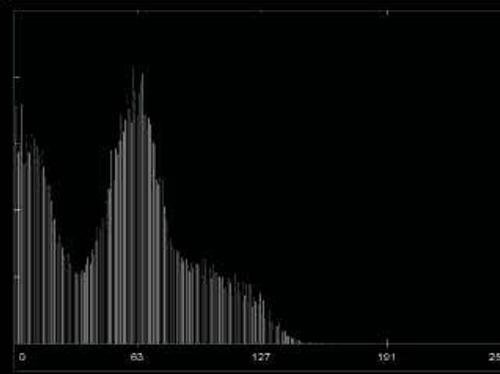
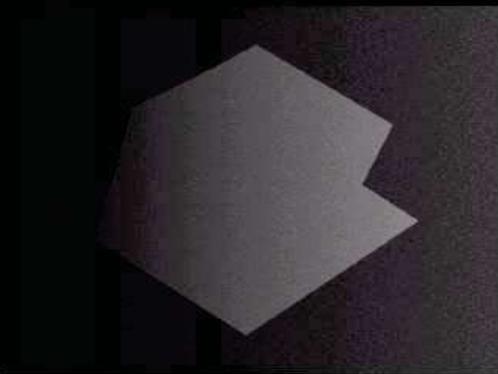


Image histogram

## Lecture 3-b

# Image Segmentation

Edge  
based

# Detection Of Discontinuities

- There are **three** basic types of grey level **discontinuities** that we tend to look for in digital images:
  1. Points
  2. Lines
  3. Edges
- We typically find discontinuities using **masks** and **correlation**

# Point Detection

❖ We can use **Laplacian** masks for point detection.

❖ Laplacian masks have the **largest coefficient** at the center of the mask while neighbor pixels have an opposite sign.

-1	-1	-1
-1	8	-1
-1	-1	-1

0	-1	0
-1	4	-1
0	-1	0

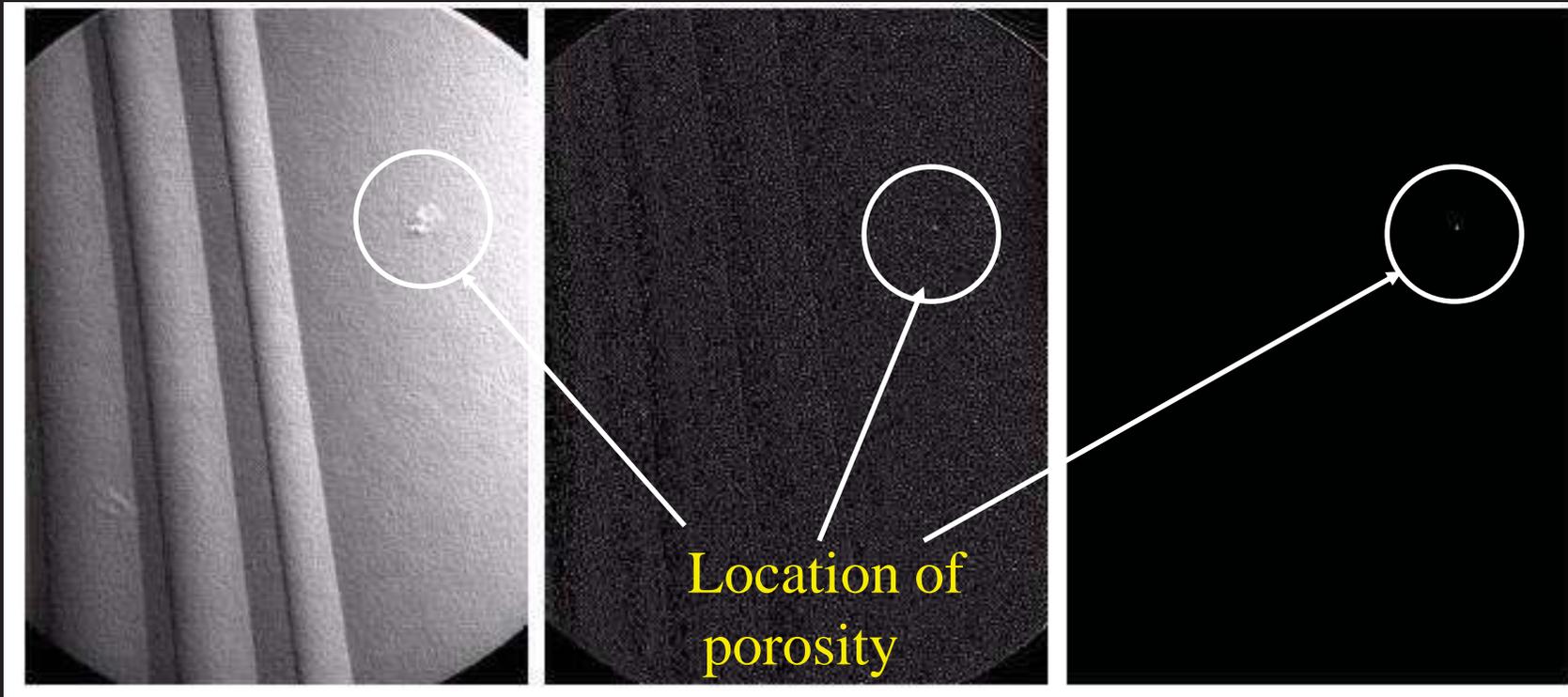
❖ This mask will give the **high response** to the object that has the **similar** shape as the mask such as **isolated points**.

❖ Notice that **sum** of all coefficients of the mask is equal to **zero**. This is due to the **need** that the response of the filter must be **zero** inside a **constant** intensity area

# Point Detection

Point detection can be done by applying the thresholding function:

$$g(x, y) = \begin{cases} 1 & |\nabla f(x, y)| \geq T \\ 0 & \text{otherwise} \end{cases}$$



X-ray image of the turbine blade with porosity

Laplacian image

After thresholding

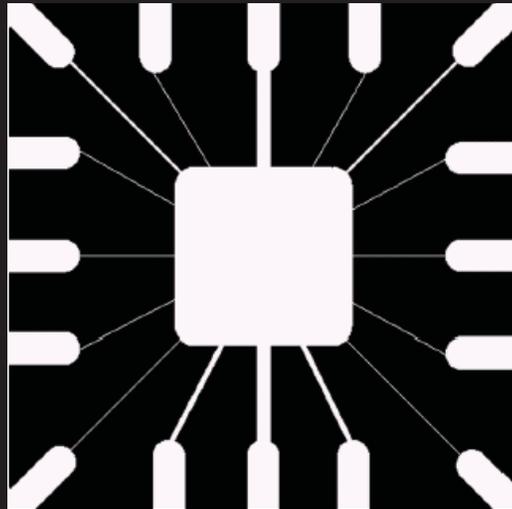
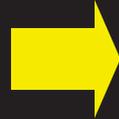
# Line Detection

- ❖ Similar to point detection, **line detection** can be performed using the mask the has the **shape look similar** to a part of a line
- ❖ There are several directions that the line in a digital image can be.
- ❖ For a simple line detection, **4 directions** that are mostly used are **Horizontal**, **+45** degree, **vertical** and **-45** degree.

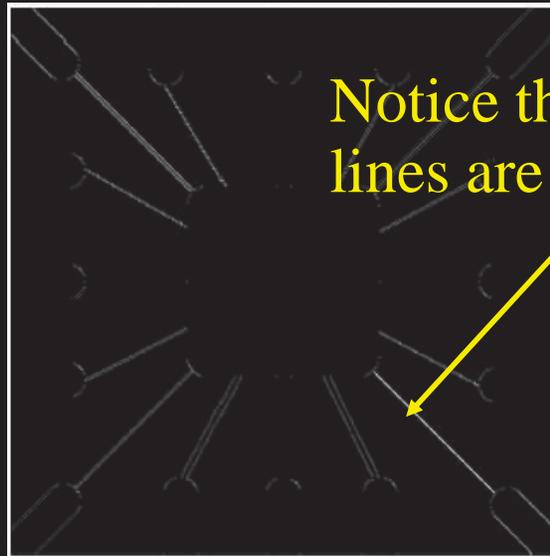
-1	-1	-1	-1	-1	2	-1	2	-1	2	-1	-1
2	2	2	-1	2	-1	-1	2	-1	-1	2	-1
-1	-1	-1	2	-1	-1	-1	2	-1	-1	-1	2
Horizontal			+45°			Vertical			-45°		

# Line Detection Example

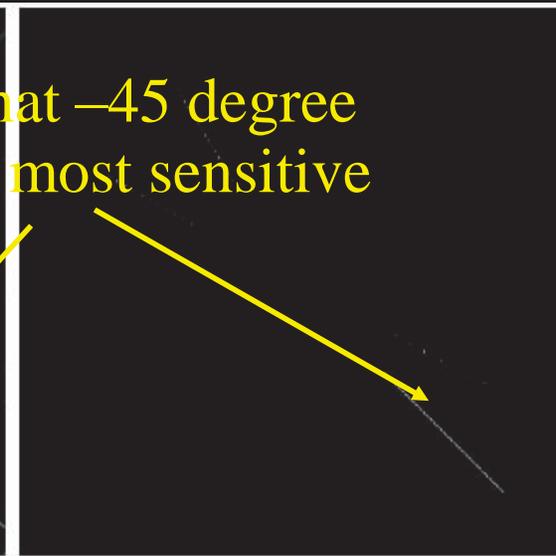
Binary wire  
bond mask  
image



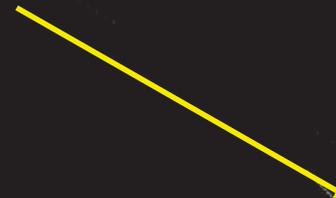
Absolute value  
of result after  
processing with  
-45 line detector



Result after  
thresholding

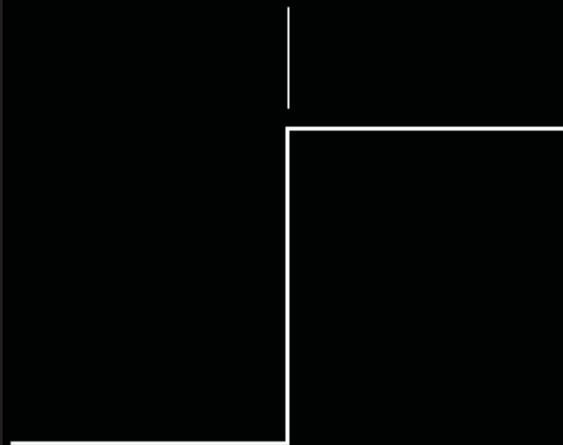


Notice that -45 degree  
lines are most sensitive



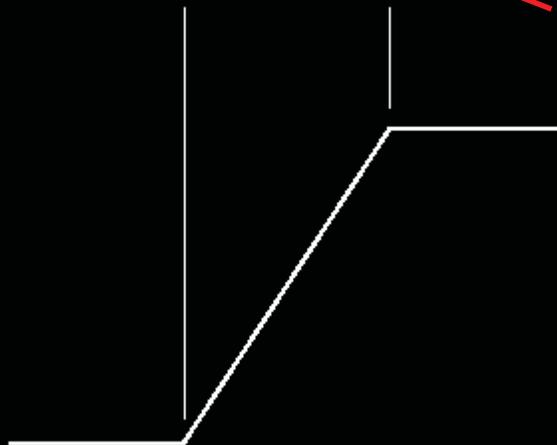
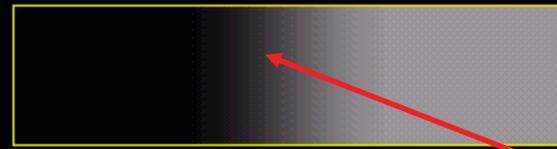
# Edges

Ideal step edge



Gray-level profile of a horizontal line through the image

Ideal ramp edge



Blurred edge

Gray-level profile of a horizontal line through the image

Generally, objects and background have different intensities. Therefore, **Edges** of the objects are the areas where **abrupt intensity changes** occur.

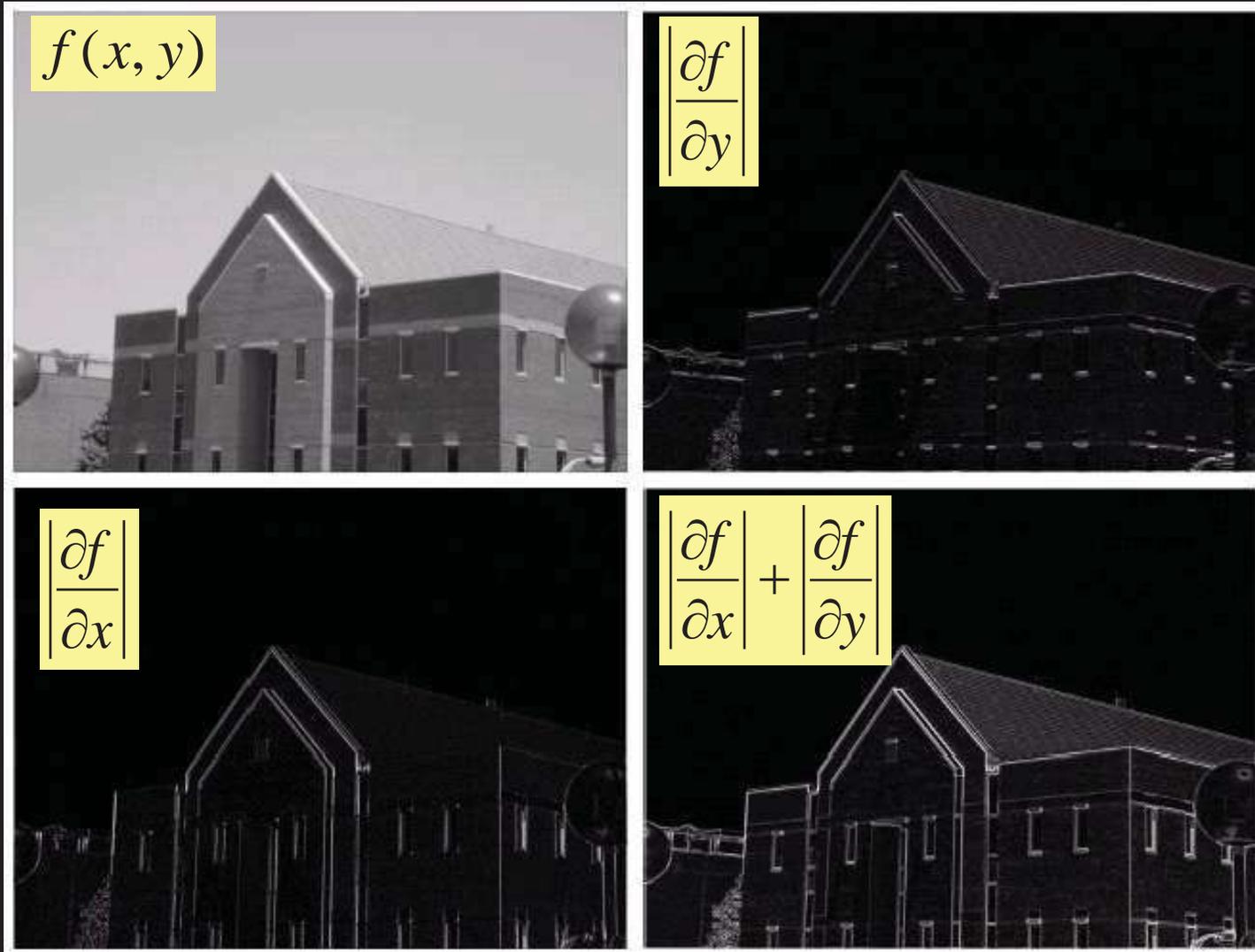
# Edge Detection Problems

Often, **problems** arise in edge detection in that there are **too much** detail

For example, the brickwork in the next example

One way to **overcome** this is **to smooth** images prior to edge detection

# Example of Image Gradient

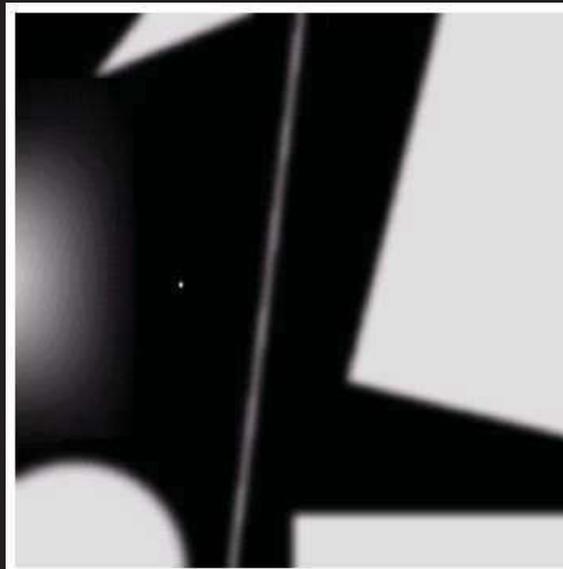


Note: the original image is smoothed by a 5x5 moving average mask first.

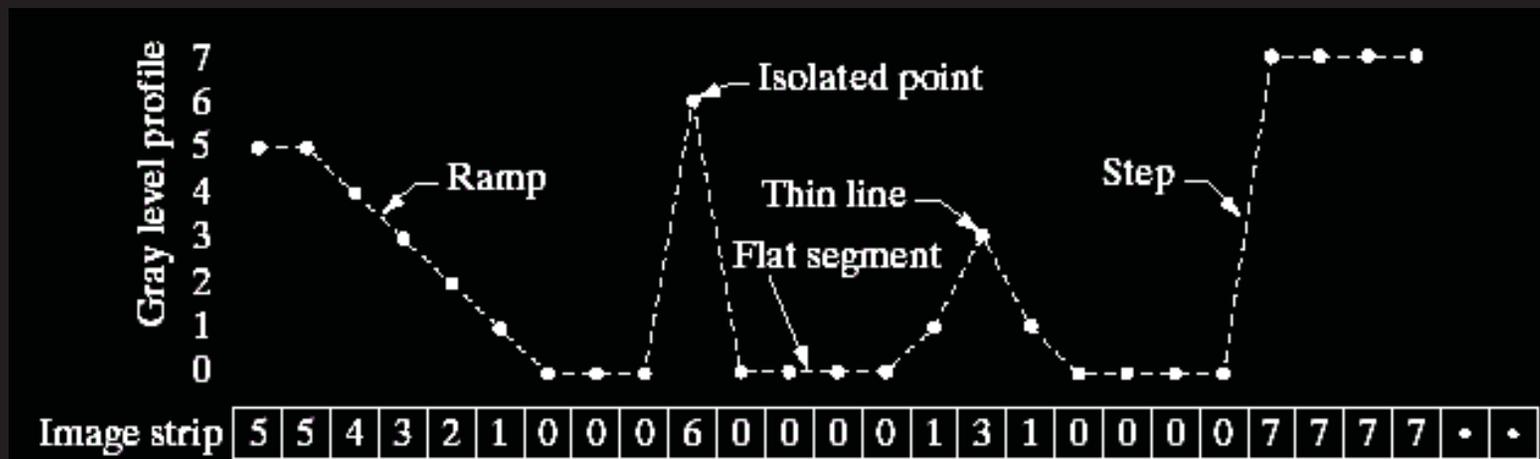
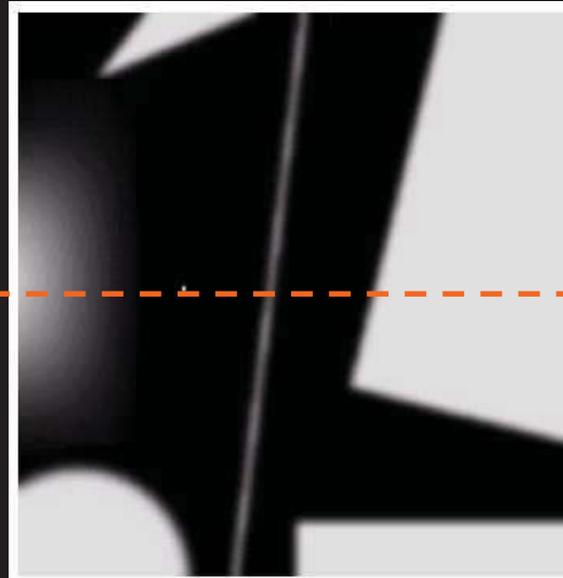
# Spatial Differentiation

Differentiation measures the *rate of change* of a function

Let's consider a simple 1 dimensional example



# Spatial Differentiation



# Spatial Differentiation – 1<sup>st</sup> Derivative

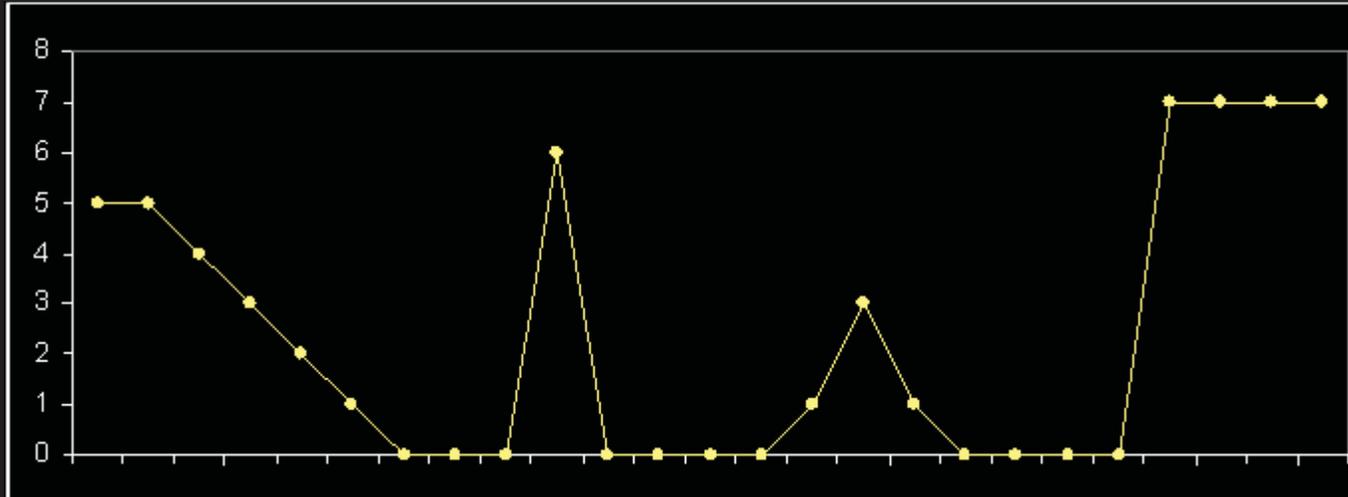
The formula for the 1<sup>st</sup> derivative of a function is as follows:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

It's just the difference between subsequent values and measures the rate of change of the function

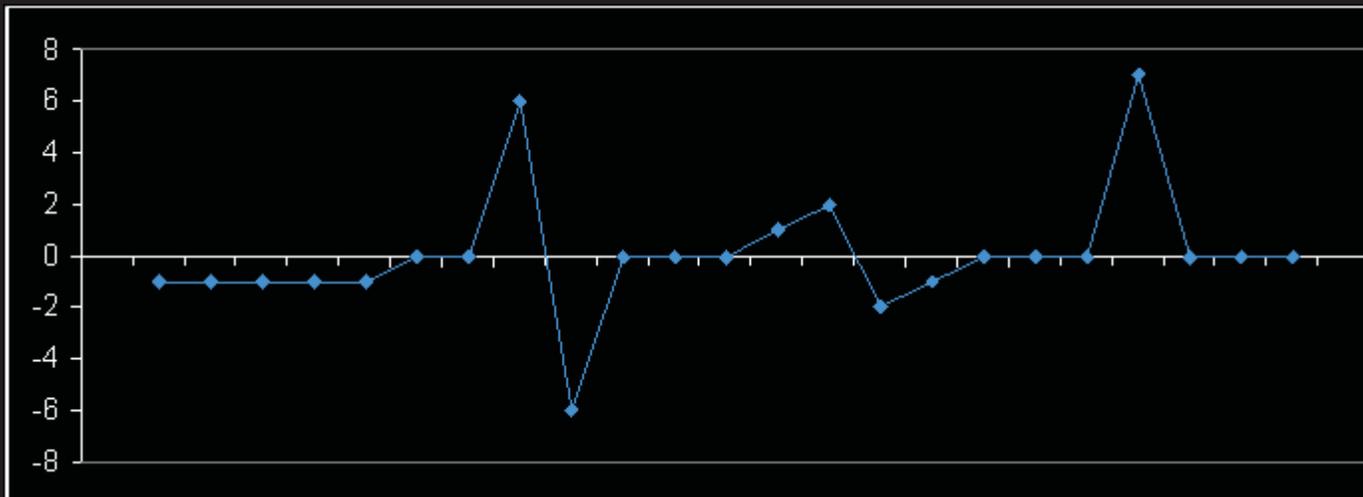
# Spatial Differentiation – 1<sup>st</sup> Derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

0	-1	-1	-1	-1	0	0	6	-6	0	0	0	0	1	2	-2	-1	0	0	0	0	7	0	0	0
---	----	----	----	----	---	---	---	----	---	---	---	---	---	---	----	----	---	---	---	---	---	---	---	---



# Spatial Differentiation – 1<sup>st</sup> Derivative

Implementing 1<sup>st</sup> derivative filters is difficult in practice

For a function  $f(x, y)$  the **gradient** of  $f$  at coordinates  $(x, y)$  is given as the column vector:

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

# Spatial Differentiation – 1<sup>st</sup> Derivative

**The magnitude** of the **gradient** vector is given by:

$$\begin{aligned}\nabla f &= \text{mag}(\nabla f) \\ &= \left[ G_x^2 + G_y^2 \right]^{1/2} \\ &= \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right]^{1/2}\end{aligned}$$

For practical reasons this can be simplified as:

$$\nabla f \approx |G_x| + |G_y|$$

## Spatial Differentiation – 1<sup>st</sup> Derivative

For practical reasons, **the magnitude** of the **gradient** vector is given by:

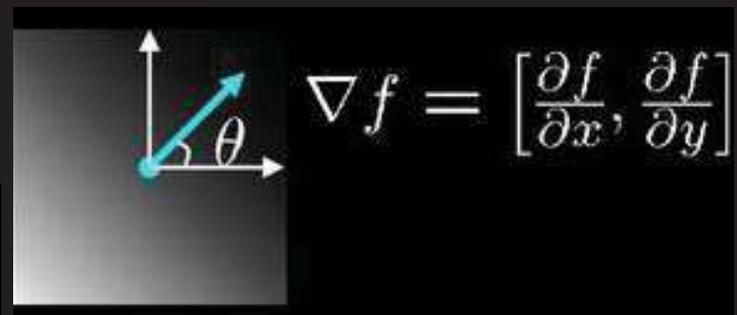
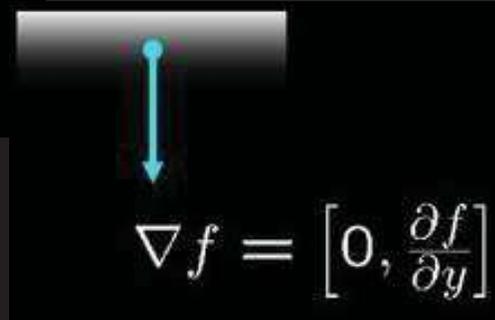
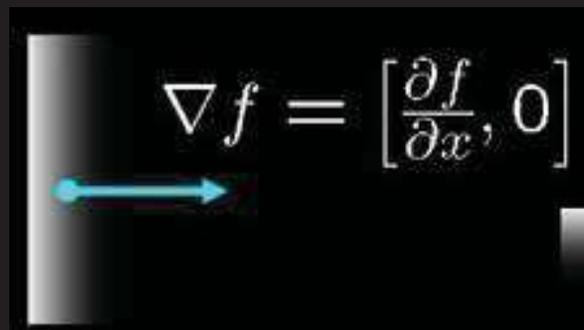
$$\nabla f \approx |G_x| + |G_y|$$

# Spatial Differentiation – 1<sup>st</sup> Derivative

**The angle (Direction)** of the **gradient vector** is given by:

$$\theta = \tan^{-1} \left( \frac{\partial f / \partial y}{\partial f / \partial x} \right) = \tan^{-1} ( \mathbf{G}_y / \mathbf{G}_x )$$

How dose this relate to the image direction ?



# Spatial Differentiation – 1<sup>st</sup> Derivative

There is some debate as to how best to calculate these gradients but we will use:

$$\nabla f \approx \left| (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \right| \\ + \left| (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \right|$$

which is based on these coordinates

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

# Spatial Differentiation – 1<sup>st</sup> Derivative

## Sobel Operators

Based on the previous equations we can derive the *Sobel Operators*

$$\mathbf{G_x} = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad \mathbf{G_y} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

To filter an image it is filtered using both operators the results of which are added together

# Sobel Operators

## Gradient detectors

For y = 2 To wid - 1: For x = 2 To hgt - 1

men = (f(x - 1, y - 1) + f(x + 1, y - 1) + f(x - 1, y + 1) + f(x + 1, y + 1) + f(x, y + 1) + f(x, y - 1) + f(x + 1, y) + f(x - 1, y)) / 8

## Sobel detector

**Gx** = -f(x - 1, y - 1) - f(x + 1, y - 1) + f(x - 1, y + 1) + f(x + 1, y + 1) + 2 \* (f(x, y + 1) - f(x, y - 1))

**Gy** = -f(x - 1, y - 1) + f(x + 1, y - 1) - f(x - 1, y + 1) + f(x + 1, y + 1) + 2 \* (f(x + 1, y) - f(x - 1, y))

gr = ((Gx \* Gx + Gy \* Gy) \* 0.5)

If Abs(gr - men) > 30 Then gr = 1 Else gr = 0

gr = gr\*255

Form1.PSet (x , y), gr

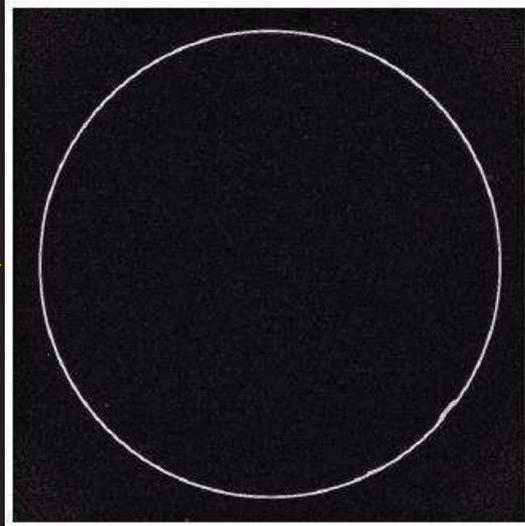
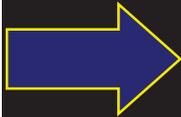
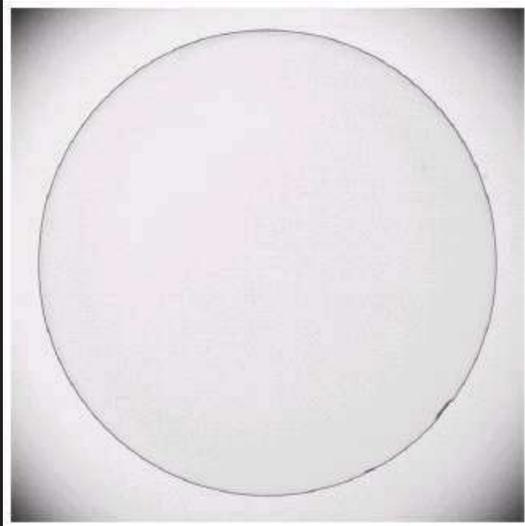
Next: Next

-1	-2	-1		-1	0	1
0	0	0		-2	0	2
1	2	1		-1	0	1

# Spatial Differentiation – 1<sup>st</sup> Derivative

## Sobel Example

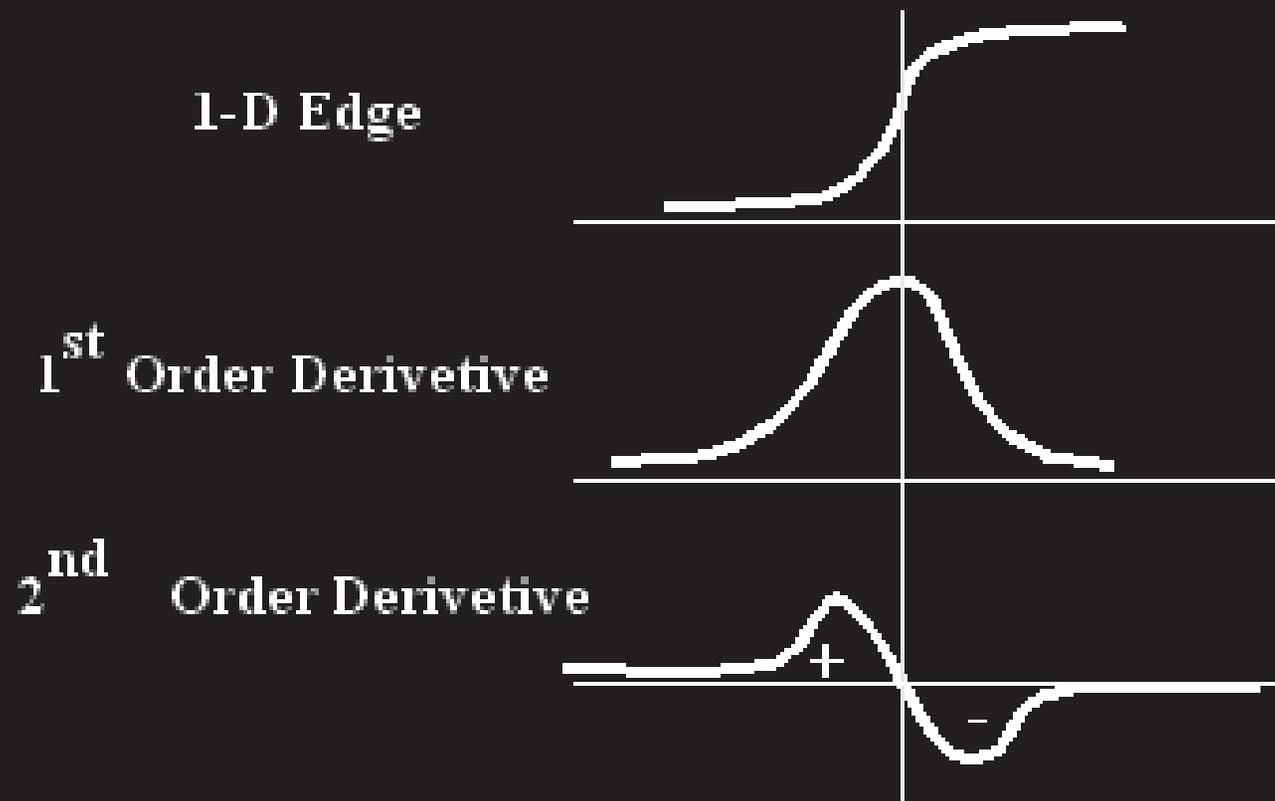
Images taken from Gonzalez & Woods, Digital Image Processing (2002)



**An image of a contact lens which is enhanced in order to make defects (at four and five o'clock in the image) more obvious**

Sobel filters are typically used for edge detection

# Spatial Differentiation – 1<sup>st</sup> Derivative



## *Edge linking and boundary detection*

- Edge detection is always followed by edge linking Local processing
- Analyze pixels in small neighbourhood  $S_{xy}$  of each edge point
- Pixels that are similar are linked
- Principal properties used for establishing similarity:

(1)  $M(x, y)$  = Magnitude of gradient vector

(2)  $\theta(x, y)$  = Direction of gradient vector

## *Edge linking and boundary detection*

$$\nabla f \approx |G_x| + |G_y|$$

$$\theta = \tan^{-1} (G_y / G_x)$$

# *Edge linking* strategy

- Edge pixel with coordinates  $(s, t)$  in  $Sxy$  is similar in **magnitude** to pixel at  $(x, y)$   
*if*  $|M(s, t) - M(x, y)| < E$
- Edge pixel with coordinates  $(s, t)$  in  $Sxy$  has an **angle** similar to pixel at  $(x, y)$   
*if*  $|\theta(s, t) - \theta(x, y)| < A$
- Edge pixel  $(s, t)$  in  $Sxy$  is **linked** with  $(x, y)$  if both criteria are satisfied

## *Edge linking and boundary detection*

- The above strategy is expensive.
- A record has to be kept of all linked points by, for example, assigning a different label to every set of linked points

## *Edge linking and boundary detection*

Simplification suitable for real-time applications:

(1) Compute  $M(x, y)$  and  $\theta(x, y)$  of input image  $f(x, y)$

(2) Form binary image

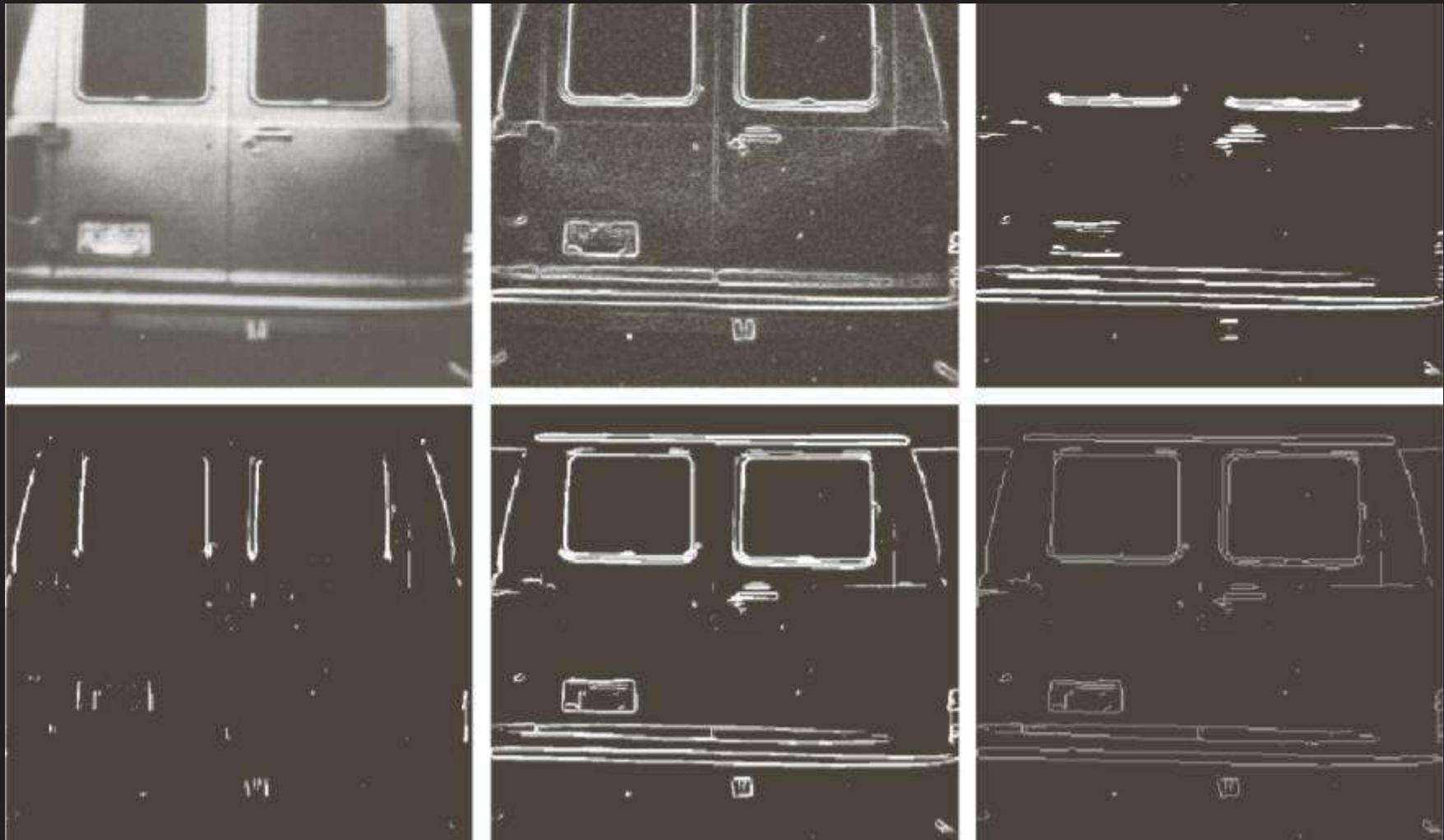
$$g(x, y) = \begin{cases} 1, & \text{if } M(x, y) > T_M \text{ AND } \alpha(x, y) \in [A - T_A, A + T_A] \\ 0, & \text{otherwise} \end{cases}$$

(3) Scan rows of  $g$  and fill (set to  $1$ ) all gaps (sets of  $0s$ ) in each row that do not exceed a specified length  $K$

(4) Rotate  $g$  by  $\theta$  and apply step (3). Rotate result back by  $-\theta$ .

## *Edge linking and boundary detection*

- Image rotation is expensive  $\Rightarrow$  when linking in numerous directions is required, steps (3) and (4) are combined into a single, radial scanning procedure.



**FIGURE 10.27** (a) A  $534 \times 566$  image of the rear of a vehicle. (b) Gradient magnitude image. (c) Horizontally connected edge pixels. (d) Vertically connected edge pixels. (e) The logical OR of the two preceding images. (f) Final result obtained using morphological thinning. (Original image courtesy of Perceptics Corporation.)

## Lecture 3-c

# Image Segmentation

## Region growing based

## Region growing

- Its techniques start with one pixel of a potential region and try to grow it by adding **adjacent** pixels till the pixels being compared are too dissimilar.
- The first pixel selected can be just the first unlabeled pixel in the image or a set of **seed pixels** can be chosen from the image.
- Usually a **statistical** test is used to decide which pixels can be added to a region.
  - Region is a population with similar statistics.
  - Use statistical test to see if neighbor on border fits into the region population.

# Region Growing : Algorithm

- Let  $R$  be the  $N$  pixel region so far and  $p$  be a neighboring pixel with gray tone  $y$ .
- Define the mean  $X$  and scatter  $S^2$  (sample **variance**) by:

$$\bar{X} = \frac{1}{N} \sum_{(r,c) \in R} I(r,c)$$

$$S^2 = \frac{1}{N} \sum_{(r,c) \in R} (I(r,c) - \bar{X})^2$$

# Region Growing : Algorithm

- The **T** statistic is defined by

$$T = \left( \frac{(N-1)N}{(N+1)} (p - \bar{X})^2 / S^2 \right)^{1/2}$$

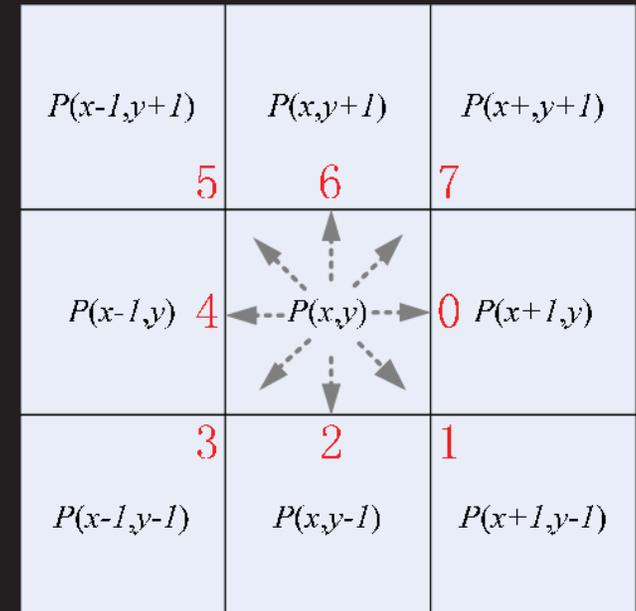
- It has a  $T_{N-1}$  distribution if all the pixels in **R** and the test pixel **p** are independent and identically distributed Gaussians (independent assumption).

# Region Growing : Algorithm

- For the  $T$  distribution, statistical tables give us the probability  $\Pr(T \leq t)$  for a given degrees of freedom and a confidence level. From this, pick a suitable threshold  $t$ .
- **If the computed  $T \leq t$**  for desired confidence level, add  $p$  to region  $R$  and update the mean and scatter using  $p$ .
- If  $T$  is too high, the value  $p$  is not likely to have arisen from the population of pixels in  $R$ .
- Start a new region.

# Region Growing : Algorithm

Start with a single (seed) and add new pixels slowly



1. choose the seed pixel
2. check the neighboring and add them to the region if they are similar to the seed
3. repeat step 2 for each of the newly added pixels; stop if no more pixels can added
4.  $ABS(f(\text{seed}) - f(\text{pixel})) < \text{threshold}$

# Example

$$|f(\text{seed}) - f(\text{neighbors})| \leq 1$$

α)

	1	2	3	4	5	6	7
1	5	6	1	1	1	0	0
2	6	7	7	1	1	1	6
3	3	6	7	7	7	6	5
4	4	3	7	7	7	5	3
5	3	3	2	1	3	3	4
6	2	2	1	2	1	2	3
7	1	1	1	1	1	2	2



α)

	1	2	3	4	5	6	7
1	5	6	1	1	1	0	0
2	6	7	7	1	1	1	6
3	3	6	7	7	7	6	5
4	4	3	7	7	7	5	3
5	3	3	2	1	3	3	4
6	2	2	1	2	1	2	3
7	1	1	1	1	1	2	2

# Example

$$|f(\text{seed}) - f(\text{neighbors})| \leq 1$$

$\beta$ )	1	2	3	4	5	6	7
1	$\beta$	$\beta$	$\alpha$	$\alpha$	$\alpha$	$\alpha$	$\alpha$
2	$\beta$	$\beta$	$\beta$	$\alpha$	$\alpha$	$\alpha$	$\beta$
3	$\gamma$	$\beta$	$\beta$	$\beta$	$\beta$	$\beta$	$\beta$
4	$\gamma$	$\gamma$	$\beta$	$\beta$	$\beta$	$\beta$	$\gamma$
5	$\gamma$	$\gamma$	$\gamma$	$\alpha$	$\gamma$	$\gamma$	$\gamma$
6	$\gamma$	$\gamma$	$\alpha$	$\gamma$	$\alpha$	$\gamma$	$\gamma$
7	$\alpha$	$\alpha$	$\alpha$	$\alpha$	$\alpha$	$\gamma$	$\gamma$

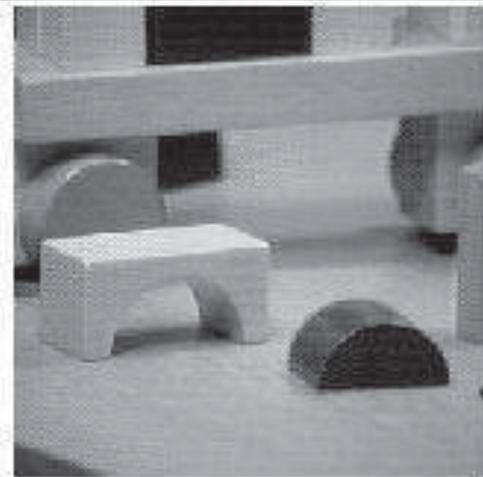
## Region Growing – seeded segmentation

**Note that a complete segmentation of an image must satisfy a number of criteria:**

- All pixels must be assigned to regions
- Each pixel must belong to a single region only
- Each region must be a connected set of pixels
- Each region must be uniform
- Any merged pair of adjacent regions must be non-uniform
- Better in noisy image where edges are hard to identify

# Region growing

image



segmentation



# Lecture 3-d

## Image Segmentation

Split-and-merge  
based

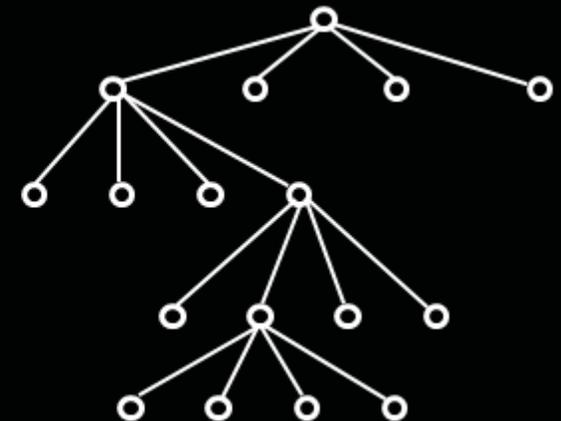
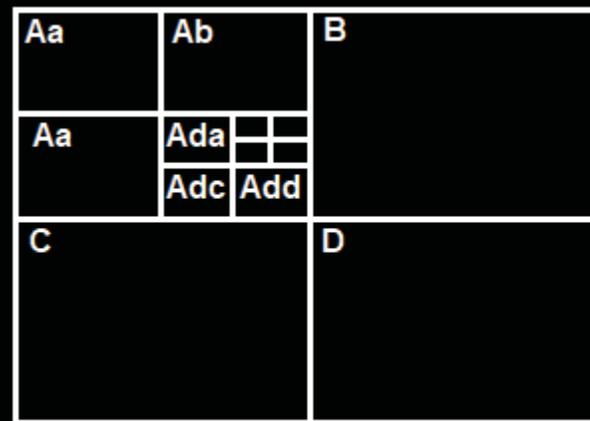
# Split-and-merge

- The opposite approach to region growing is region shrinking ( **splitting** ).
- It is a top-down approach and it starts with the assumption that the entire image is homogeneous
- If this is not true , the image is split into four sub images
- This splitting procedure is repeated recursively until we split the image into homogeneous regions

# Split-and-merge

1. Start with the whole image.
2. If the variance is too high, **break** into quadrants.
3. **Merge** any adjacent regions that are similar enough.
4. Repeat steps 2 and 3, iteratively until no more **splitting** or **merging** occur.

→ Idea: good  
Results: blocky

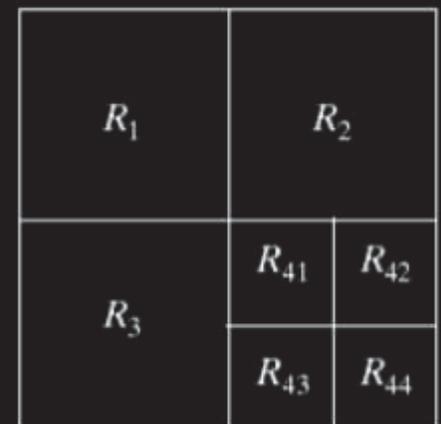


## Region Splitting

- Region growing starts from a set of seed point
- An alternative is to start with whole image as single region and subdivided the region that do not satisfy a condition of homogeneity

# Region Merging

- Region merging is the opposite of region splitting
- starts with small regions (e.g.  $2 \times 2$  or  $4 \times 4$  regions) and merge the region that have similar characteristics (such as gray level, variance)
- Typically, splitting and merging are used iteratively



# Split and Merge Approach

This is a 2 step procedure:

- **top-down:** split image into homogeneous quadrant regions
- **bottom-up:** merge similar adjacent regions

The algorithm includes:

## Top-down

- Successively subdivide image into quadrant regions  $R_i$
- Stop when all regions are homogeneous:  $P(R_i) = \text{TRUE}$  obtain quadtree structure

## Bottom-up

- At each level, merge adjacent regions  $R_i$  and  $R_j$  if  $P(R_i \cup R_j) = \text{TRUE}$
- Iterate until no further splitting/merging is possible

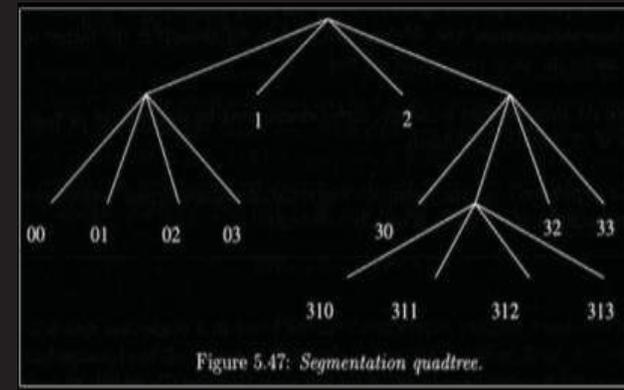
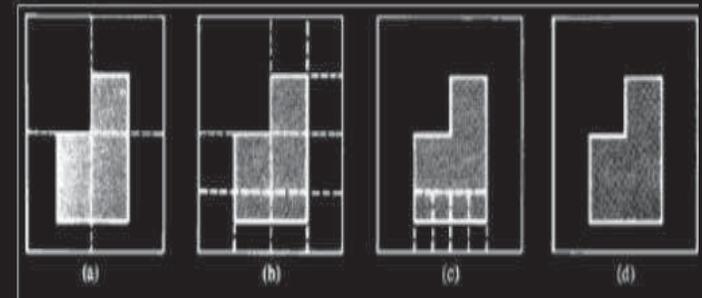
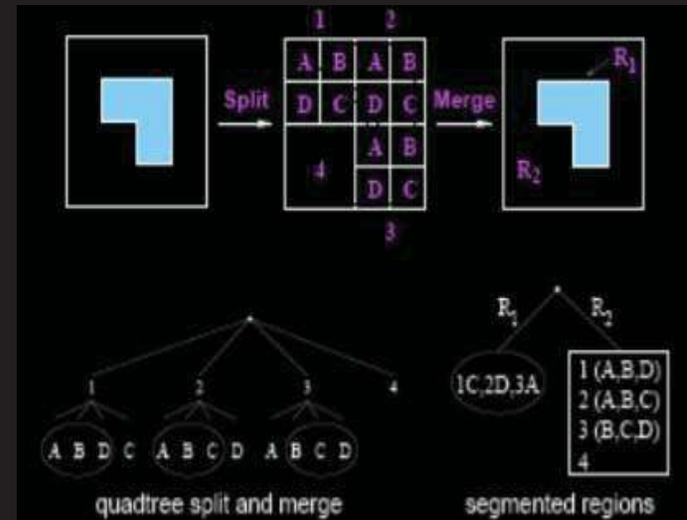
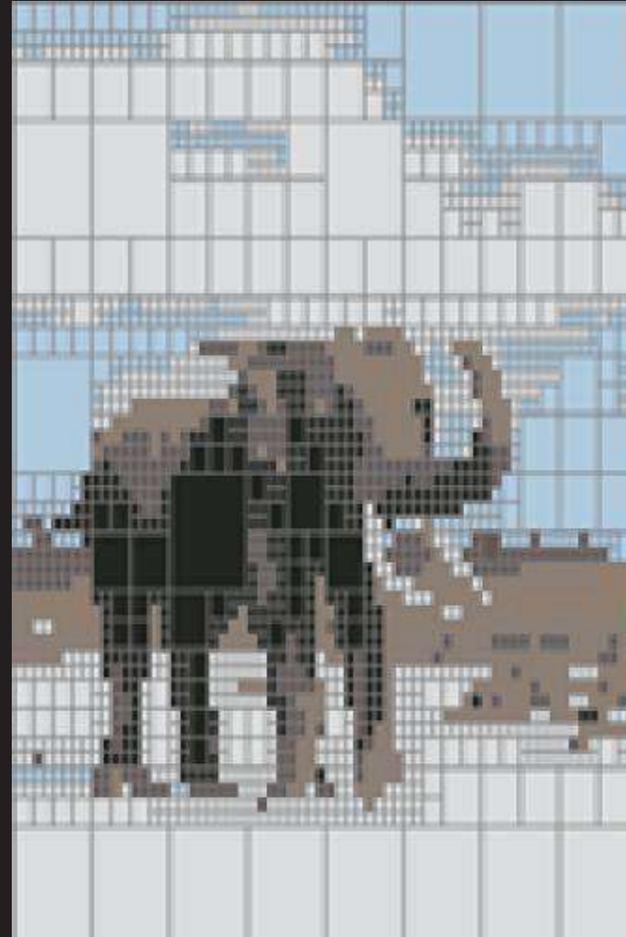


Figure 5.47: Segmentation quadtree.

# Split-and-merge



- Split-and-merge algorithm combines these two ideas
  - Split image into quadtree, where each region satisfies homogeneity criterion
  - Merge neighboring regions if their union satisfies criterion (like connected components)



**image**



**after split**

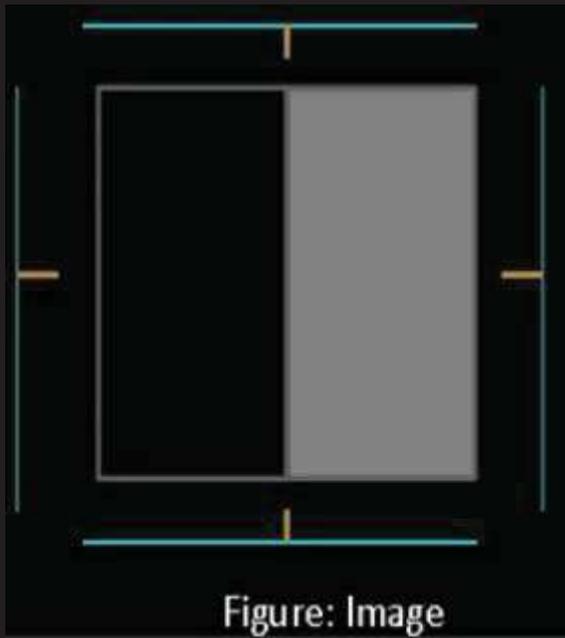


**after merge**

S. L. Horowitz and T. Pavlidis, Picture Segmentation by a Tree Traversal Algorithm, 1976

S. Birchfield, Clemson Univ., ECE 847, <http://www.ces.clemson.edu/~stb/ece847>

# Split-and-merge - Example



# Split-and-merge - Example

Sample image



1<sup>st</sup> split

1	1	1	1	1	1	1	2
1	1	1	1	1	1	1	0
3	1	4	9	9	8	1	0
1	1	8	8	8	4	1	0
1	1	6	6	6	3	1	0
1	1	5	6	6	3	1	0
1	1	5	6	6	2	1	0
1	1	1	1	1	1	0	0

1	1	1	1	1	1	1	2
1	1	1	1	1	1	1	0
3	1	4	9	9	8	1	0
1	1	8	8	8	4	1	0
1	1	6	6	6	3	1	0
1	1	5	6	6	3	1	0
1	1	5	6	6	2	1	0
1	1	1	1	1	1	0	0

# Split-and-merge - Example

2<sup>nd</sup> split



3<sup>rd</sup> split

1	1	1	1	1	1	1	2
1	1	1	1	1	1	1	0
3	1	4	9	9	8	1	0
1	1	8	8	8	4	1	0
1	1	6	6	6	3	1	0
1	1	5	6	6	3	1	0
1	1	5	6	6	2	1	0
1	1	1	1	1	1	0	0

1	1	1	1	1	1	1	2
1	1	1	1	1	1	1	0
3	1	4	9	9	8	1	0
1	1	8	8	8	4	1	0
1	1	6	6	6	3	1	0
1	1	5	6	6	3	1	0
1	1	5	6	6	2	1	0
1	1	1	1	1	1	0	0

# Split-and-merge - Example

Merge



Final result

1	1	1	1	1	1	1	2
1	1	1	1	1	1	1	0
3	1	4	9	9	8	1	0
1	1	8	8	8	4	1	0
1	1	6	6	6	3	1	0
1	1	5	6	6	3	1	0
1	1	5	6	6	2	1	0
1	1	1	1	1	1	0	0

1	1	1	1	1	1	1	2
1	1	1	1	1	1	1	0
3	1	4	9	9	8	1	0
1	1	8	8	8	4	1	0
1	1	6	6	6	3	1	0
1	1	5	6	6	3	1	0
1	1	5	6	6	2	1	0
1	1	1	1	1	1	0	0

# Algorithm: Region Splitting

- Form initial region in the image
- For each region in an image, recursively perform:
  1. Compute the variance in the gray values for the region
  2. If the variance is above a threshold, split the region along the appropriate boundary
  3. If some property of a region is not constant
  4. Regular decomposition Methods: divide the region into a fixed number of equal-sized regions.

# Algorithm: Region Merging

- (1) Form initial regions in the image using thresholding ( or a similar approach) followed by component labeling.
- (2) Prepare a region adjacency graph (RAG) for the image.
- (3) **For** each region in an image, perform the following steps:
  - (a) **Consider** its adjacent region and test to see if they are similar.
  - (b) For regions that are similar, merge them and modify the RAG.
- (1) Repeat step 3 until no regions are merged.

# Applications

1. In image compression
2. Object recognition
3. Computer graphics
4. Medical Imaging
5. MPEG-4 video object (VO) segmentation

Thank you

Any Questions ?

**END**

**Of Lecture**

# Quiz

<https://docs.google.com/forms/d/e/1FAIpQLScjoh9e6E1HRInJ8jn0wsNtrBzvCE4PdQ75Dmhv2IQJ6IVH7Q/viewform>

- A simple approach to image segmentation is to start from some pixels (**seeds**) representing distinct image regions and to grow them, until they cover the entire image
- For region growing we need a **rule describing a growth mechanism** and a **rule checking the homogeneity** of the regions after each growth step

- The growth mechanism
- at each stage  $k$  and for each region  $R_i(k)$ ,  $i = 1, \dots, N$ , we check if there are unclassified pixels in the 8-neighbourhood of each pixel of the region border
- Before assigning such a pixel  $x$  to a region  $R_i(k)$ , we check if the region homogeneity:
  - $P(R_i(k) \cup \{x\}) = \text{TRUE}$  , is valid

The arithmetic mean  $m$  and standard deviation  $sd$  of a class  $R_i$  having  $n$  pixels:

$$M = (1/n) \sum_{(r,c) \in R(i)} I(r,c)$$
$$s.d = \text{Square root} \left( (1/n) \sum_{(r,c) \in R(i)} [I(r,c) - M]^2 \right)$$

Can be used to decide if the merging of the two regions  $R_1, R_2$  is allowed, if  $|M_1 - M_2| < (k)s.d(i)$ ,  $i = 1, 2$ , two regions are merged

**Homogeneity test:** if the pixel intensity is close to the region mean value

$$|I(r,c) - M(i)| \leq T(i)$$

Threshold  $T_i$  varies depending on the region  $R_n$  and the intensity of the pixel  $I(r,c)$ . It can be chosen this way:

$$T(i) = \{ 1 - [s.d(i)/M(i)] \} T$$

# Segmentation as partitioning

- A *partition* of image is collection of sets  $S_1, \dots, S_N$  such that

$$I = S_1 \cup S_2 \dots \cup S_N \quad (\text{sets cover entire image})$$

$$S_i \cap S_j = \emptyset \text{ for all } i \neq j \quad (\text{sets do not overlap})$$

- A *predicate*  $H(S_i)$  measures region *homogeneity*

$$H(R) = \begin{cases} \text{true} & \text{if pixels in region } R \text{ are similar} \\ \text{false} & \text{otherwise} \end{cases}$$

- We want
  1. Regions to be homogeneous

$$H(S_i) = \text{true} \text{ for all } i$$

2. Adjacent regions to be different from each other

$$H(S_i \cup S_j) = \text{false} \text{ for all adjacent } S_i, S_j$$

# Two approaches

- Splitting  
(Divisive clustering)

- start with single region covering entire image
- repeat: split inhomogeneous regions
- even better:  
repeat: split cluster to yield two distant components (difficult)

*Property 2 is always true:*

$H(S_i \cup S_j) = \text{false}$  for adjacent regions

*Goal is to satisfy Property 1:*

$H(S_i) = \text{true}$  for every region

- Merging  
(Agglomerative clustering)

- start with each pixel as a separate region
- repeat: merge adjacent regions if union is homogeneous
- even better:  
repeat: merge two closest clusters

*Property 1 is always true:*

$H(S_i) = \text{true}$  for every region

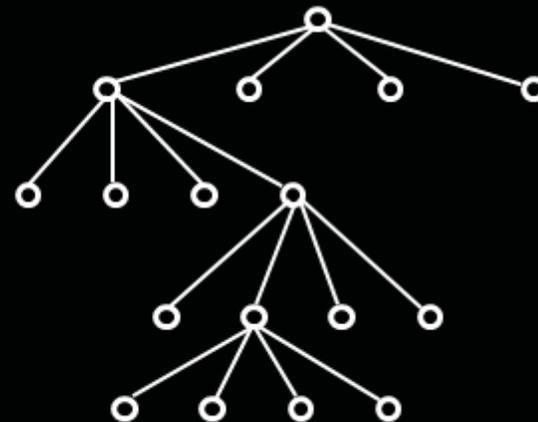
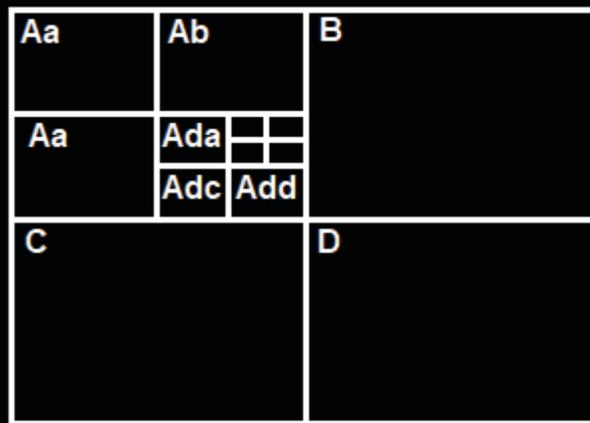
*Goal is to satisfy Property 2:*

$H(S_i \cup S_j) = \text{false}$  for adjacent regions

**In practice, merging works much better than splitting**

# Region splitting

- Start with entire image as a single region
- Repeat:
  - Split any region that does not satisfy homogeneity criterion into subregions
- Quad-tree representation is convenient
- Then need to merge regions that have been split



## Region Growing : Algorithm 2

1. Chose or determined a group of seed pixel which can correctly represent the required region;
2. Fixed the formula which can contain the adjacent pixels in the growth;
3. Made rules or conditions to stop the growth process



## Spatial Differentiation – 2<sup>nd</sup> Derivative

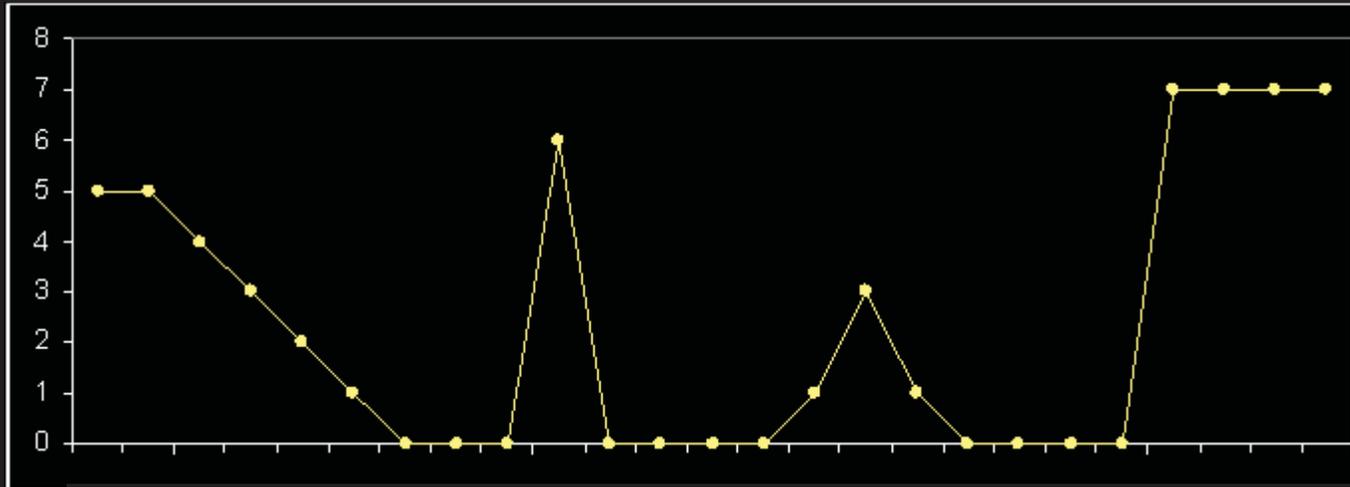
The formula for the 2<sup>nd</sup> derivative of a function is as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

Simply takes into account the values both before and after the current value

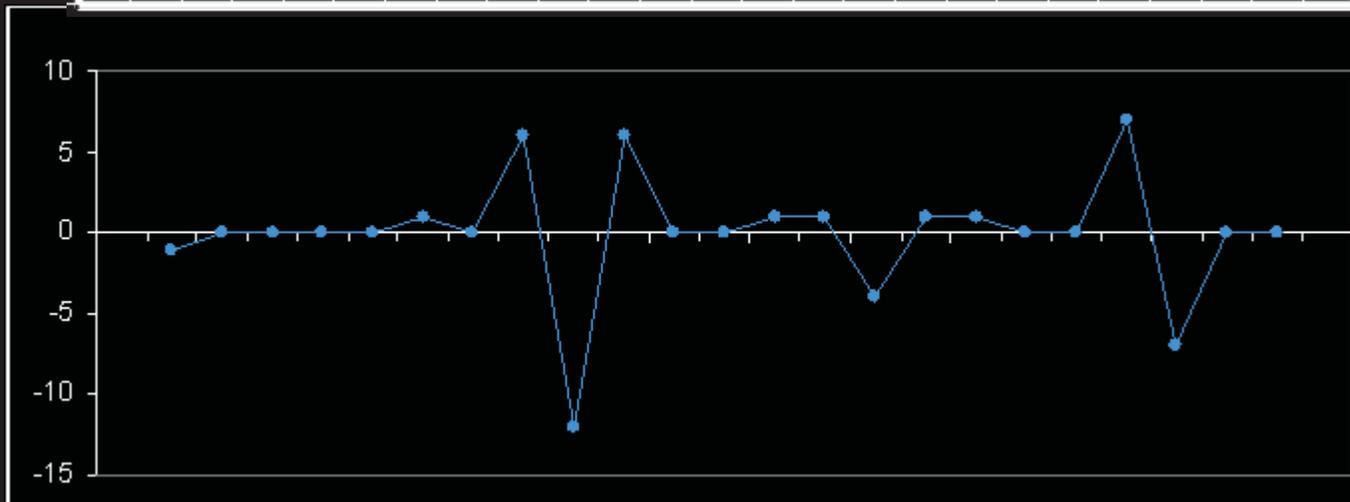
# Spatial Differentiation – 2<sup>nd</sup> Derivative

$$\frac{\partial^2 f}{\partial x^2} = \frac{f(x+1) + f(x-1) - 2f(x)}{\Delta x^2}$$



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	0	0
----	---	---	---	---	---	---	---	-----	---	---	---	---	---	----	---	---	---	---	---	----	---	---	---	---



# Spatial Differentiation – 2<sup>nd</sup> Derivative

## Using 2<sup>nd</sup> Derivatives For Image Enhancement

The 2<sup>nd</sup> derivative is **more useful** for image enhancement than the 1<sup>st</sup> derivative

- Stronger response to fine detail
- Simpler implementation
- We will come back to the 1<sup>st</sup> order derivative later on

The first sharpening filter we will look at is the ***Laplacian***

- Isotropic
- One of the simplest sharpening filters
- We will look at a digital implementation

# Spatial Differentiation – 2<sup>nd</sup> Derivative

## The Laplacian

The Laplacian is defined as follows:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

where the partial 1<sup>st</sup> order derivative in the  $x$  direction is defined as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

and in the  $y$  direction as follows:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

# Spatial Differentiation – 2<sup>nd</sup> Derivative

## The Laplacian

So, the Laplacian can be given as follows:

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

We can easily build a filter based on this:

0	1	0
1	-4	1
0	1	0

# Spatial Differentiation – 2<sup>nd</sup> Derivative

## The Laplacian

Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities



Original  
Image

Laplacian  
Filtered Image

Laplacian  
Filtered Image  
Scaled for Display

# Spatial Differentiation – 2<sup>nd</sup> Derivative

## The Laplacian

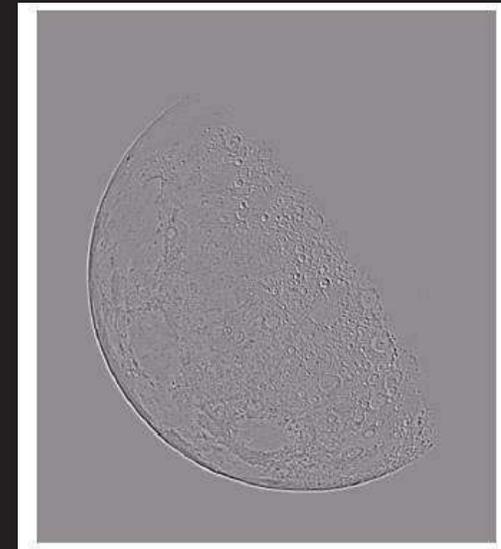
### But That Is Not Very Enhanced!

The result of a Laplacian filtering is not an enhanced image

We have to do more work in order to get our final image

Subtract the Laplacian result from the original image to generate our final sharpen

$$g(x, y) = f(x, y) - \nabla^2 f$$



Laplacian  
Filtered Image  
Scaled for Display

# Spatial Differentiation – 2<sup>nd</sup> Derivative

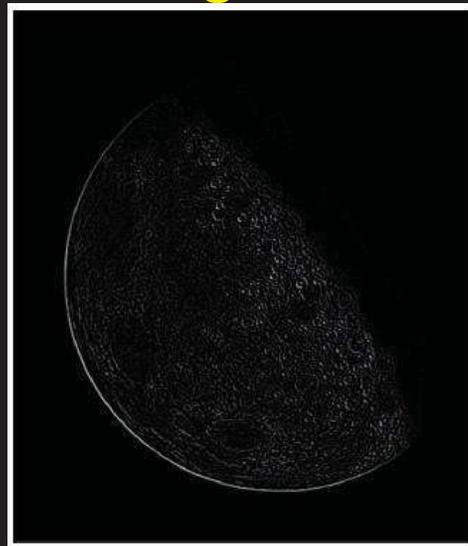
## The Laplacian

### Laplacian Image Enhancement



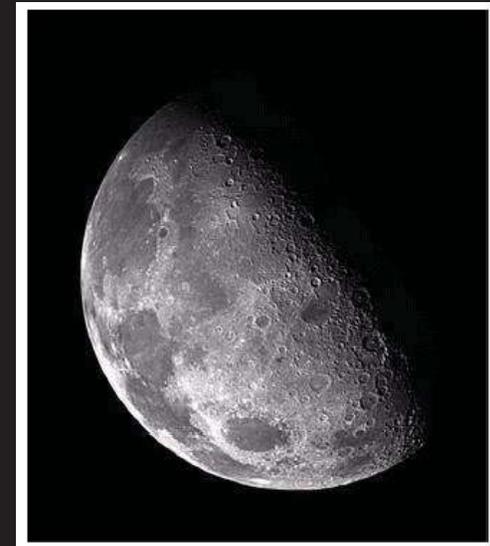
Original  
Image

-



Laplacian  
Filtered Image

=



Sharpened  
Image

In the final sharpened image edges and fine detail are much more obvious

# Spatial Differentiation – 2<sup>nd</sup> Derivative

## The Laplacian

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

## Lecture 4

# Advanced Image Processing

## Image Segmentation

# Morphological Image Processing methods

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**2022-2021**

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# Contents

Once segmentation is complete, morphological operations can be used to **remove imperfections** in the segmented image and provide information on the **form** and **structure** of the image

In this lecture we will consider

1. What is morphology ?
2. Simple morphological operations
3. **Compound operations**
4. **Morphological algorithms**

# *Contents*

Today we will continue to look at the problem of Morphological operation:

In particular we will look at:

1. Binary image morphology
2. Dilation and Erosion
3. Opening and Closing

Chapter **13.3** in

*Image Processing Analysis, and Machine Vision*

# Learning Outcomes

At the end of this lecture, you should be able to;

1. Describe the importance of morphological features in an image.
2. Describe the operation of erosion, dilation, open and close operations.
3. Identify the practical advantage of the morphological operations.
4. Apply morphological operations for problem solving.

# Learning Objectives

1. Define and apply the primary morphological operations: erosion and dilation.
2. Explain the use of relational and logical operators in the context of binary image processing.
3. Define and apply compound morphological operations, such as opening and closing.
4. Explain the effect of using structuring elements of different shapes and sizes for each morphological operation.
5. Identify and apply the appropriate morphological operations and structuring elements to achieve a given processing outcome.
6. Use simple shapes to filter objects in an image.

## What is Morphological Analysis?

- The term “morphological analysis” describes a range of non-linear image processing techniques that deal with the shape or morphology of features in an image.
- The word morphology refers to form and structure.
- Known as “mathematical morphology”
- Most morphological analysis techniques operate on binary images

## Uses for morphological analysis include:

1. Noise reduction and feature detection:- The objective is that noise be reduced as much as possible without eliminating essential features.
2. Analysis of connectivity of components
3. Object selection using geometric features



1. Dilation - grow image regions



2. Erosion - shrink image regions



3. Opening - structured removal of image region boundary pixels



4. Closing - structured filling in of image region boundary pixels



5. Hit and Miss Transform - image pattern matching and marking



6. Thinning - structured erosion using image pattern matching



7. Thickening - structured dilation using image pattern matching



8. Skeletonization/Medial Axis Transform - finding skeletons of binary regions

## Morphological Skeleton Representation and Coding of Binary Images

PETROS A. MARAGOS, MEMBER, IEEE, AND RONALD W. SCHAFFER, FELLOW, IEEE

**Abstract**—This paper presents the results of a study on the use of morphological set operations to represent and encode a discrete binary image by parts of its skeleton, a thinned version of the image containing complete information about its shape and size. Using morphological erosions and openings, a finite image can be uniquely decomposed into a finite number of skeleton subsets and then the image can be exactly reconstructed by dilating the skeleton subsets. The morphological skeleton is shown to unify many previous approaches to skeletonization, and some of its theoretical properties are investigated. Fast algorithms that reduce the original quadratic complexity to linear are developed for skeleton decomposition and reconstruction. Partial reconstructions of the image are quantified through the omission of subsets of skeleton points. The concepts of a globally and locally minimal skeleton are introduced and fast algorithms are developed for obtaining minimal skeletons.

For images containing blobs and large areas, the skeleton subsets are much thinner than the original image. Therefore, encoding of the skeleton information results in lower information rates than optimum block-Huffman or optimum runlength-Huffman coding of the original image. The highest level of image compression was obtained by using Elias coding of the skeleton.

### I. INTRODUCTION

**A**UTOREGRESSIVE modeling and orthogonal transforms such as Fourier or Karhunen-Loève transforms have provided the theoretical basis for most of the research in digital image coding during the past decade. Both of these approaches exploit primarily the algebraic structure of signals. However, in the case of binary image signals, which are mainly perceived as geometrical patterns, there is a need for representations that emphasize geometric rather than algebraic structure. One such geometric representation is the *skeleton*. In general, the term skeleton has been used to describe a line-thinned caricature of the binary image which summarizes its shape and conveys information about its size, orientation, and connectivity. The skeleton has already been applied in biological shape description [1]–[3], pattern recognition [3], [9], [25], image coding [7], [18], [22], quantitative metallography [13], [14], and automated industrial in-

roduced by Blum [1], who originally called it the “*medial axis*” and later the “*symmetric axis*” [2]. Blum’s initial procedure for obtaining the medial axis was to set up “*grassfires*” at time  $t = 0$  along all the points of the object boundary, and to let these grassfires propagate as wavefronts toward the center of the object at uniform speed following Huygen’s principle. The medial axis points, where these wavefronts would intersect and extinguish, together with their arrival times defined the “*medial (symmetric) axis function*.” A very important property of this symmetric axis function is the ability to reconstruct the object boundary by propagating the wavefronts backward. Subsequently, a number of people developed a mathematical theory for the skeleton: Kotelly [12] and Calabi [4] (with Hartnett [5] later) for continuous images, and Rosenfeld and Pfaltz [27], [24], Mott-Smith [21], and Montanari [20] for discrete images. Influenced by all the above contributions and many others referenced in his epitomizing work [2], Blum considered two new approaches to find the medial axis. First, he used the “*symmetric point distance*” from a skeleton point to the boundary. Second, he showed that the symmetric axis is the locus of the centers of the “*maximal disks*” inscribable inside a filled-in image object. Blum’s second interpretation motivated Frank et al. [7] to use the concepts of a point and a *growth* in a progressive binary image transmission scheme.

Parallel to and independently from the evolution of all the above skeletonization ideas, *mathematical morphology* evolved as a set-theoretical method for image analysis whose purpose is the quantitative description of geometrical structures. Mathematical morphology, after its first introduction by Matheron [19] and Serra [28] in 1964, has found numerous applications [28], [32]. Some contributions to the morphology of graytone functions were also made by Sternberg [31], [32]. One important feature of mathematical morphology is the fact that it unifies the re-



**Petros A. Maragos** (S’80–M’85) was born in Kalymnos, Greece, on November 4, 1957. He received the Diploma degree in electrical engineering from the National Technical University of Athens, Athens, Greece, in 1980, and the M.S. and Ph.D. degrees both in electrical engineering from the Georgia Institute of Technology, Atlanta, in 1982 and 1985, respectively.

From 1980 to 1985 he was a Research Assistant at the Digital Signal Processing Laboratory of the Electrical Engineering School at Georgia Tech,

where he was engaged in research on image modeling, coding, segmentation, and shape analysis. Since August 1985 he has been an Assistant Professor of Electrical Engineering in the Division of Applied Sciences, Harvard University, Cambridge, MA. His current research interests include signal and image processing, computer vision, and pattern recognition.

Dr. Maragos received the M.S. thesis research award from Sigma Xi (Georgia Tech Chapter) in 1983.



**Ronald W. Schaffer** (S’62–M’67–SM’74–F’77) received the B.S.E.E. and M.S.E.E. degrees from the University of Nebraska, Lincoln, in 1961 and 1962, respectively, and the Ph.D. degree from the Massachusetts Institute of Technology, Cambridge, in 1968.

From 1968 to 1974 he was a member of the Acoustics Research Department, Bell Laboratories, Murray Hill, NJ, where he was engaged in research on speech analysis and synthesis, digital signal processing techniques, and digital waveform coding. Since 1974 he has been on the Faculty of the Georgia Institute of Technology, Atlanta, as John O. McCarty/Audichron Professor and Regents’ Professor of Electrical Engineering. He is the co-author of the widely used textbooks, *Digital Signal Processing* and *Digital Processing of Speech Signals*.

Dr. Schaffer has served as an Associate Editor of the TRANSACTIONS, member of several committees, Vice-President and President of the Society, and Chairman of the 1981 ICASSP. He is a Fellow of the Acoustical Society of America and a member of Sigma Xi, Eta Kappa Nu, and Phi Kappa Phi. He was awarded the Achievement Award and the Society Award of the IEEE ASSP Society in 1979 and 1983, respectively; the 1983 IEEE Region III Outstanding Engineer Award; and he shared the 1980 Emanuel R. Piore Award with L. R. Rabiner. In 1985 he received the Class of 1934 Distinguished Professor Award at Georgia Tech.

IEEE TRANSACTIONS ON ACOUSTICS, SPEECH, AND SIGNAL PROCESSING, VOL. ASSP-34, NO. 5, OCTOBER 1986 Morphological Skeleton Representation and Coding of Binary Images PETROS A. MARAGOS, MEMBER, IEEE, AND RONALD W. SCHAFFER, FELLOW, IEEE

## *Scope of mathematical morphology*

Mathematical morphology framework is used for:

1. Image filtering (shape simplification, enhancing object structure,...)
2. Image segmentation (watersheds)
3. Image measurements (area, perimeter, granulometry)
4. Pattern recognition
5. Texture analysis

# What Is Morphology?

- **Morphological** image processing (or *morphology*) describes a range of image processing techniques that deal with the **shape** (or morphology) of features in an image
- Morphological operations are typically applied to **remove imperfections** introduced during segmentation, and so typically operate on bi-level images

# What Is Morphology?

**Morphology:** a branch of biology that deals with the form and structure of animals and plants

Morphological image processing is used to extract image **components** for **representation** and **description** of region shape, such as **boundaries**, **skeletons**, and the **convex hull**

# Quick Example

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



Image after segmentation



Image after segmentation and morphological processing

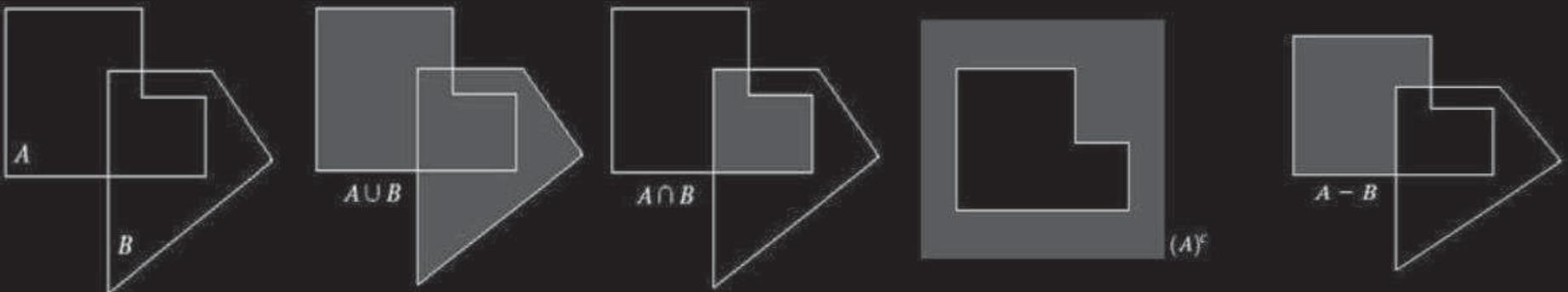
If **A** and **B** are two sets then

1. UNION =  $A \cup B$

2. INTERSECTION =  $A \cap B$

3. COMPLIMENT =  $(A)^c$

4. DIFFERENCE =  $A - B$

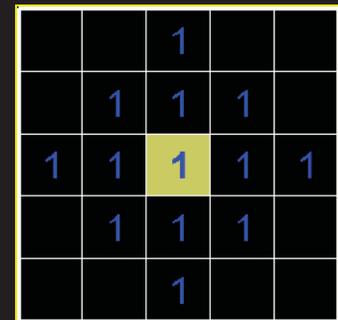
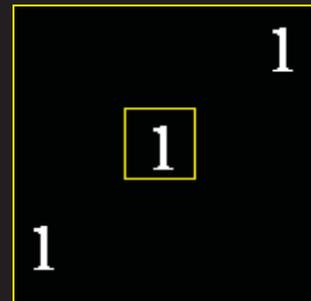
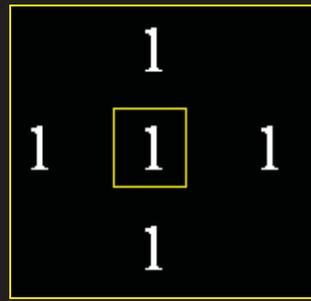
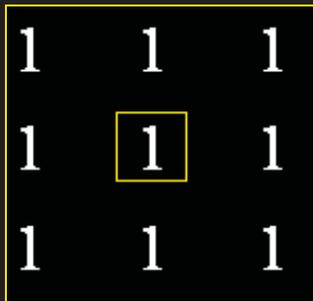
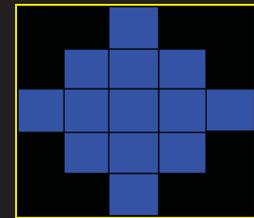
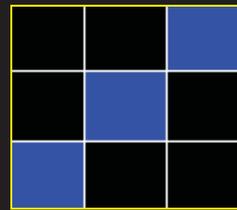
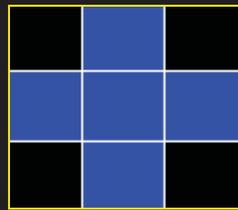
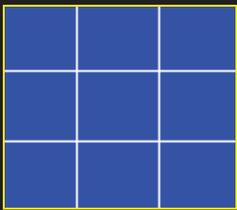


# Structuring elements

- Similarly to the case of **spatial** linear filtering, a binary image can be processed by another binary image called **structuring element**.
- In an **analogy** between spatial linear filtering and morphological image processing, the structuring element can be seen as the equivalent of the **mask** and its origin as the equivalent of the **centre** of a mask.

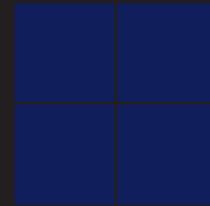
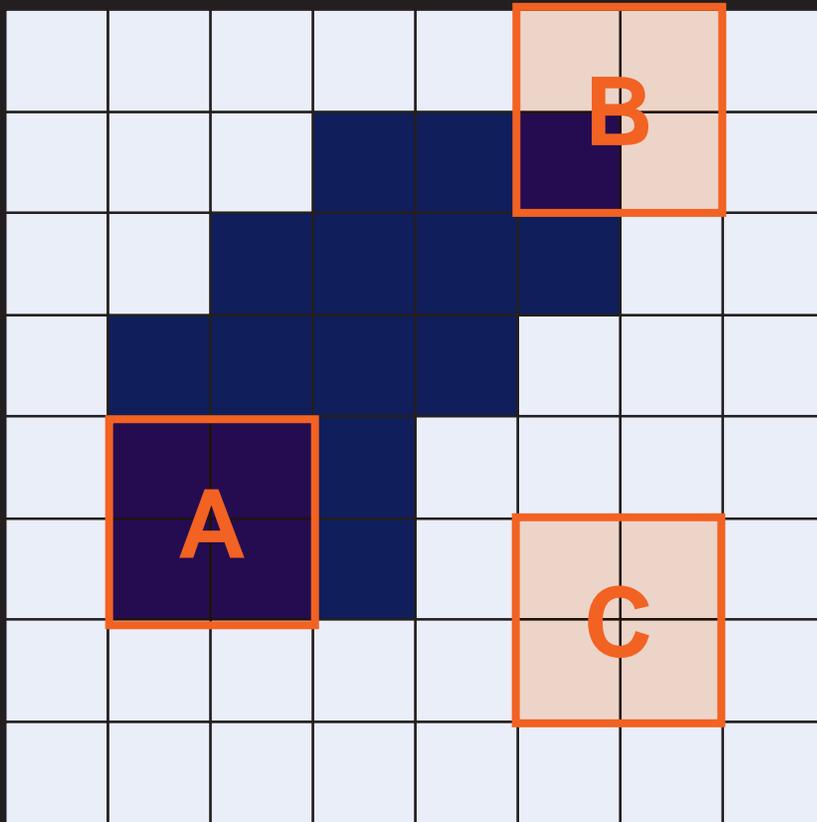
# Structuring elements

By convention, we sometimes describe structuring elements by showing only their **1**s and their origin.



Four typical structuring elements. The origin is shown with a black border.

# Structuring Elements, Hits & Fits



Structuring Element

**Fit:** All *on pixels* in the structuring element cover *on pixels* in the image

**Hit:** Any *on pixel* in the structuring element covers an *on pixel* in the image

All morphological processing operations are based on these simple ideas

# Fundamental Operations

- Fundamentally morphological image processing is very like spatial filtering
- The structuring element is **moved** across every pixel in the original image to give a pixel in a new processed image
- The value of this new pixel depends on the operation performed
- There are two basic morphological operations: **erosion** and **dilation**
  1. **Dilation** adds pixels to the boundaries of objects in an image
  2. **Erosion** removes pixels on object boundaries

## Dilation $\oplus$ – (Dilate, Grow, Expand)

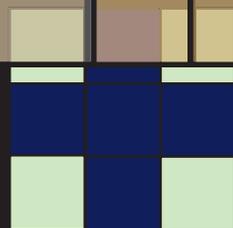
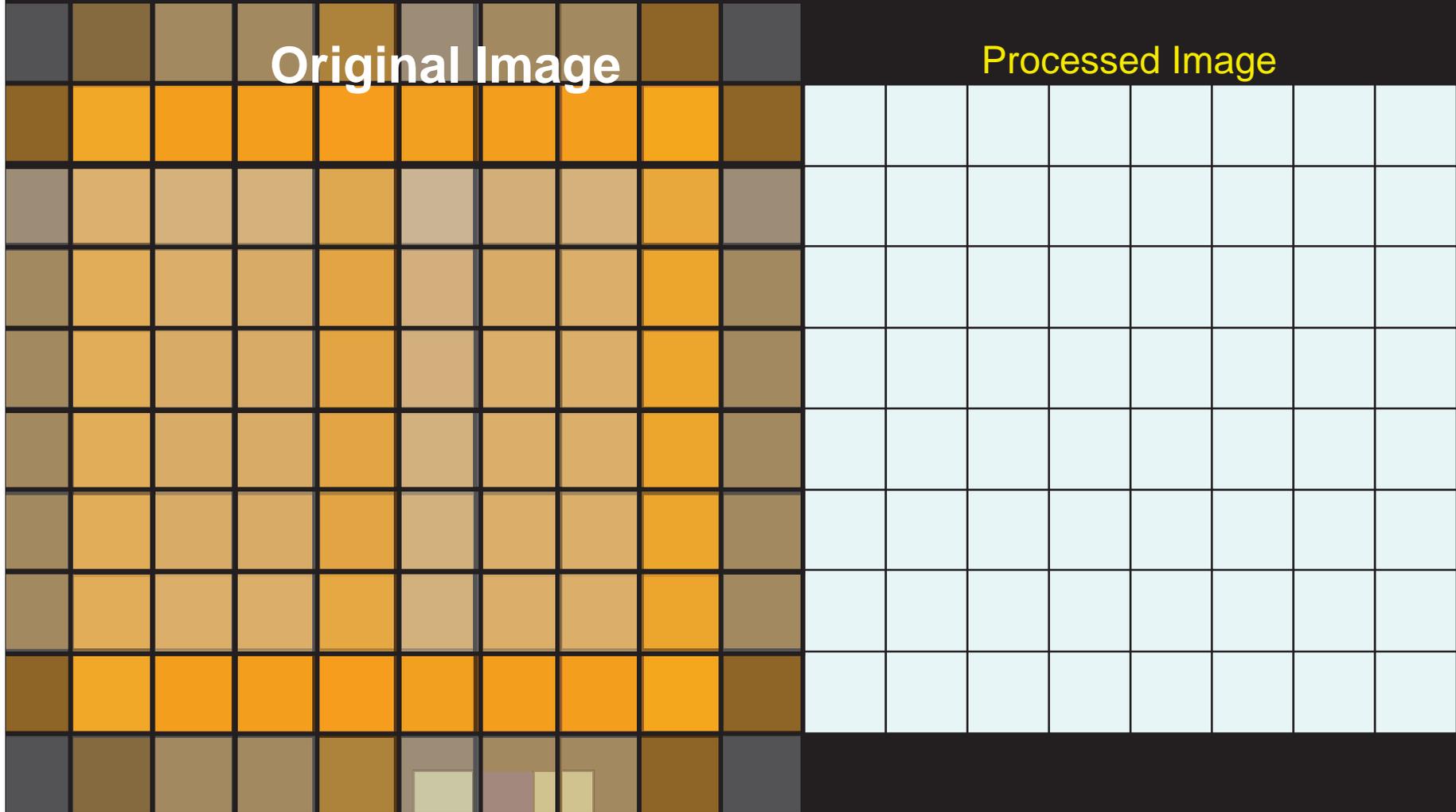
Dilation of image  $f$  by structuring element  $s$  is given by  $(f \oplus s)$

The structuring element  $s$  is positioned with its origin at  $(x, y)$  and the new pixel value is determined using the rule:

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ hits } f \\ 0 & \text{otherwise} \end{cases}$$

# Dilation Example

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ hits } f \\ 0 & \text{otherwise} \end{cases}$$

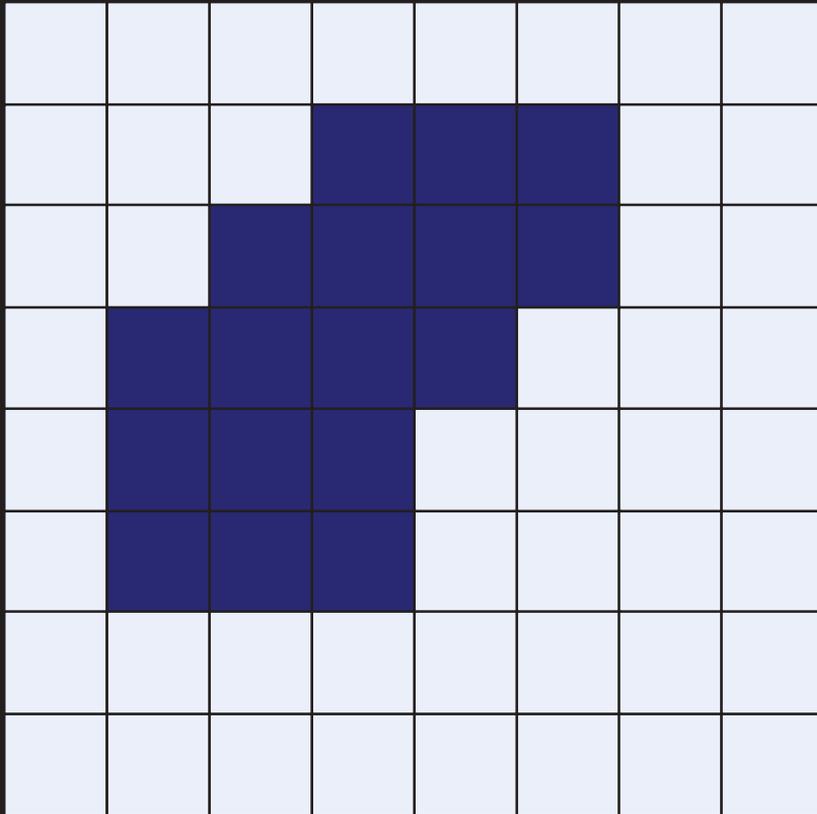


Structuring Element

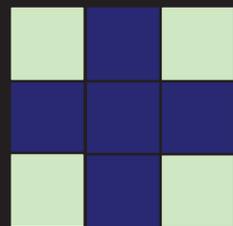
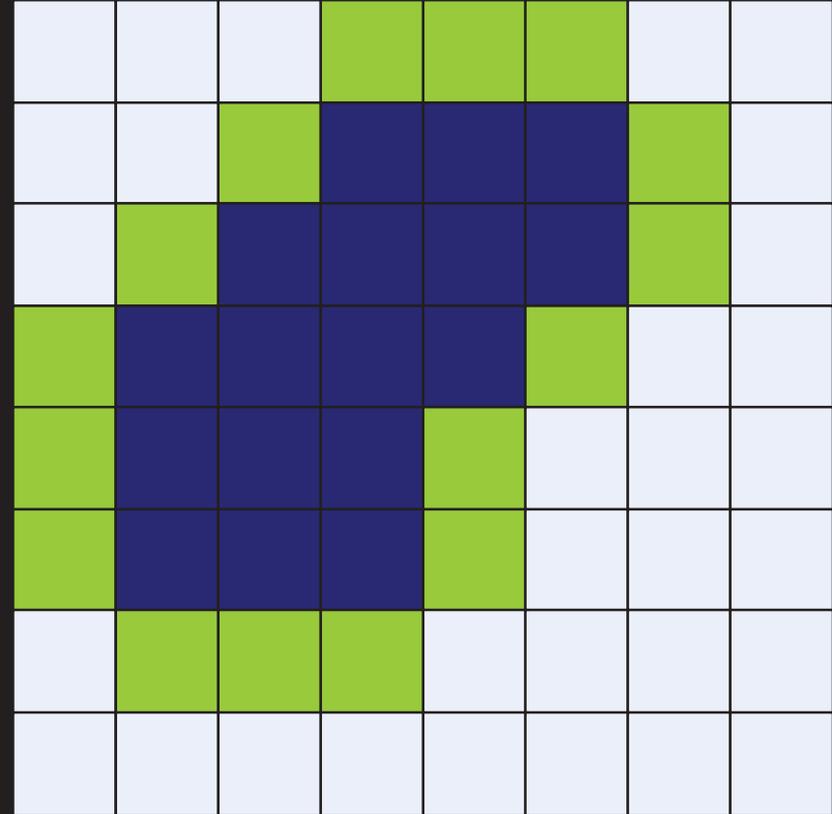
# Dilation Example

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ hits } f \\ 0 & \text{otherwise} \end{cases}$$

Original Image



Processed Image With Dilated Pixels

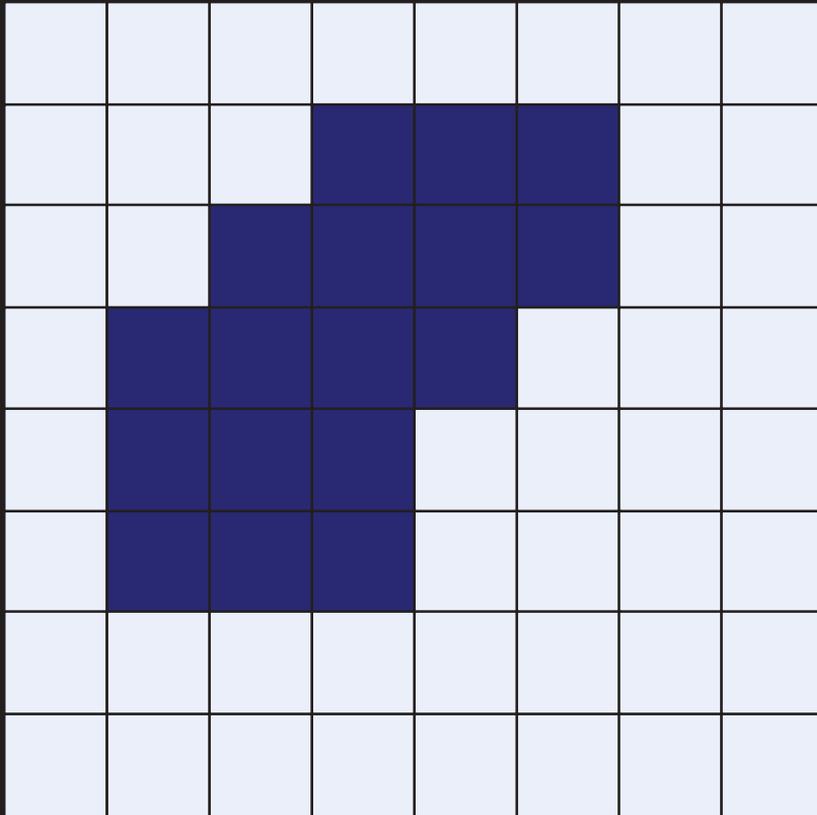


Structuring Element

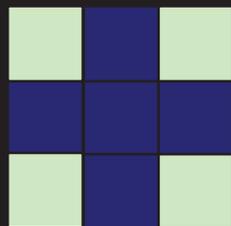
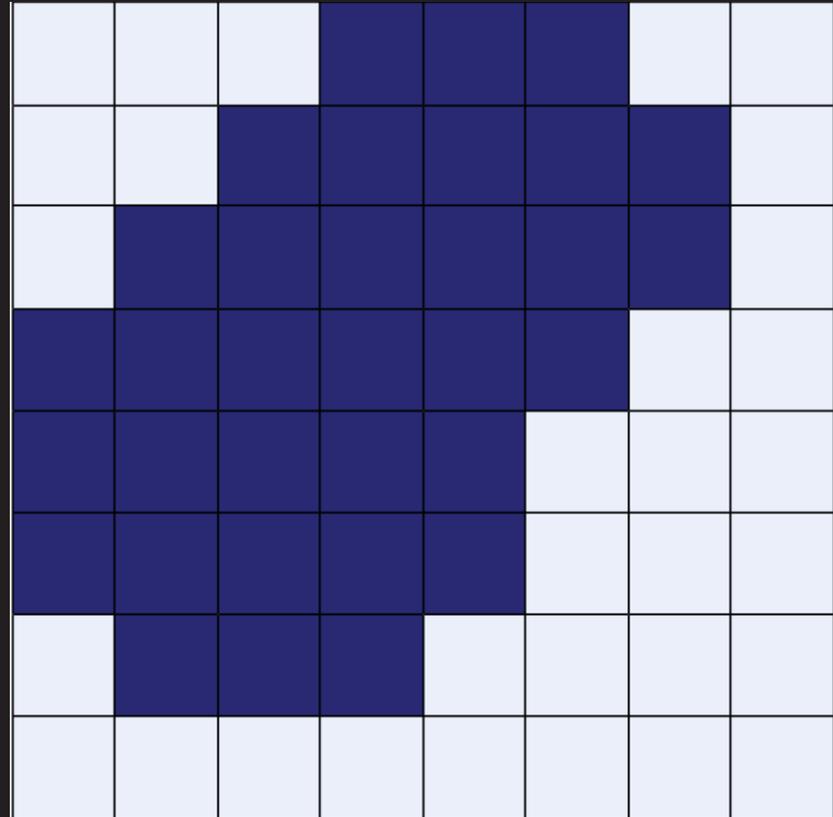
# Dilation Example

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ hits } f \\ 0 & \text{otherwise} \end{cases}$$

Original Image



Processed Image With Dilated Pixels



Structuring Element

# Dilation Example 1



Original image



Dilation by 3\*3  
square structuring  
element



Dilation by 5\*5  
square structuring  
element

# Dilation Example 2

Original image

After dilation

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

**Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.**

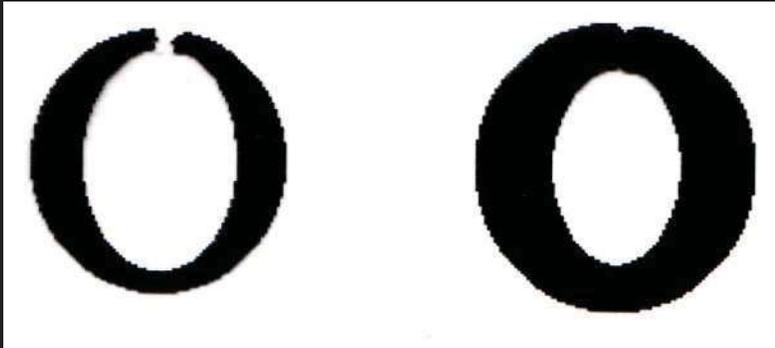


0	1	0
1	1	1
0	1	0

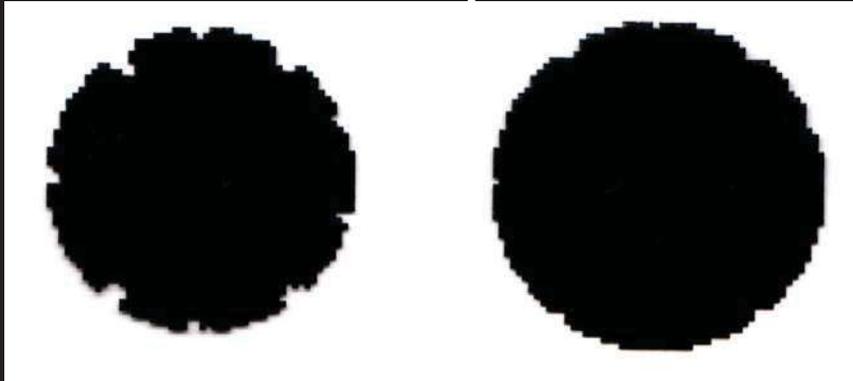
Structuring element

# What Is Dilation For?

Dilation can repair breaks

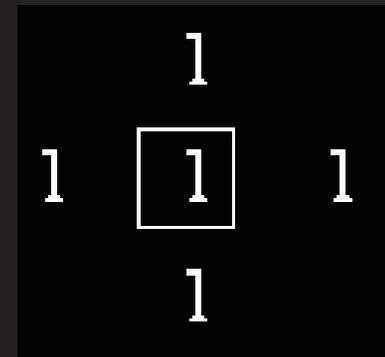


Dilation can repair intrusions



**Watch out:** Dilation enlarges objects

# Example 1



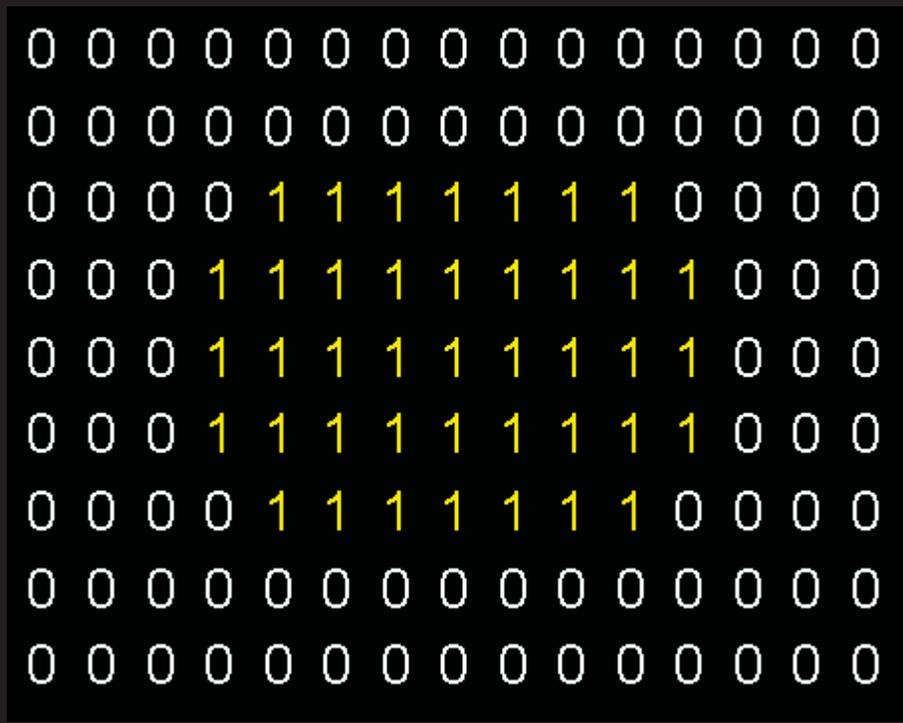
Find the dilation of the binary image on the left by the structuring element on the right.

# Example 1

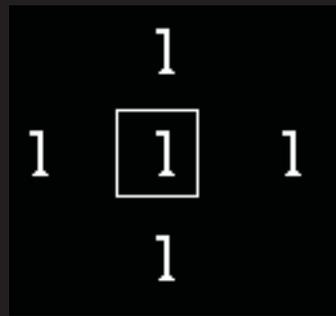
```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 1 1 1 1 1 1 0 0 0 0
0 0 0 1 1 1 1 1 1 1 1 1 0 0 0
0 0 0 1 1 1 1 1 1 1 1 1 0 0 0
0 0 0 1 1 1 1 1 1 1 1 1 0 0 0
0 0 0 0 1 1 1 1 1 1 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

The pixels of the dilation that do not belong to the original image are shown in red.

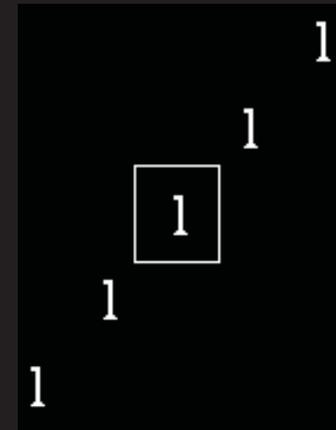
# Example 1



The dilation has dilated the image.

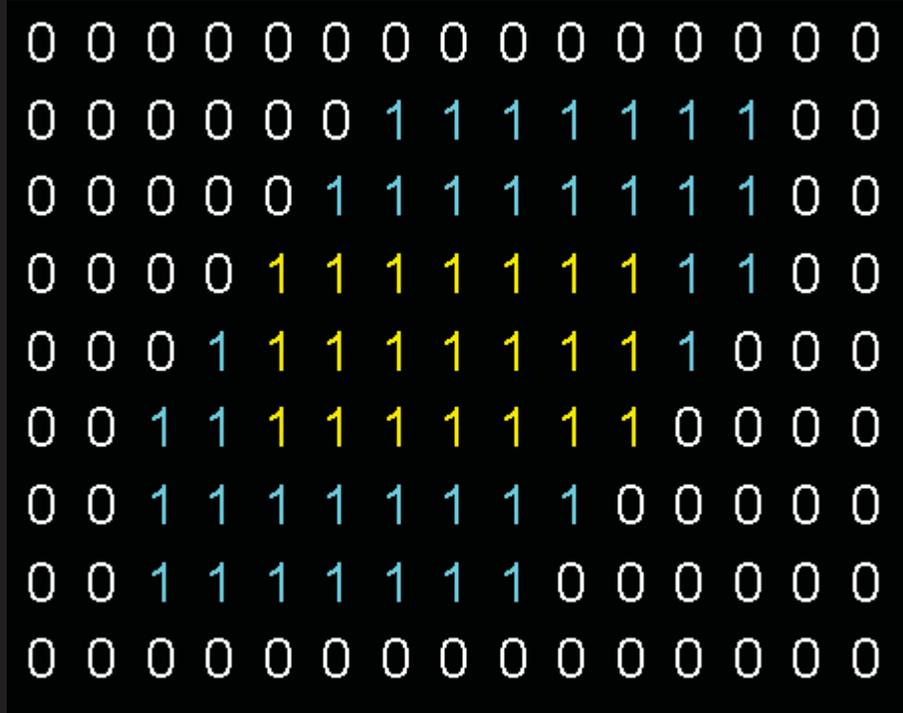


# Example 2

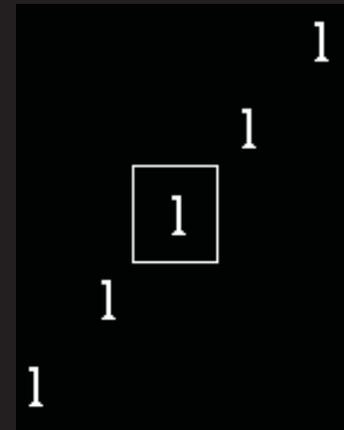


Find the dilation of the binary image on the left by the structuring element on the right.

# Example 2



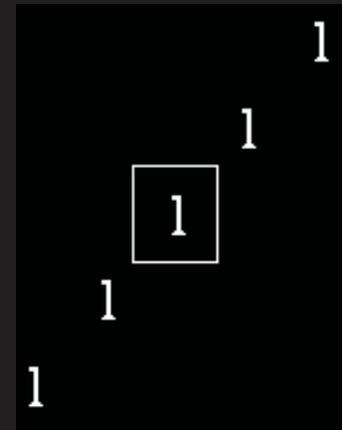
The pixels of the dilation that do not belong to the original image are shown in red.



# Example 2

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	1	0	0
0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	1	1	1	1	1	1	1	1	1	0	0
0	0	0	1	1	1	1	1	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1	1	1	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	0	0	0	0	0
0	0	1	1	1	1	1	1	1	0	0	0	0	0	0
0	0	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

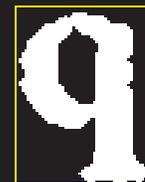
The dilation has dilated the image in the direction suggested by the structuring element.



# Example 3

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

**Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.**



The dilation of a binary image by the structuring element on the right.

## Erosion $\ominus$ - (shrinking)

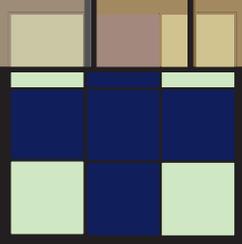
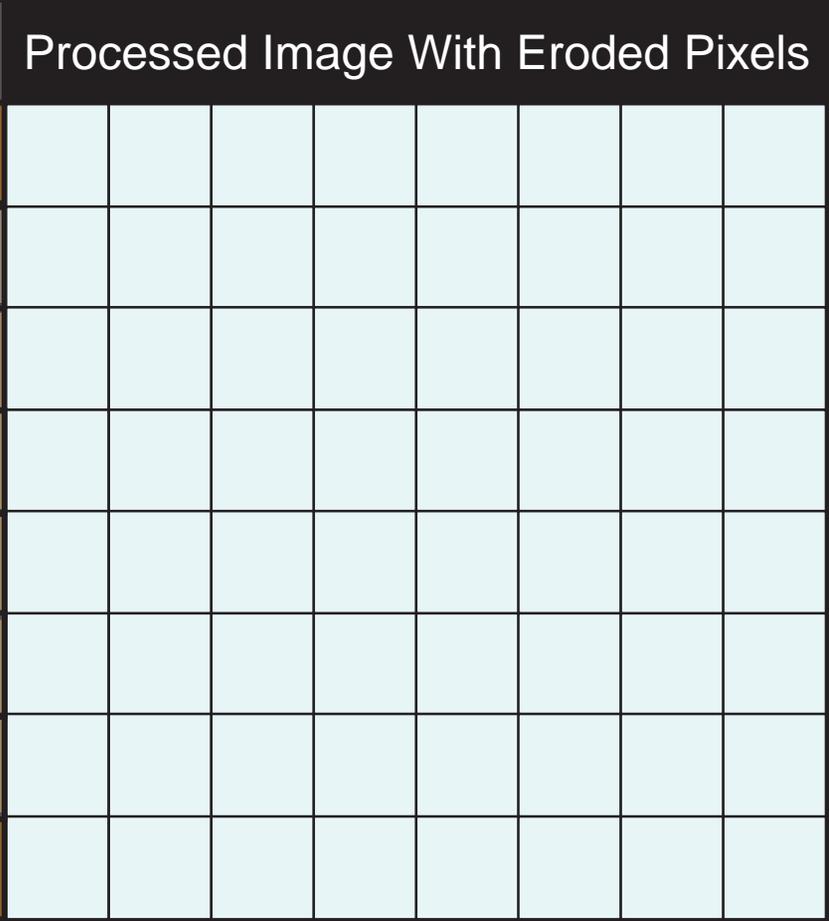
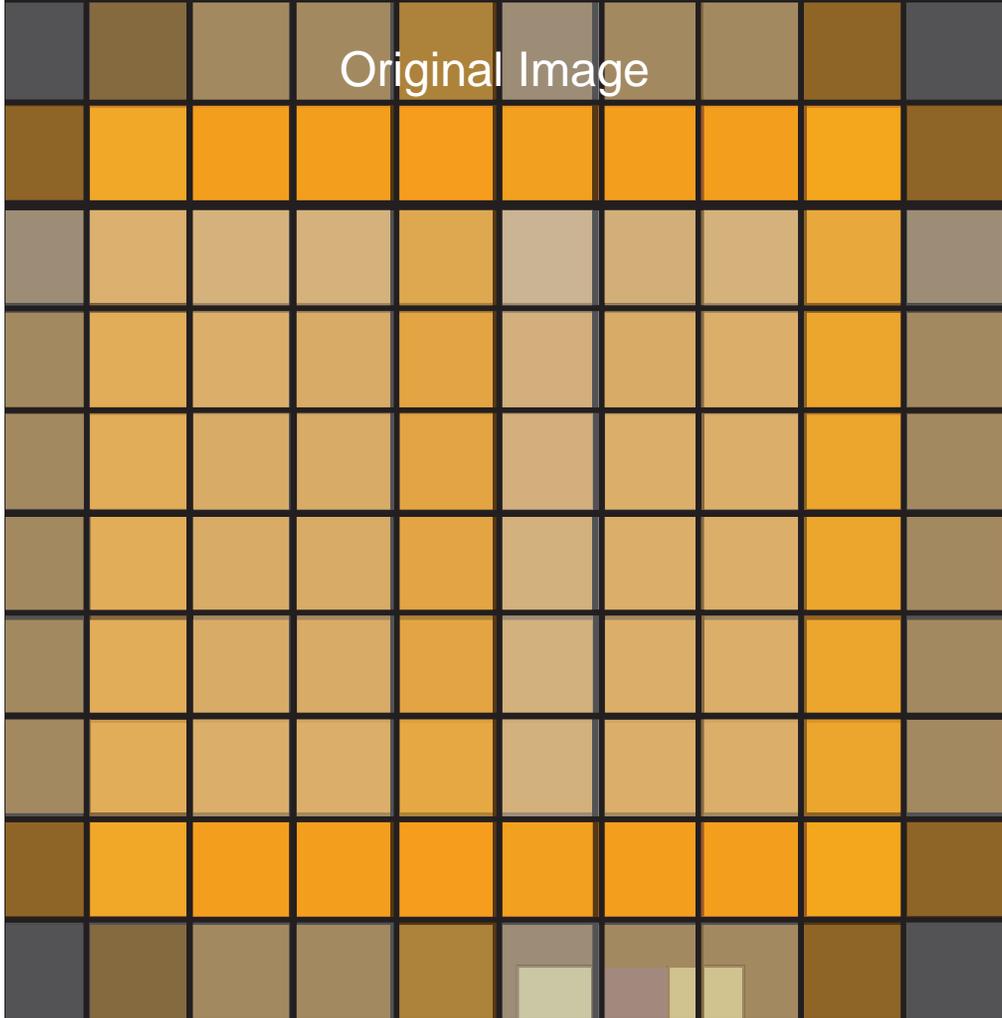
Erosion of image  $f$  by structuring element  $s$  is given by  $(f \ominus s)$

The structuring element  $s$  is positioned with its origin at  $(x, y)$  and the new pixel value is determined using the rule:

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ fits } f \\ 0 & \text{otherwise} \end{cases}$$

# Erosion Example

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ fits } f \\ 0 & \text{otherwise} \end{cases}$$

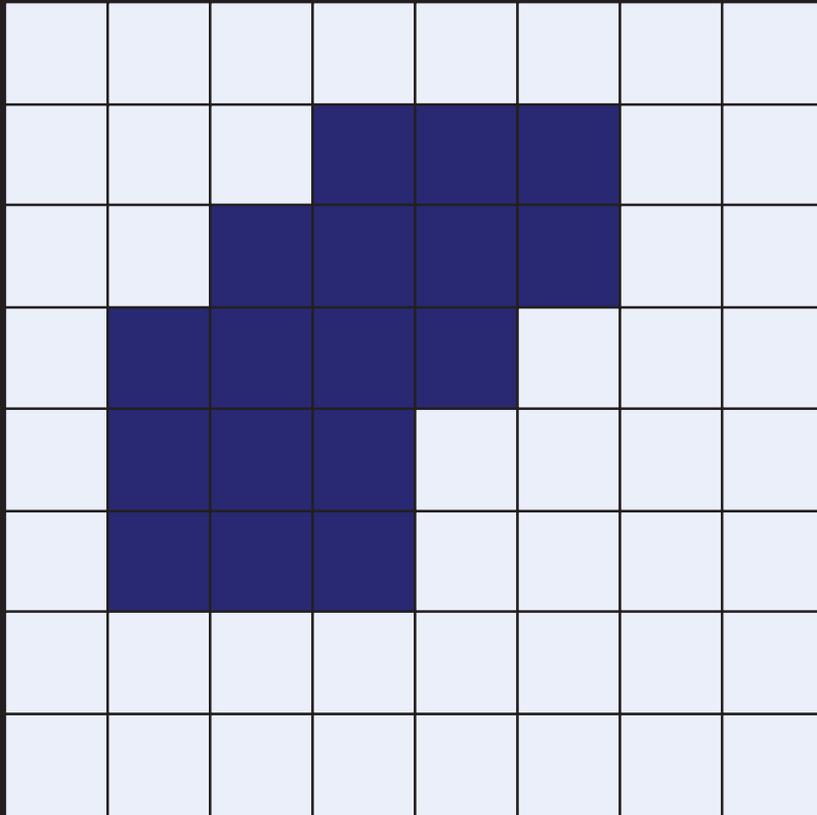


Structuring Element

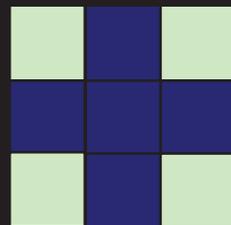
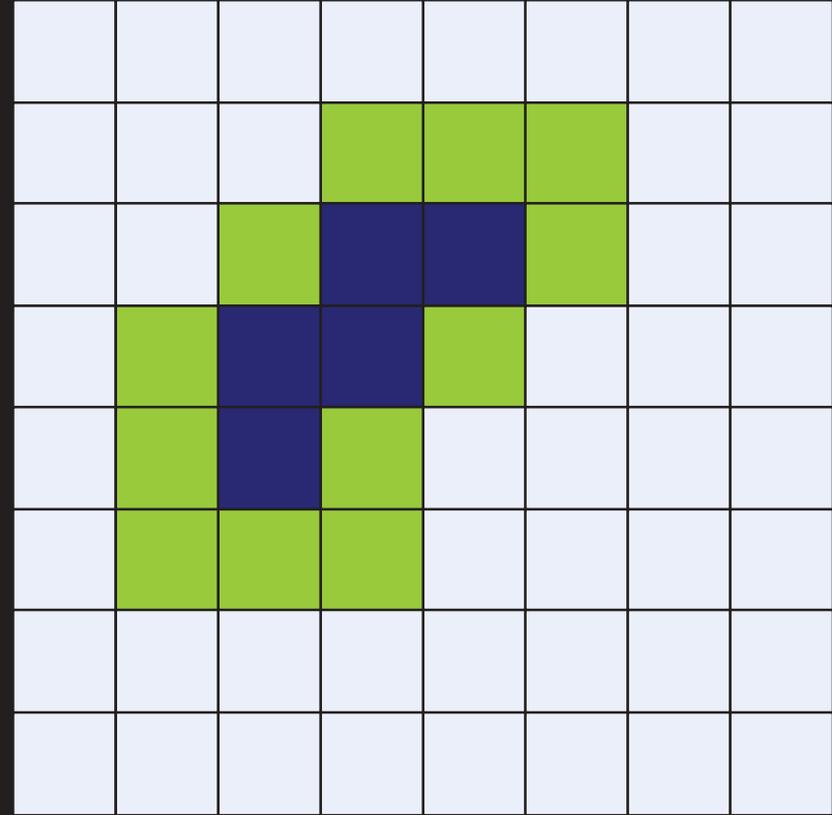
# Erosion Example

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ fits } f \\ 0 & \text{otherwise} \end{cases}$$

Original Image



Processed Image

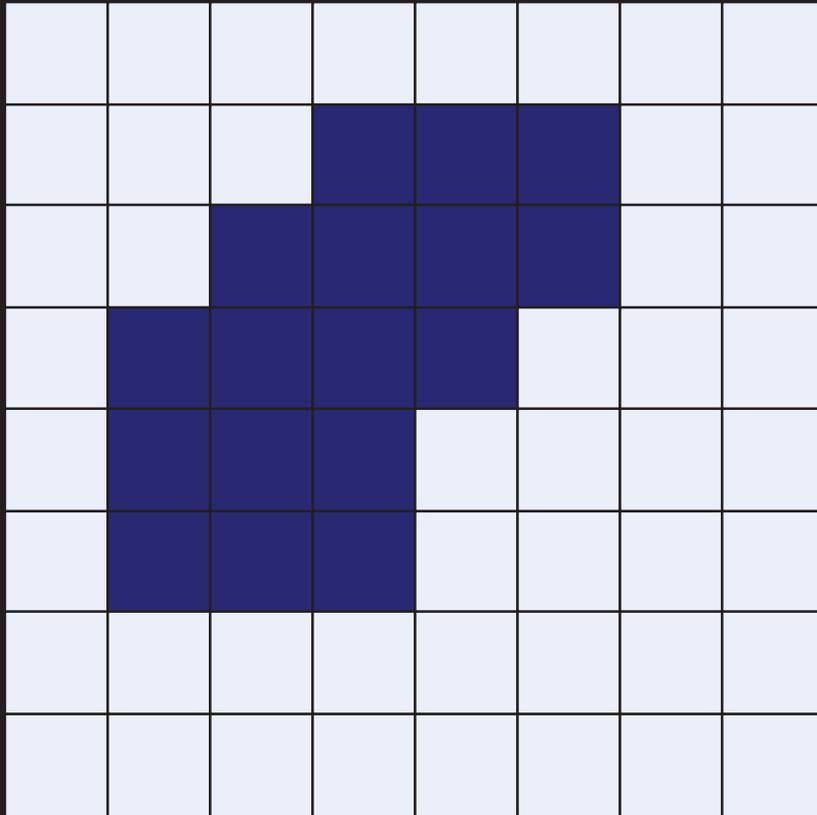


Structuring Element

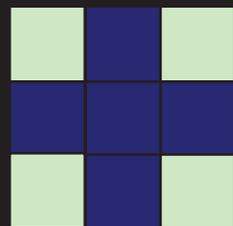
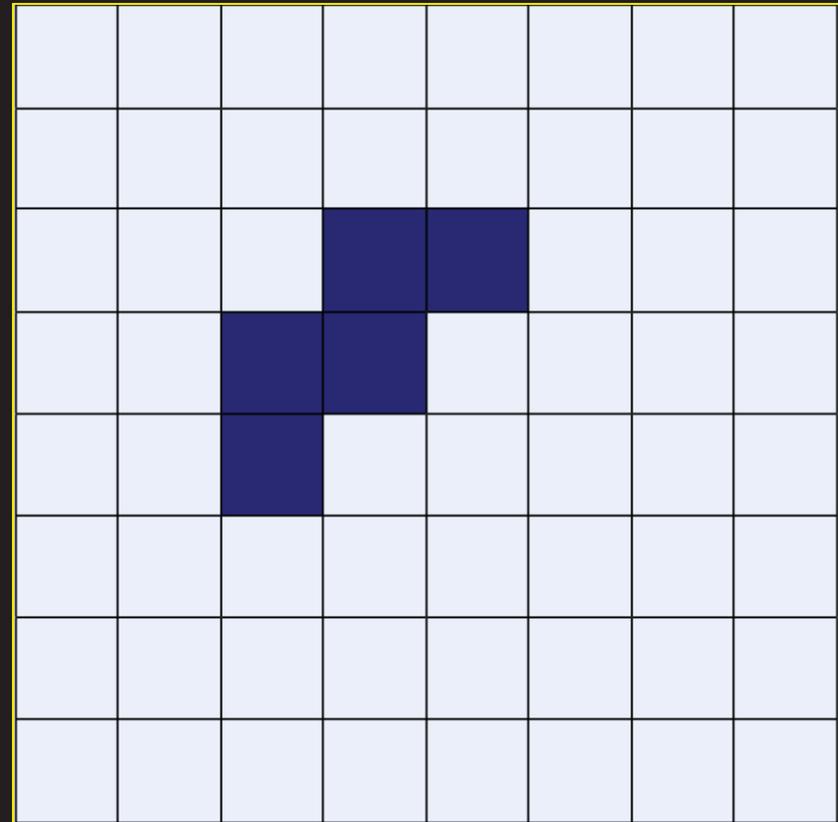
# Erosion Example

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ fits } f \\ 0 & \text{otherwise} \end{cases}$$

Original Image



Processed Image



Structuring Element

# Erosion Example 1



Original image

Erosion by 3\*3  
square structuring  
element

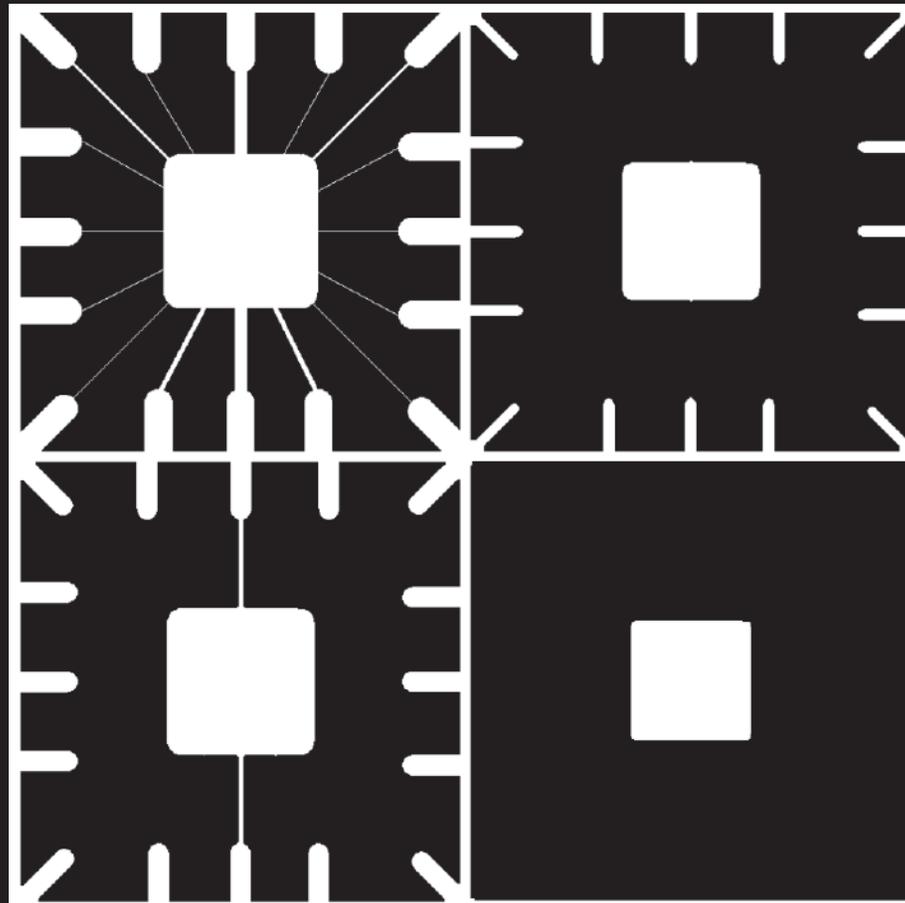
Erosion by 5\*5  
square structuring  
element

**Watch out:** In these examples a 1 refers to a black pixel!

# Erosion Example 2

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Original image



After erosion with a disc of radius 10

After erosion with a disc of radius 5

After erosion with a disc of radius 20



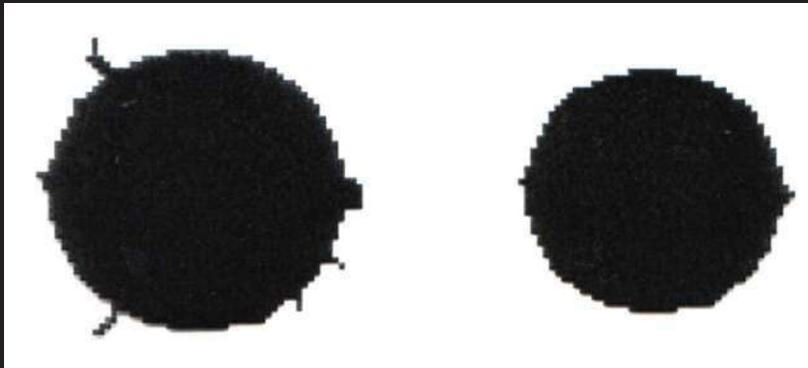
# *What Is Erosion For?*

Erosion can split apart joined objects



Erosion can split apart

Erosion can strip away extrusions



**Watch out:** Erosion shrinks objects

# *Compound Operations*

More interesting morphological operations can be performed by performing combinations of erosions and dilations

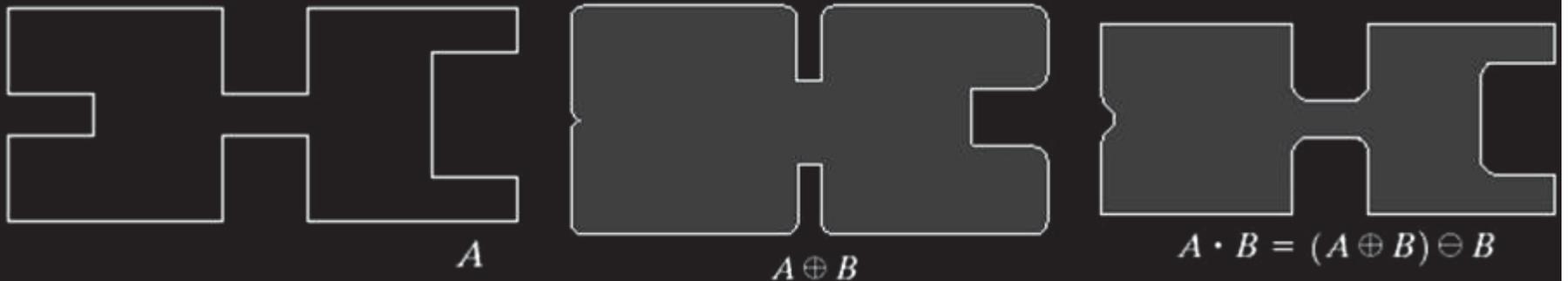
The most widely used of these *compound operations* are:

1. Opening
2. Closing

# Closing (*DE*)

The closing of image  $f$  by structuring element  $s$ , denoted  $f \bullet s$  is simply a dilation followed by an erosion

$$f \bullet s = (f \oplus s) \ominus s$$



# Opening ( $ED$ )

The opening of image  $f$  by structuring element  $s$ , denoted  $f \circ s$  is simply an erosion followed by a dilation:

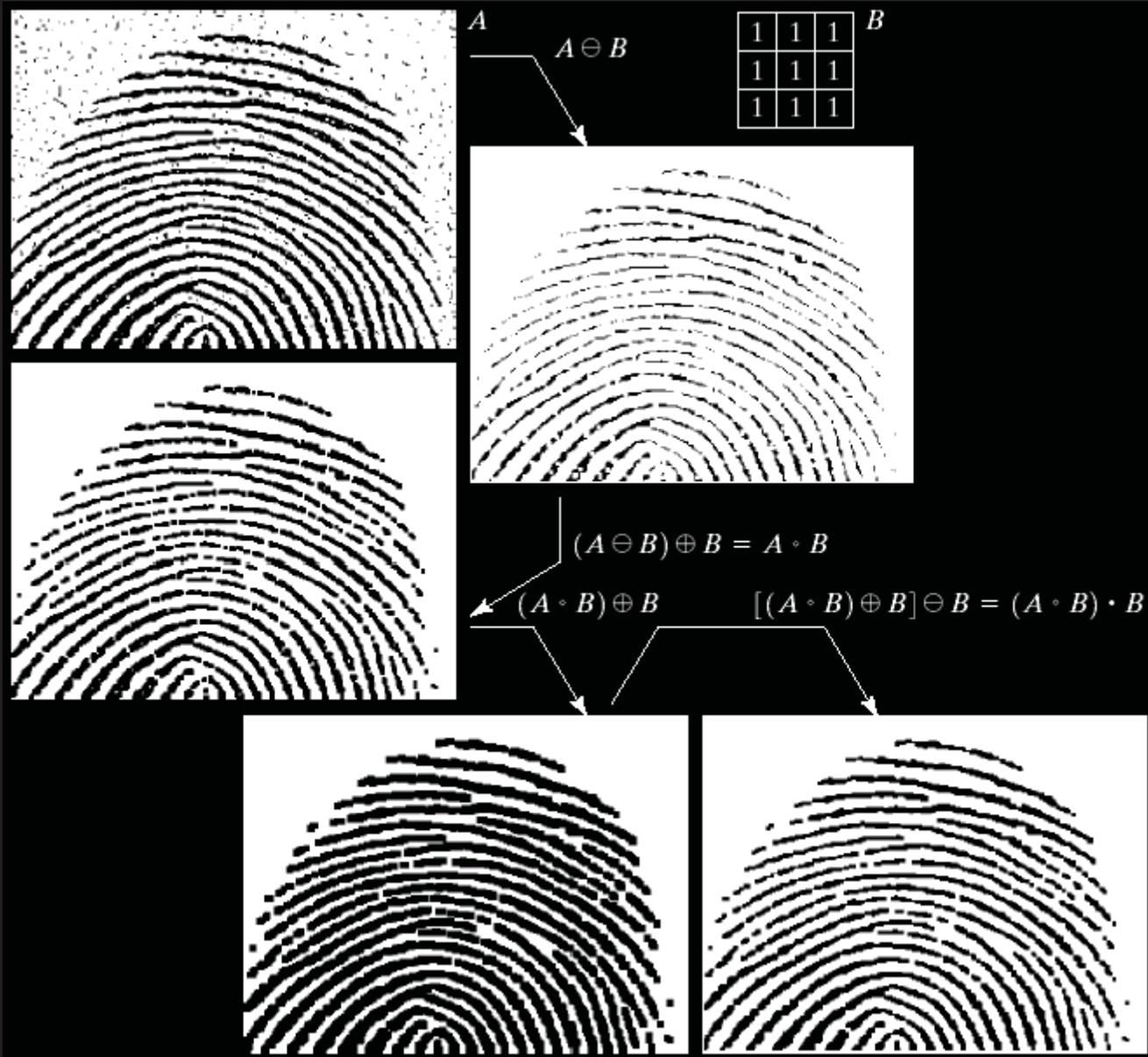
$$f \circ s = (f \ominus s) \oplus s$$

- Eliminates protrusions
- Breaks connections
- Smoothest contour



# Morphological Processing Example

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



Thank you

Any Questions ?

**END**

**Of Lecture**

# Homework

- Morphological operators often take a [binary image](#) and a [structuring element](#) as input and combine them using a set operator (intersection, union, inclusion, complement).
- They process objects in the input image based on characteristics of its shape, which are encoded in the structuring element.
- The mathematical details are explained in [Mathematical Morphology](#).

- Usually, the structuring element is sized  $3 \times 3$  and has its origin at the center pixel.
- It is shifted over the image and at each pixel of the image its elements are compared with the set of the underlying pixels.
- If the two sets of elements match the condition defined by the set operator (*e.g.* if the set of pixels in the structuring element is a subset of the underlying image pixels), the pixel underneath the origin of the structuring element is set to a pre-defined value (0 or 1 for binary images).
- A morphological operator is therefore defined by its structuring element and the applied set operator.

- For the basic morphological operators the structuring element contains only foreground pixels (*i.e.* ones) and `don't care's'.
- These operators, which are all a combination of erosion and dilation, are often used to select or suppress features of a certain shape, *e.g.* removing noise from images or selecting objects with a particular direction.

- The more sophisticated operators take zeros as well as ones and 'don't care's' in the structuring element. The most general operator is the hit and miss, in fact, all the other morphological operators can be deduced from it.
- Its variations are often used to simplify the representation of objects in a (binary) image while preserving their structure, *e.g.* producing a skeleton of an object using skeletonization and tidying up the result using thinning.

- Morphological operators can also be applied to graylevel images, *e.g.* to reduce noise or to brighten the image. However, for many applications, other methods like a more general spatial filter produces better results.

# *Set Theory*

- The language of mathematical morphology is set theory
- We will mostly work in  $\mathbf{Z}^2$
- Easy to extend to  $\mathbf{Z}^n$
- Can be extended to continuous domain
- If  $\mathbf{x} = (x_1, x_2)$  is an element in  $\mathbf{X}: \mathbf{x} \in \mathbf{X}$
- Today's lecture covers only binary mathematical morphology: (*gray-scale mathematical morphology in Image analysis 2*)

# *Some Set Theory*

- Empty set:  $\emptyset$
- Every element in  $A$  is also in  $B$  (subset):  $A \subset B$
- Union of  $A$  and  $B$ :  $C = A \cup B = \{x/x \in A \text{ or } x \in B\}$
- Intersection of  $A$  and  $B$ :  $C = A \cap B = \{x/x \in A \text{ and } x \in B\}$
- Disjoint / mutually exclusive:  $A \cap B = \emptyset$

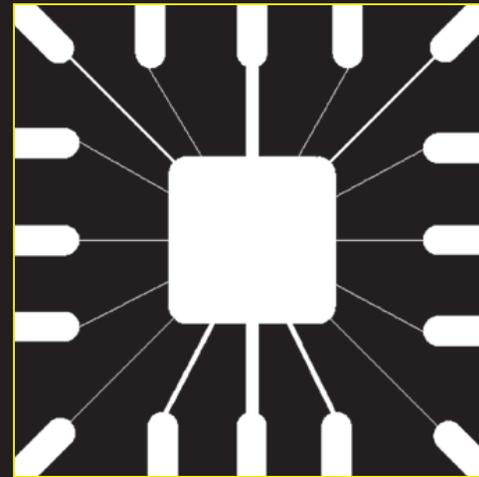
# *Some More Set Theory*

- Complement of  $A$ :  $A^c = \{x/x \notin A\}$   
(OBS! with respect to which universe)
- Difference of  $A$  and  $B$ :  
 $A \setminus B = \{x/x \in A \text{ and } x \notin B\} = A \cap B^c$
- Reflection (Transposition) of  $A$ :  $\hat{A} = \{-a/a \in A\}$   
The set  $A$  is symmetric if and only if  $A = \hat{A}$
- Translation of  $A$  by a vector  $z = (z_1, z_2)$ :  
 $(A)_Z = \{x/x = a + z, a \in A\}$

# *Binary images*

The value of a pixel  $(x,y)$  of a binary image can be either 0 or 1.

A binary image is usually visualised as a black and white image.



# *Binary images*

A binary image can be described by the **set** of its **1**-valued pixels.

The set theoretic operations **union**, **intersection** and **difference** can be applied on binary images.

We may assume that all binary images are subsets of  $\Omega$  and define the **complement** of a binary image.

## $\ominus$ *EROSION - PROPERTIES*

1. It is commutative.
2. It is increasing, i.e., if  $A \subset C$ , then  $A \ominus B \subset C \ominus B$
3. If the origin belongs to the structuring element  $B$ , then erosion is:  
anti-extensive, i.e.,  $A \ominus B \subset A$
4. It is distributive over set intersection, i.e.,  
 $(A1 \cap A2) \ominus B = (A1 \ominus B) \cap (A2 \ominus B)$
5. It is translation invariant

## $\oplus$ DILATION - PROPERTIES

1. It is commutative
2. It is increasing, i.e., if  $A \subset C$ , then

$$A \oplus B \subset C \oplus B$$

1. If the origin belongs to the structuring element  $B$ , then dilation is: extensive, i.e.,  $A \subset A \oplus B$

2. It is distributive over set union, i.e.,  $(A1 \cup A2) \oplus B = (A1 \oplus B) \cup (A2 \oplus B)$

3. It is translation invariant

# *EROSION - DILATION DUALITY*

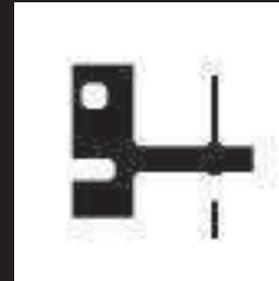
- Erosion and dilation are dual with respect to complementation and reflection



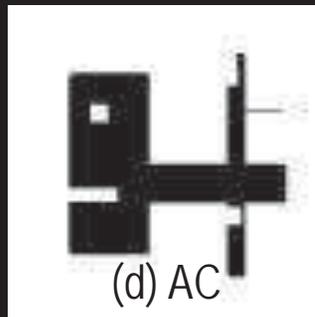
(a) A



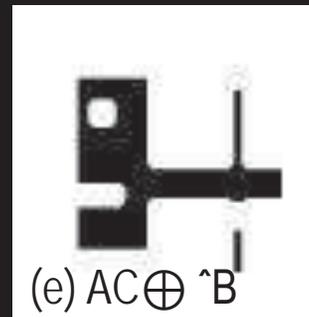
(b)  $A \ominus B$



(c)  $(A \ominus B)C$



(d) AC



(e)  $AC \oplus \hat{B}$

# Morphological Algorithms

Using the simple technique we have looked at so far we can begin to consider some more interesting morphological algorithms

We will look at:

1. Boundary extraction
2. Region filling

There are lots of others as well though:

- Extraction of connected components
- Thinning/thickening
- Skeletonization

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

# *Aerial Photography & Photogrammetry*

## Lecture 4-1

# Principles of Stereoscopic Vision

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[faiselgm73@gmail.com](mailto:faiselgm73@gmail.com)

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*All rights reserved*



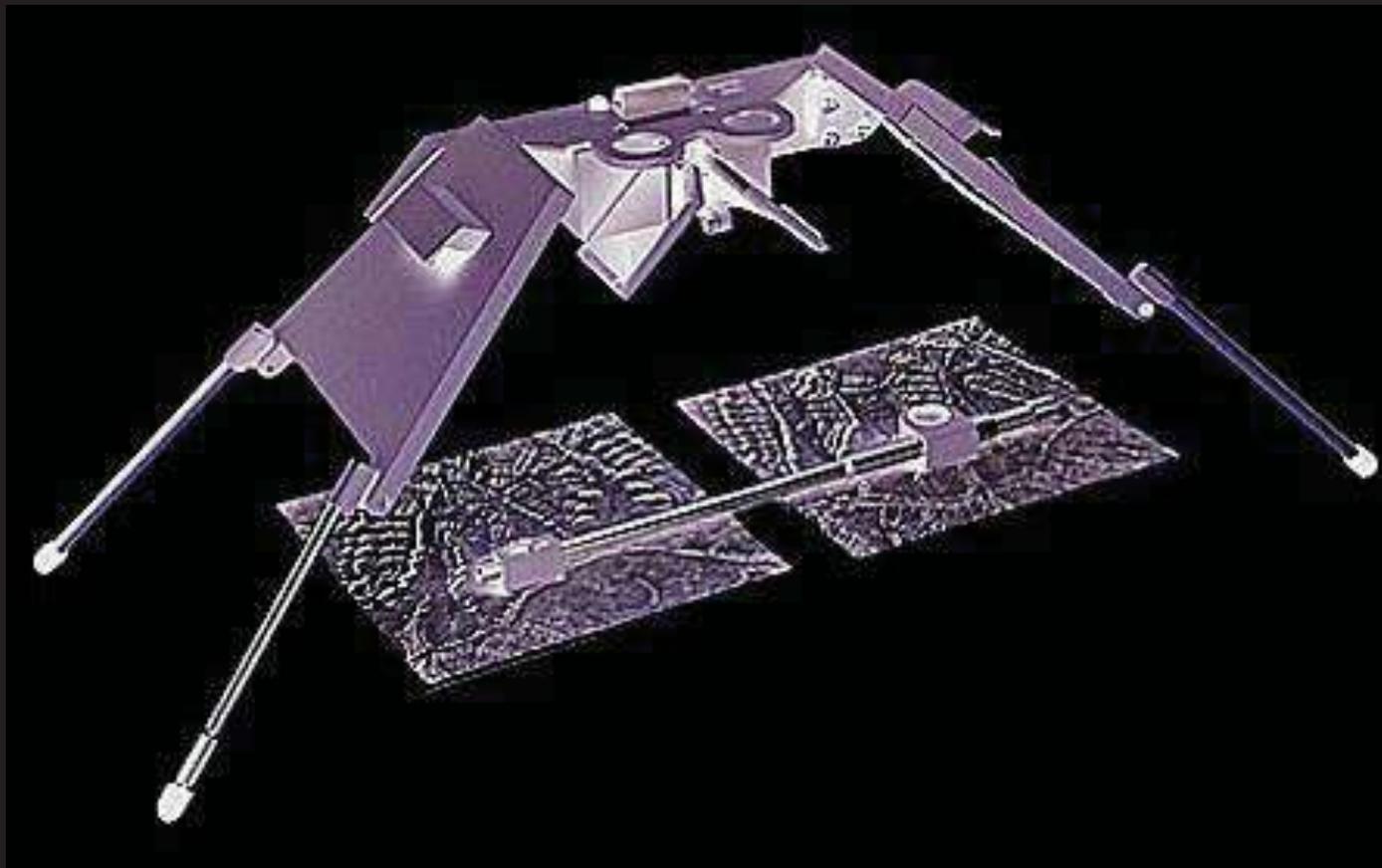
Satellite image in map coordinate space



Satellite image displayed in perspective mode, in image coordinate space







# Reading Chapters

*“Elements of Photogrammetry with Applications in GIS”, by Förstner etl, 2016.*

	<b>Material</b>	<b>Chapter Sections</b>	<b>Page</b>	<b>Exercises page</b>	<b>LABORATORY exercises Page</b>
<u>1</u>	Introduction		1	23	
<u>2</u>	Geometry of a Vertical Aerial Photograph		27	43	
<u>3</u>	<b>Principles of Stereoscopic Vision</b>		44	66	
<u>4</u>	Scale of a Vertical Aerial Photograph		68		83
<u>5</u>	Horizontal Measurements - Distance, Bearings and Areas		86		101
<u>6</u>	Vertical Measurements		105		127
<u>7</u>	Acquisition of Aerial Photography		131		154

# Principles of Stereoscopic Vision

- Up to this point, you have probably considered **topographic displacement** in aerial photographs as nothing but an obstacle that you wish didn't exist.
- However, **without it** we would not be able to view a **stereoscopic** model in the **3-Dimension**.
- This 3-D interpretation of aerial photographs allows the interpreter **to see much more** than can be seen in a single (2-D photograph).
- This lecture will deal with the **theory of stereoscopy**, discuss related topics, and lay the **groundwork for Chapter 6** (Text), which discusses **measurements in the third dimension**.

# Lecture Contents

## 3.1 Definitions

- 3.1.1 Stereoscopy
- 3.1.2 Stereoscopic Pair of Photographs
- 3.1.3 Stereogram
- 3.1.4 Stereoscope

## 3.2 Geometry of Stereoscopy

- 3.2.2 Absolute Parallax
- 3.2.3 Flight-Line Location

## 3.3 Theory of Stereoscopy

- 3.3.2 Depth Perception
- 3.3.3 The Floating-Dot Principle
- 3.3.4 Vertical Exaggeration
- 3.3.5 The Pseudoscopic Stereo Model

## 3.4 PROPER ORIENTATION OF A STEREO MODEL

## Laboratory Exercises 1

### 3.6 A Test for Stereoscopic Perception

- 3.6.1 How to Take the Test
- 3.6.2 How to Grade the Test

# OBJECTIVES

After a thorough understanding of this chapter, you will be able to:

- Define stereoscopy, stereoscopic pair, stereogram, stereoscope, and absolute parallax of a point.
- List four types of stereoscopes and state the primary advantage of each.
- Explain how the  $x$  and  $y$  axes are defined on a stereoscopic pair of aerial photographs, as compared to a single photo.
- Determine the absolute parallax of a single point on a stereopair.

# OBJECTIVES

After a thorough understanding of this chapter, you will be able to:

- Explain why **two eyes** are needed to see depth on a stereopair.
- Define **vertical exaggeration** and state two ways of increasing or decreasing the exaggeration.
- **Calculate** the vertical **exaggeration** of a specific stereoscopic pair of aerial photos given the proper equation and the necessary data.

# OBJECTIVES

In order to understand the principles of stereoscopic vision, it is necessary to become familiar with the terminology of stereoscopic vision.

# 1- Definitions Stereoscopy

- **Stereoscopy** (also called **stereoscopies**, or **stereo imaging**) is a technique for creating or enhancing the illusion of depth in an image by means of stereopsis for binocular vision.
- The word *stereoscopy* derives from Greek στερεός (*stereos*), meaning 'firm, solid', and σκοπέω (*skopeō*), meaning 'to look, to see'.
- Any stereoscopic image is called a **stereogram**.
- Originally, stereogram referred to a **pair of stereo** images which could be viewed using a stereoscope.

# 1- Definitions Stereoscopy



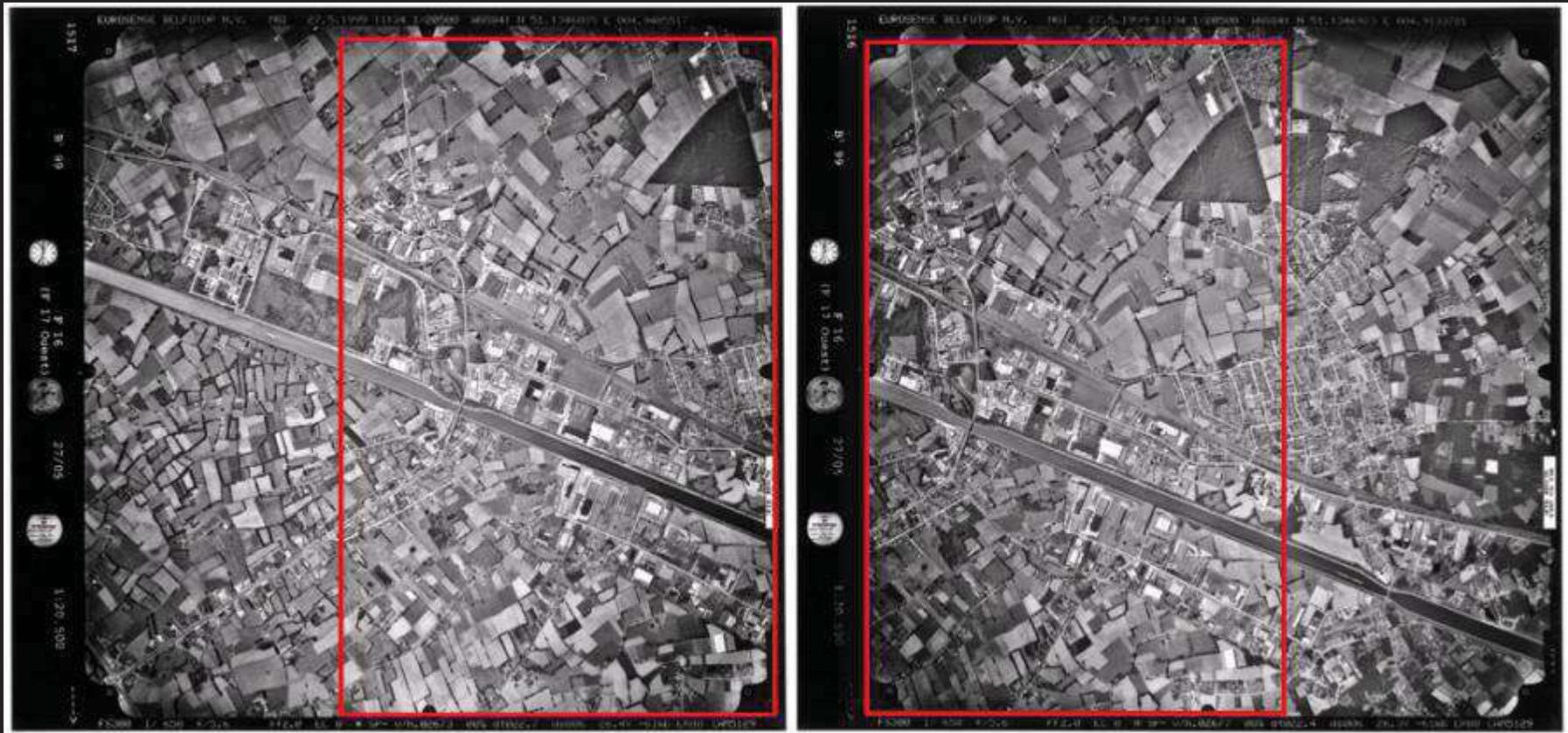
# 1- Definitions Stereoscopy

- A stereoscopic pair of images combined after coloring one red and the other cyan.
- It can be viewed in 3D by using simple anaglyph 3D glasses. (See Appendix-3-1)



# 1- Definitions Stereoscopic Pair of Photographs

Two images of the same area, taken from different points of view are called a stereopair.



# 1- Definitions **Stereoscopic Pair of Photographs**

- A ***stereoscopic pair*** of aerial photographs consists of two adjacent, overlapping photos in the same flight line.
- The stereoscopic view is seen only in the overlapped portion of the photos.
- Therefore, a minimum of **50%** endlap is necessary for complete stereoscopic viewing of the area photographed.
- As a safety factor, photo missions are usually designed for **60%** endlap.

# 1- Definitions Stereogram

- If we take a stereoscopic pair of aerial photos, cut out of each photo the part that shows the same area of interest on the ground, then correctly orient and mount them side by side, we have a *stereogram* like the one in **Figure 3.1**. See [Appendix-3-2](#)
- This is an unusual stereogram because there is very little evidence of topography when viewed without a stereoscope.
- This is due to the lack of definite drainage patterns and the arrangement of agricultural fields.
- Stereoscopic examination, however, reveals several drumlin mounds.

# 1- Definitions Stereogram



**Figure 3.1.** Stereogram of what appears to be flat agricultural ground. When viewed stereoscopically, several drumlin mounds are evident. Photo scale = 1:20,400. (Courtesy of Illinois Photographic Service.)

# 1- Definitions Stereogram



*Underwood & Underwood, Publishers  
New York, London, Toronto-Canada, Ottawa-Albany.*

From: Boston Public Library  
1 / 2 stereographs for 'Tigers'



*Photographs and  
Studios  
underwood*

(60) 3504-Famous "man-eater" at Calcutta—devoured 200 men, women and children before capture—India. Copyright 1902 by Underwood & Underwood.

From: Boston Public Library  
1 / 2 stereographs for 'Tigers'

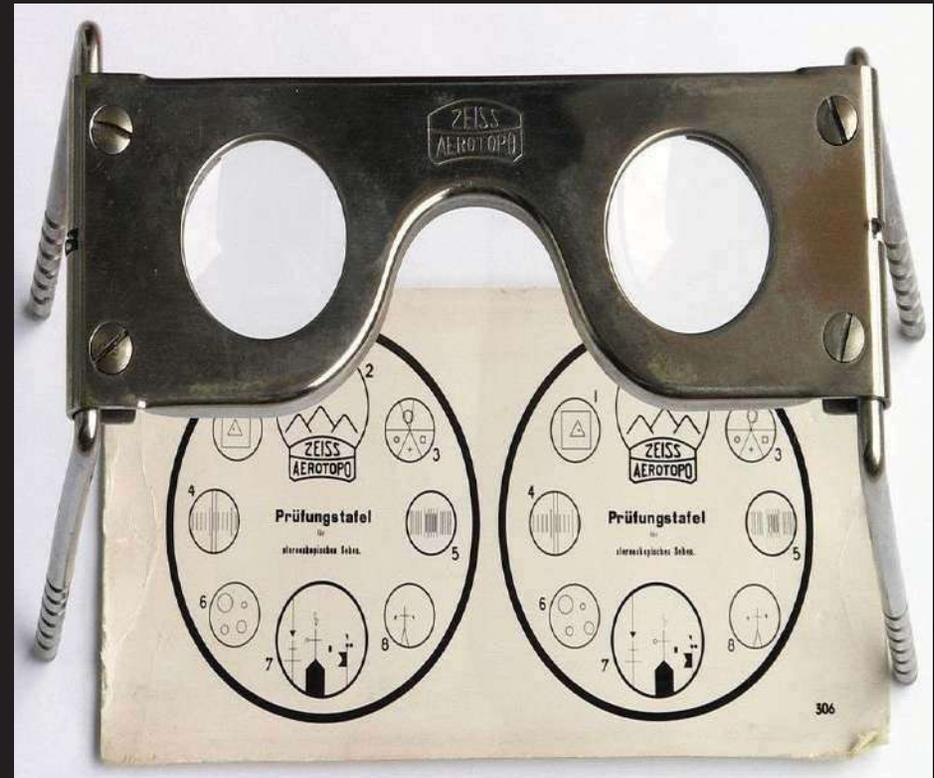
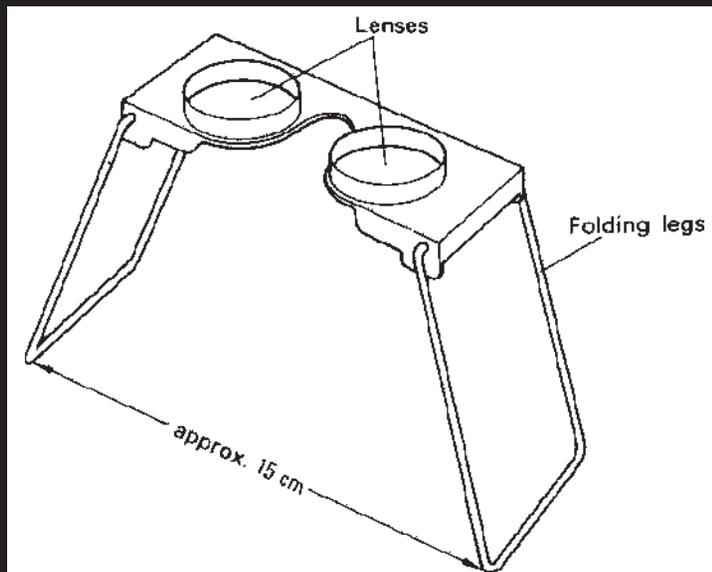
# 1- Definitions Stereoscope

- To achieve our stereoscopic image, we use a stereoscope, which is a **binocular** optical instrument that helps us view two **properly oriented photographs** to obtain the **mental impression** of a 3-D model.
- Most stereoscopes also magnify the images.
- The four types of stereoscopes with which we will be concerned are (1) the **lens** stereoscope, (2) the **mirror** stereoscope, (3) the **scanning** stereoscope, and (4) the **zoom** stereoscope. (for more details, see text book, page 46-49)

# 1- Definitions Stereoscope

## lens stereoscope

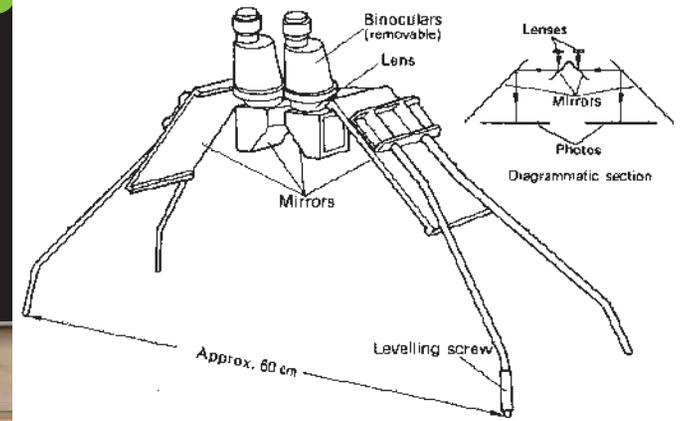
1. Simplest
2. Least expensive
3. Small
4. 2-4 x magnification
5. Used in the field



# 1- Definitions Stereoscope

## mirror stereoscope

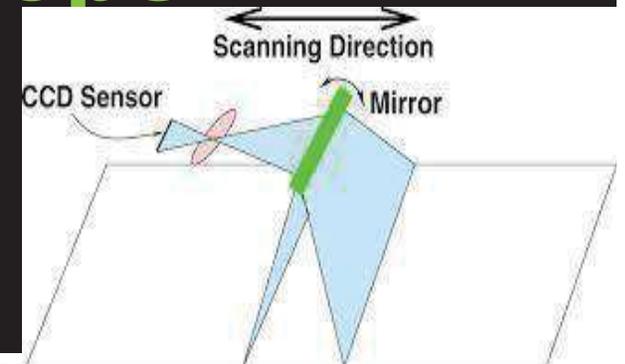
1. Photos can be placed separately
2. for viewing Used in the field?



# 1- Definitions Stereoscope

## scanning stereoscope

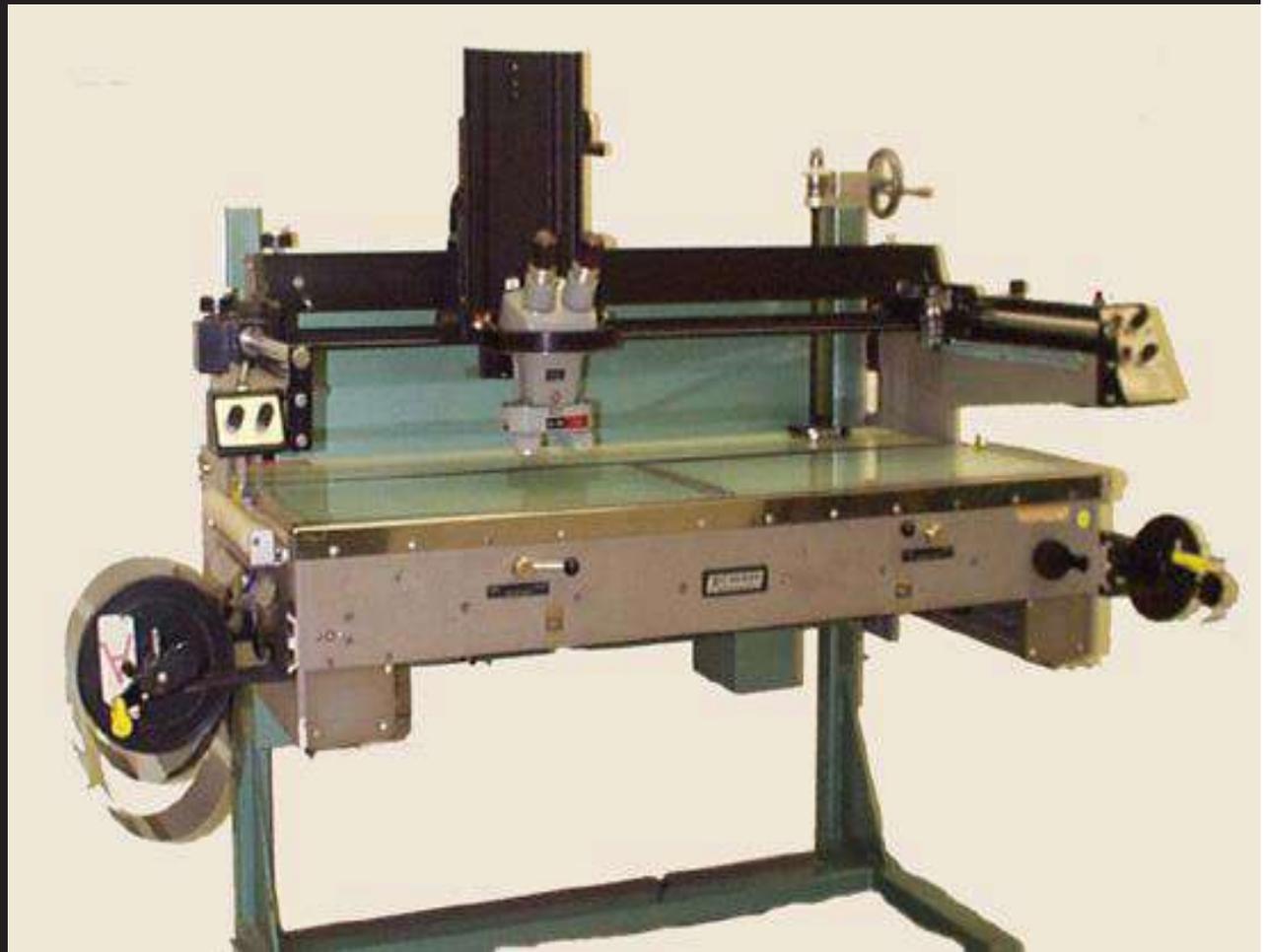
1. A series of lenses and prisms
2. Relatively expensive
3. Not used in the field



# 1- Definitions Stereoscope

## Zooming stereoscope

1. Variable magnification:  
2.5 - 20 x
2. Very Expensive
3. Not used in the field



## 2- Geometry of Stereoscopy

- In order to obtain **proper stereoscopic** vision, it is important to have **properly oriented photos** that are **aligned** in the geometry of when the images were taken.
- The next slides explains how this geometry is obtained.

# 2- Geometry of Stereoscopy

## 2-1 The Coordinate Axes

- Geometry of overlapping vertical aerial photos somewhat **different** than geometry of single photos:  $x$  and  $y$  axes not defined by the fiducial marks when **crab** exists (see appendix 3-4 )
- **Flight-line system of Coordinates** – The  $x$  axis is the line that passes through the  $PP$  and the  $CPP$ . The  $y$  axis is the line that passes through the  $PP$  perpendicular to the  $x$ .
- **Fiducial mark system** – uses just the  $PP$  to align the axes on a single photo

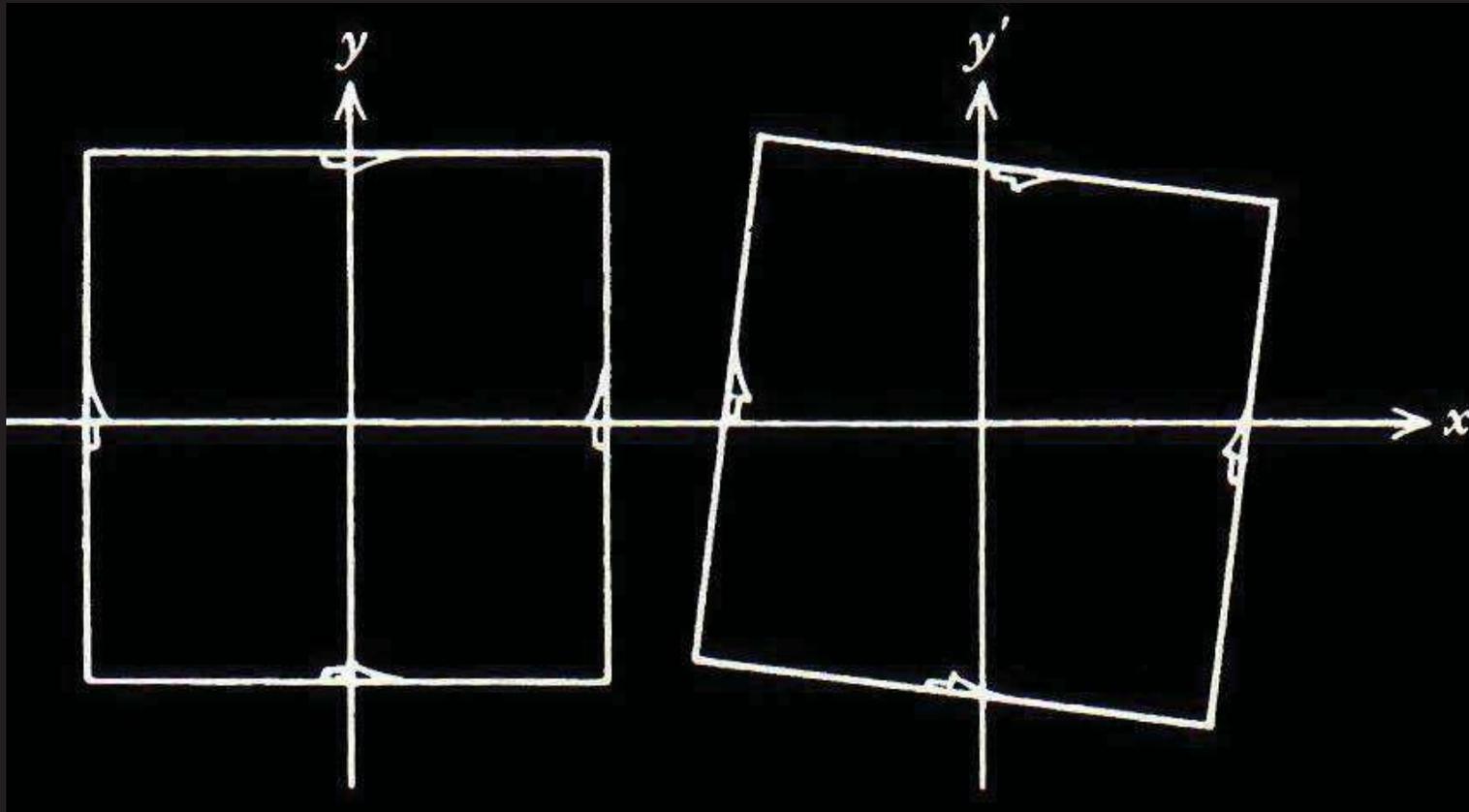
# 2- Geometry of Stereoscopy

## 2-1 The Coordinate Axes

- Only if both photos are free of tilt, drift and crab will the x and y axes pass through the side fiducial marks. ([Figure 3.6](#))
- In a stereo triplicate the center photo may have two different sets of x and y coordinates if aircraft does not fly a perfect line. ([Figure 3.7](#))

# 2- Geometry of Stereoscopy

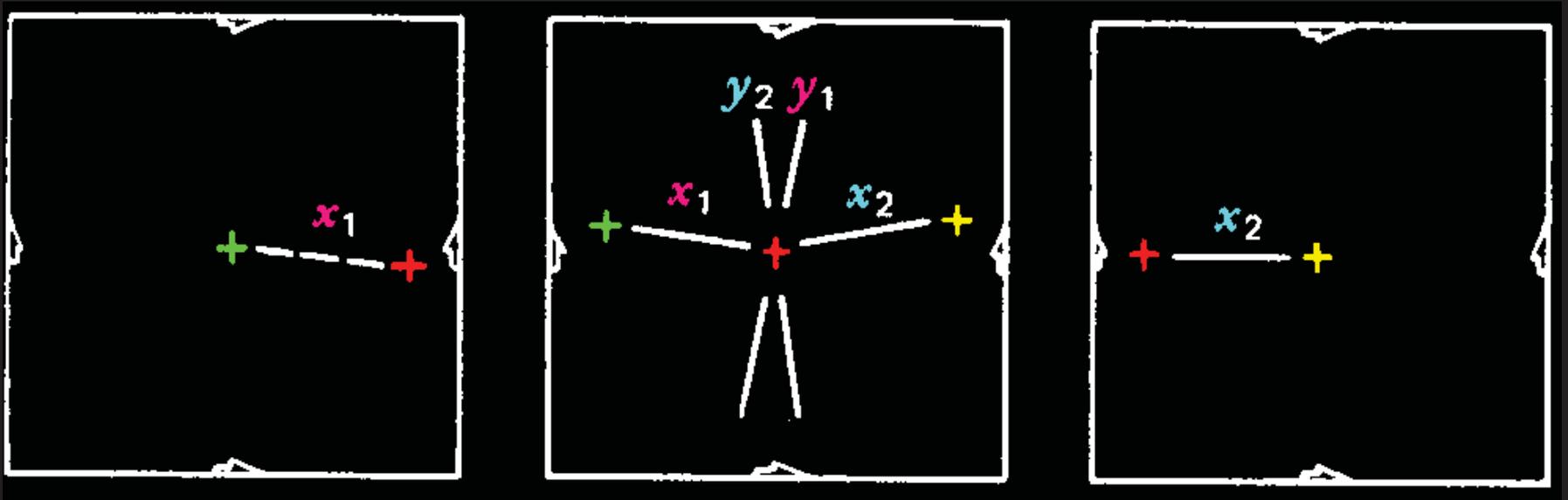
## 2-1 The Coordinate Axes



**Figure 3.6.** Coordinate axes of a stereoscopic pair of photographs. The  $x$  and  $y$  axes don't pass through the fiducial marks on the crabbed (rotated) photo on the right.

# 2- Geometry of Stereoscopy

## 2-1 The Coordinate Axes



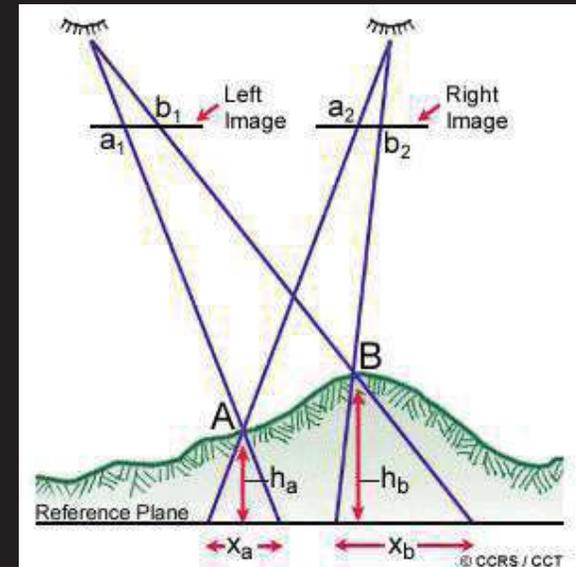
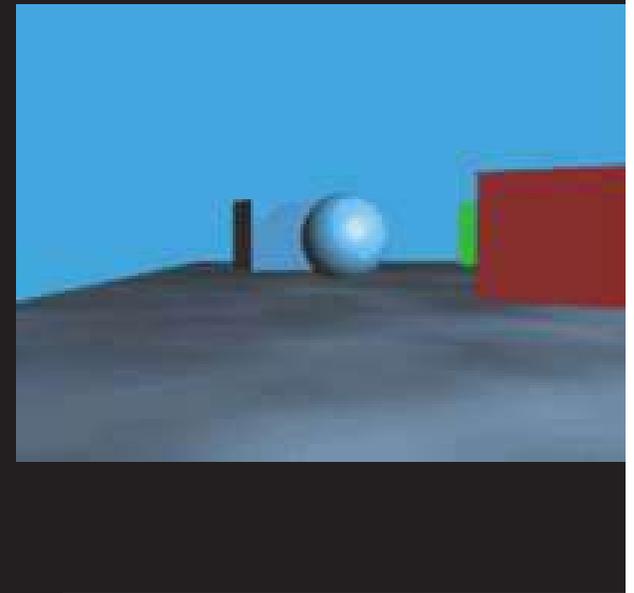
**Figure 3.7.** Two different sets of coordinates axes on the same (center) photograph. The  $x_1y_1$  axes are used with the two photos on the left, and the  $x_2y_2$  axes are used with the two photos on the right.

Conjugate principal points (CPPs) are the principal points of adjacent photos in the same flight line transferred to the photo being considered.

## 2- Geometry of Stereoscopy

### 2-2 Parallax

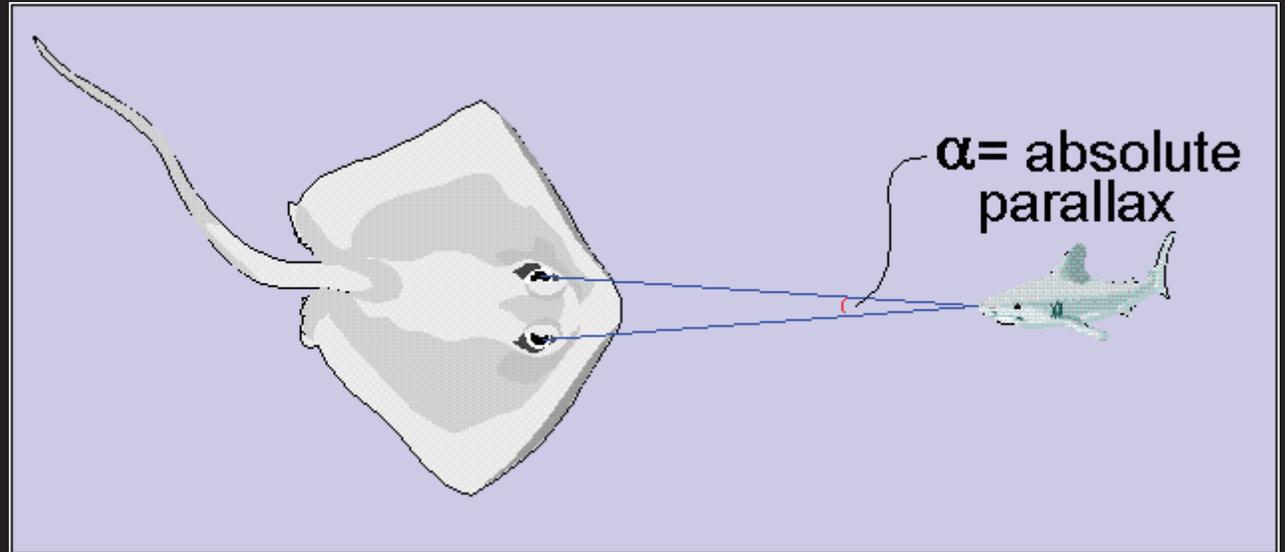
- **Parallax:** The displacement of an object caused by a change in the point of observation is called parallax.
- **Stereoscopic parallax** is caused by taking photographs of the same object but from different points of observation.



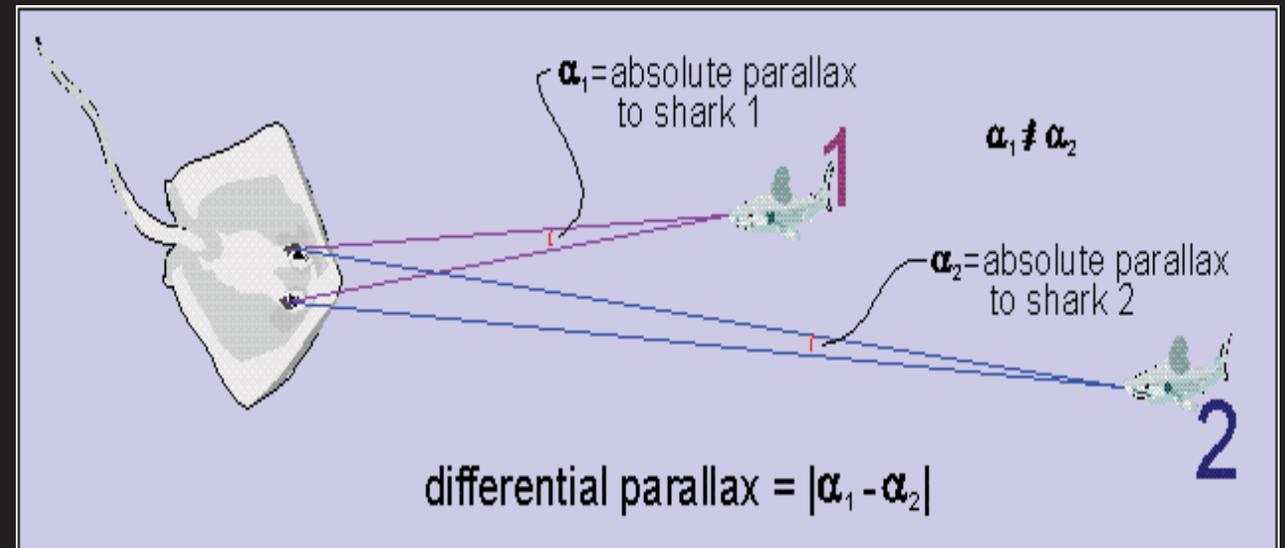
## 2- Geometry of Stereoscopy

### Types of Parallax:

1. **Absolute** parallax: It is the sum of the distances of corresponding images from their respective nadirs. It is measured parallel to the flight line.



2. **Differential** parallax: It is the difference in the absolute stereoscopic parallax at the top and base of the object measured.



## 2- Geometry of Stereoscopy

### 2-2 Parallax

- Different amounts along the  $x$ -axis on successive photographs. This difference in displacement is called the *difference in absolute parallax (dP)*.
- The **absolute** parallax of a **point on a pair** of overlapping vertical photographs is equal to the  $x$  coordinate of the point measured on the **left**-hand photograph **minus** the  $x$  coordinate of the point measured on the **right**-hand photograph.

## 2- Geometry of Stereoscopy

### 2-2 Parallax

A complete definition of absolute parallax (or  $x$  parallax) of a point is as follows:

- **Absolute parallax** is the algebraic difference, measured parallel to the line of flight ( $x$ -axis) from the corresponding  $y$ -axis to the two images of the point on a stereoscopic pair of aerial photographs.

## 2- Geometry of Stereoscopy

### 2-2 Parallax Determine the absolute parallax of a single point

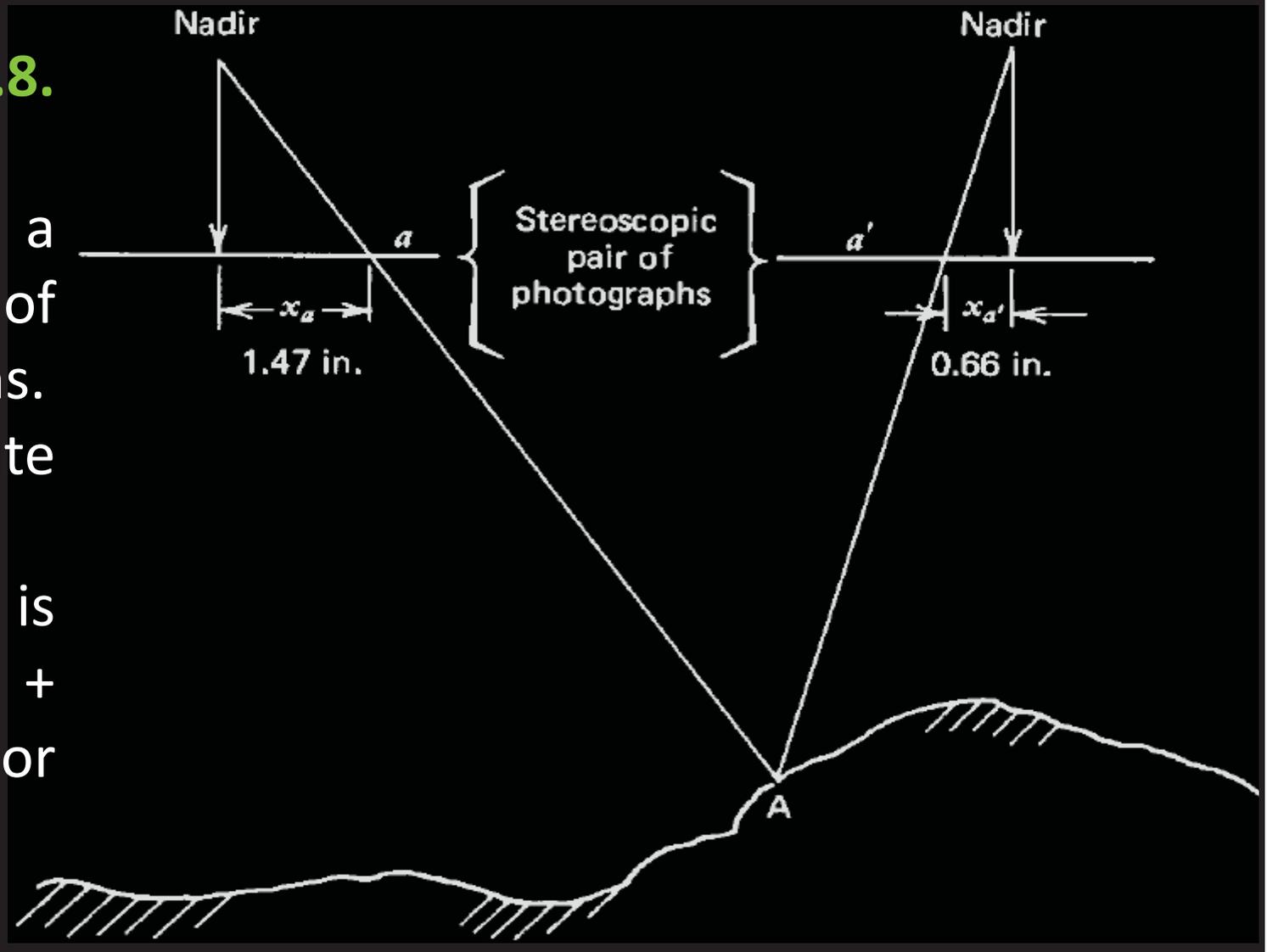
- This definition **assumes perfectly vertical** photographs taken at the same altitude.
- In **Figure 3.8** ground point **A** is imaged as points ***a*** and ***a'*** on the left- and right-hand photos, respectively.
- The ***x*** coordinate of point **A** on the left photo is ***xa*** and the ***x*** coordinate of point **A** on the right photo is ***xá***.
- Notice that ***xá*** is positive (to the right) and ***xa*** is negative (to the left).

## 2- Geometry of Stereoscopy

### 2-2 Parallax Determine the absolute parallax of a single point

**Figure 3.8.**

Absolute parallax of a stereopair of photographs. The absolute parallax of point  $a$  is 1.47 in. + 0.66 in., or 2.13 in.



## 2- Geometry of Stereoscopy

### 2-2 Parallax Determine the absolute parallax of a single point

- Therefore, by our definition, the absolute parallax of point  $A$  is  $x_a$  minus negative  $x'_a$ , or  $x_a + x'_a = 1.47 + 0.66 = 2.13$  in.
- Understanding the absolute parallax of a point is essential to understanding how we can make vertical measurements on a stereoscopic pair of vertical aerial photographs. This is the subject of Lecture 6.

## 2- Geometry of Stereoscopy

### 2-3 Flight-Line Location

- Flight line passes through the *PP* of each photo.
- *PP* of one photo located on the successive point on the photo is the Conjugate Principal Point (*CPP*) ([Figure 3.9](#))
- Connecting the *PP* and *CPP* on a photo gives the flight line.

## 2- Geometry of Stereoscopy

### 2-3 Flight-Line Location

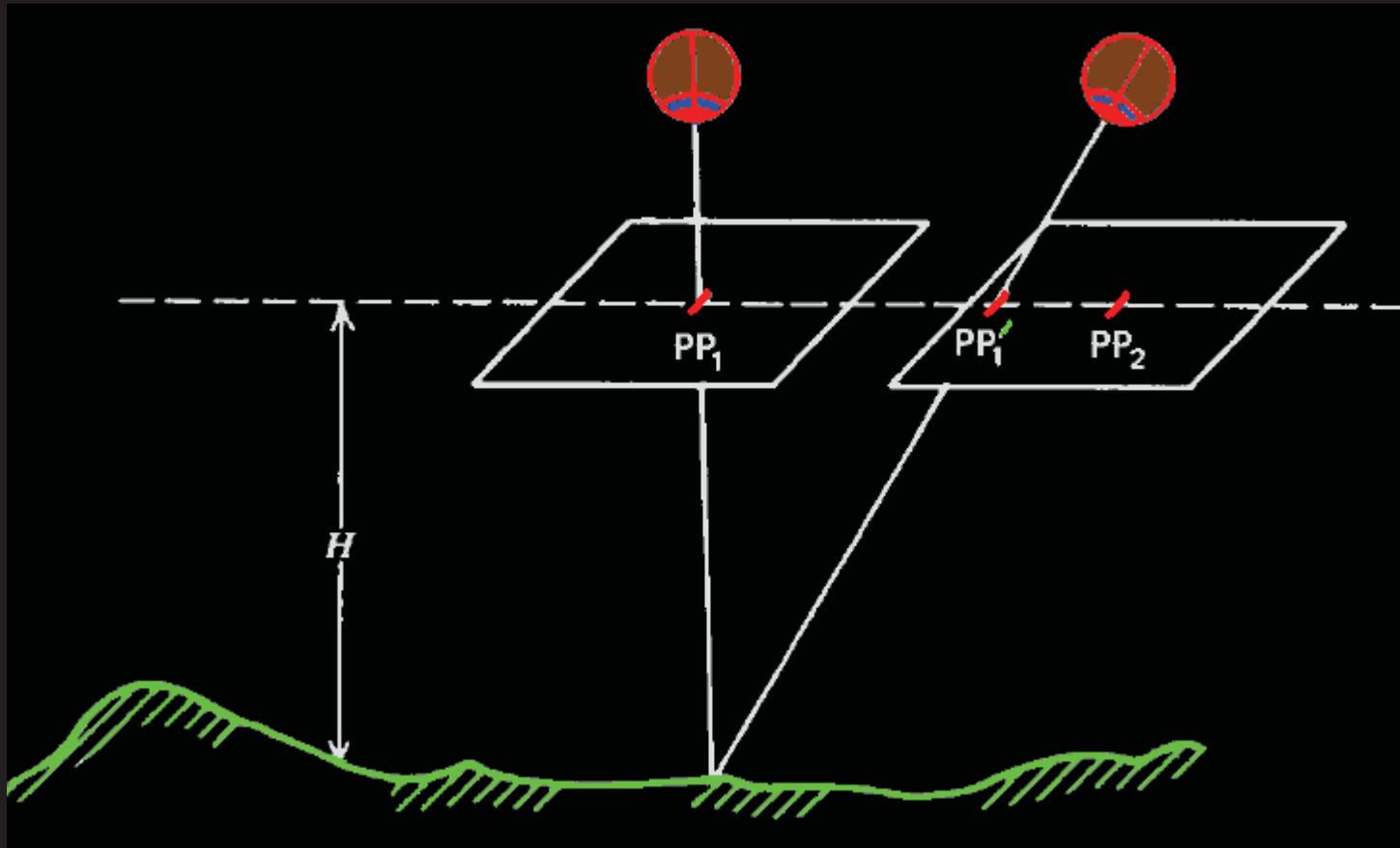


Figure 3.9. Locating the conjugate principal point (PP, or CPP).

*Thank you*

Any Questions ?



**END**

**of Lecture**

# Appendix-3-1

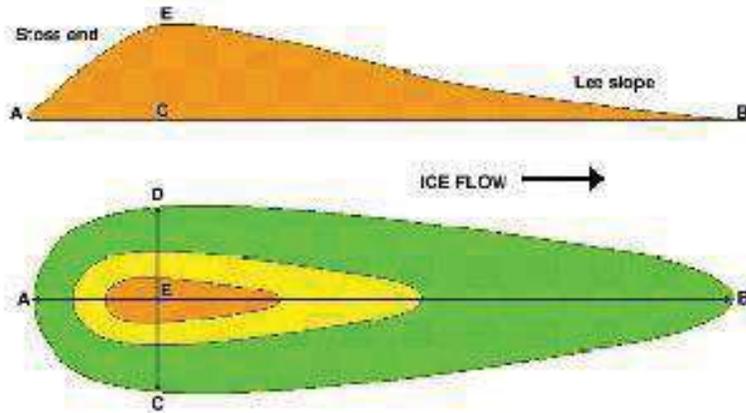
## Anaglyph 3D

**Anaglyph 3D** is the name given to the stereoscopic 3D effect achieved by means of encoding each eye's image using filters of different (usually chromatically opposite) colors, typically red and cyan. Anaglyph 3D images contain two differently filtered colored images, one for each eye. When viewed through the "color-coded" "anaglyph glasses", each of the two images reaches the eye it's intended for, revealing an integrated stereoscopic image. The visual cortex of the brain fuses this into the perception of a three-dimensional scene or composition.

[https://en.wikipedia.org/wiki/Anaglyph\\_3D](https://en.wikipedia.org/wiki/Anaglyph_3D)

# Appendix-3-2

## DRUMLINS



### Characteristics:

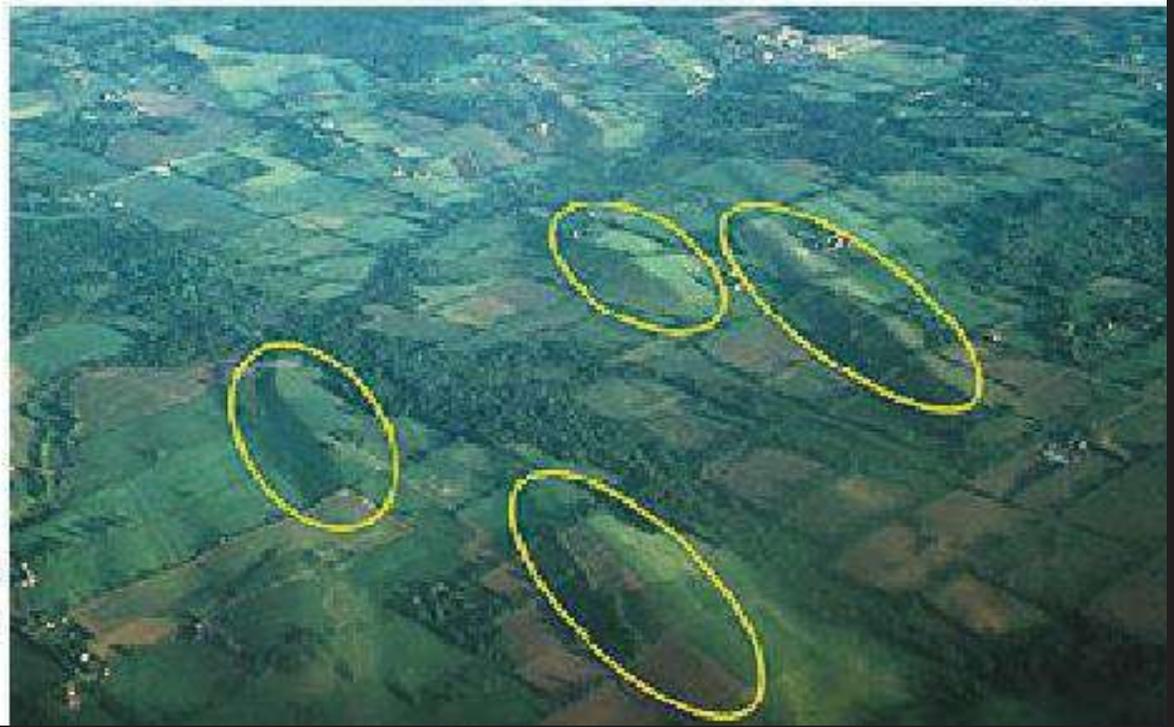
- Smooth elongated mounds of till – long axis parallel to the direction of the ice movement (mounds of glacial debris streamlined into elongated hills)
- Where found in clusters – called “**drumlin swarm**” – classic ‘basket of eggs topography’

- Range in size from small mounds (2m high / 10m long) to huge hills – can be over a km long and 100m in height.

- shape of drumlin measured using elongation ratio =

$$\frac{\text{Length of drumlin}}{\text{max width}}$$

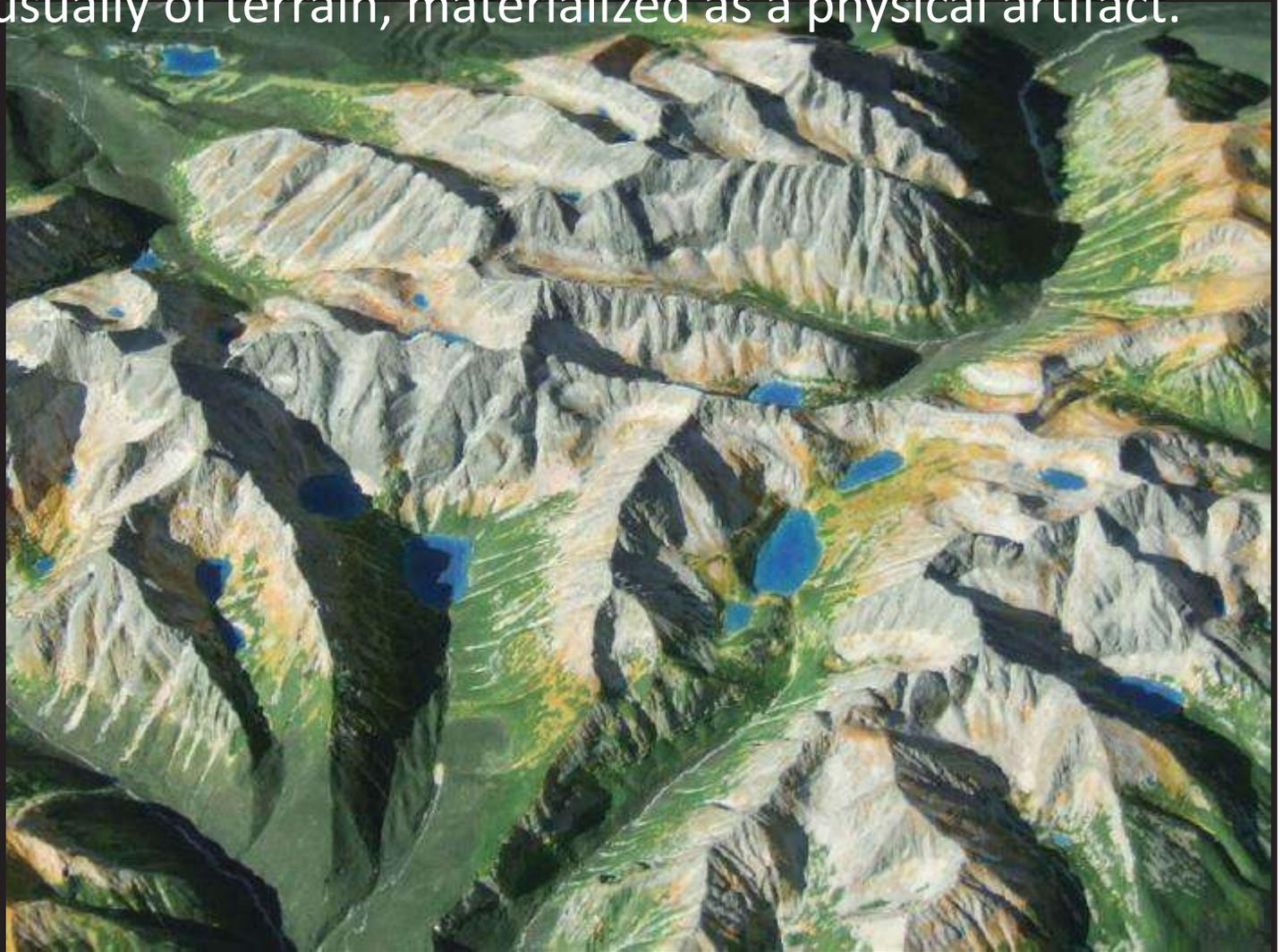
- Usually between 25:1 and 4:1 – greater elongation suggests more powerful ice flow.



# Appendix-3-2

A **raised-relief map** or **terrain model** is a three dimensional representation, usually of terrain, materialized as a physical artifact.

When representing terrain, the vertical dimension is usually exaggerated by a factor between five and ten; this facilitates the visual recognition of terrain features.

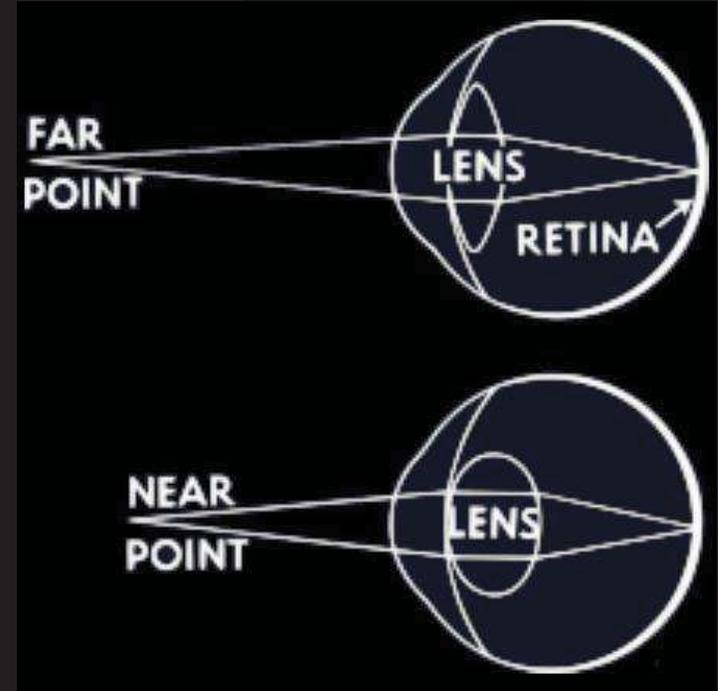
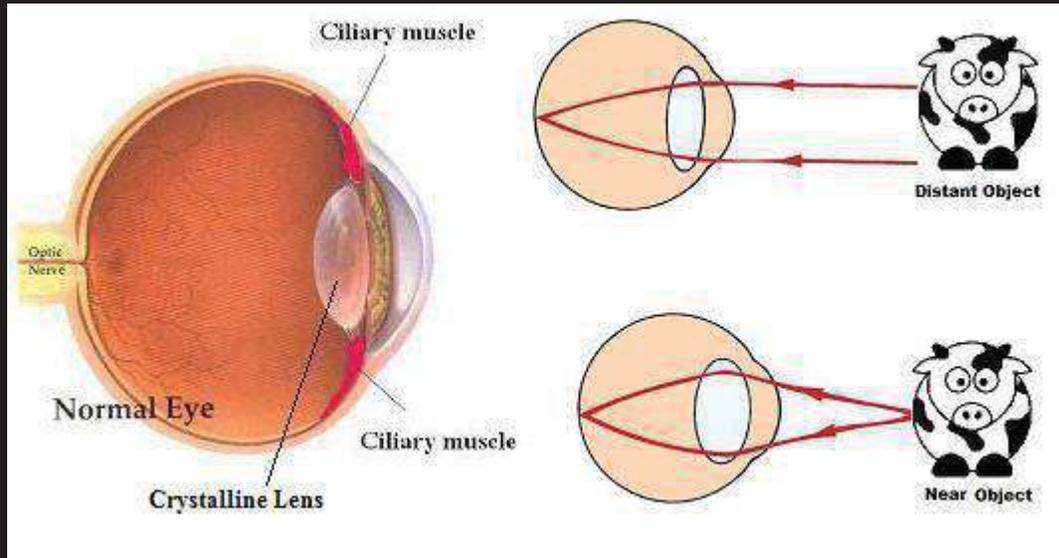


Hand-made **raised-relief map** of the High Tetras in scale 1: 50 000

# Appendix-3-3

Moore Vision Skills saved to Accommodation/Eye Focusing

An eye focusing problems is an inability to easily refocus eyes or maintain clear focus.

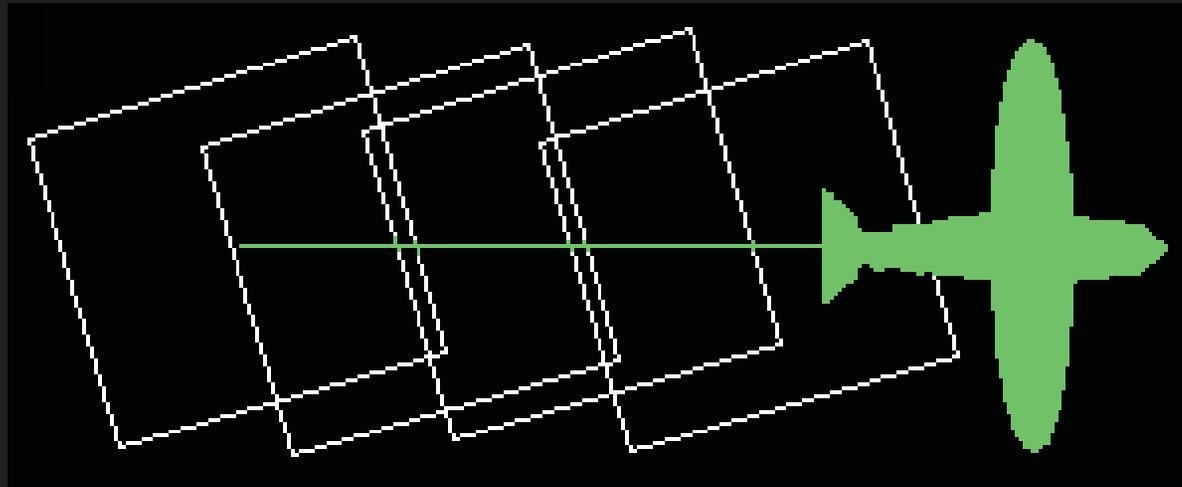


ضوء من نقطة واحدة من كائن بعيد وضوء من نقطة واحدة لكائن قريب تم نقلها للبؤرة عن طريق تغيير تقوس العدسة .

# Appendix-3-4

## Crab

- Crab is the condition caused by the failure to orient the camera with respect to the planned flight line.
- Crab should not affect more than 5% of the photograph.



# Appendix-3-5

## 3-4 Vertical exaggeration

- **Apparent scale disparity** between horizontal and vertical scales
- **Primary cause:** lack of equivalence between **photographic** base-height ratio (  $B/H'$  ) and **stereo-viewing** base-height ratio (  $b_e/h$  )

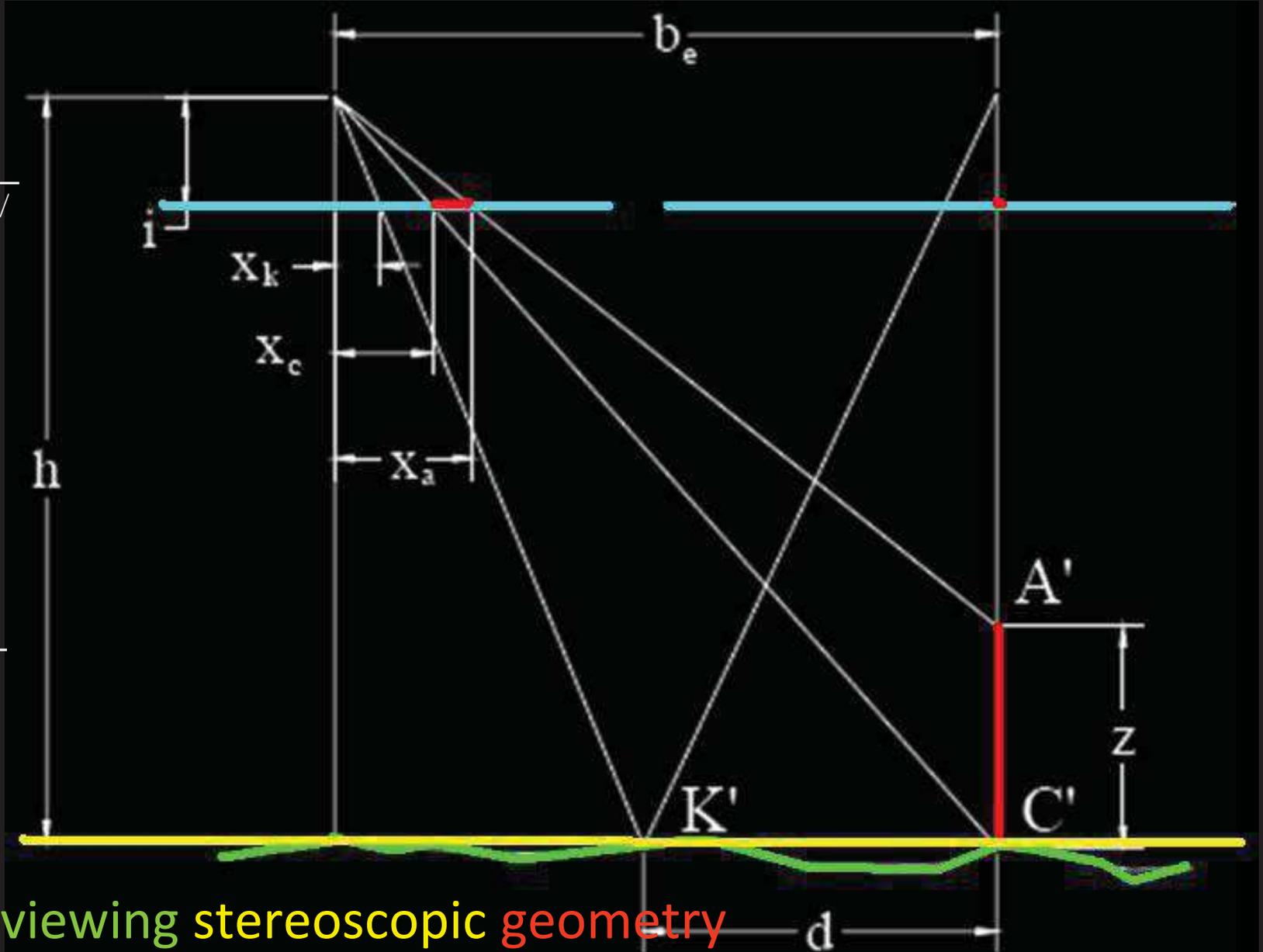


# Appendix-3-5

$$\frac{X_c}{B} = \frac{f}{H'}$$



$$X_c = \frac{Bf}{H'}$$



stereo-viewing stereoscopic geometry

# Appendix-3-5

From aerial geometry:

$$\frac{X_a}{B} = \frac{f}{H' - Z} \implies X_a = \frac{Bf}{H' - Z}$$

From stereoscopic geometry:

$$\frac{X_c}{B} = \frac{f}{H'} \implies X_c = \frac{Bf}{H'}$$

Subtracting:

$$X_a - X_c = Bf \frac{Z}{(H')^2 - H'Z}$$

# Appendix-3-5

- From similar triangles:

$$\frac{X_a}{b_e} = \frac{i}{h-z} \Rightarrow X_a = \frac{b_e i}{h-z}$$

$$\frac{X_c}{b_e} = \frac{i}{h} \Rightarrow X_c = \frac{b_e i}{h}$$

- Subtracting,

$$X_a - X_c = b_e i \frac{z}{h^2 - hz}$$

# Appendix-3-5

Equating the 2 equations for  $X_a - X_c$

$$Bf \frac{Z}{(H')^2 - H'Z} = b_e i \frac{z}{h^2 - hz}$$

• But  $Z$  and  $z$  are considerably smaller than  $H'$  and  $h$ , thus

$$\frac{BfZ}{(H')^2} \approx \frac{b_e iz}{h^2} \quad \Rightarrow \quad \frac{z}{Z} = \frac{fh}{H'i} \frac{Bh}{H'b_e}$$

# Appendix-3-5

- From similar triangles in 2 diagrams

$$\frac{x_c - x_k}{D} = \frac{f}{H'} \quad \Rightarrow \quad D = (x_c - x_k) \frac{H'}{f}$$

$$\frac{x_c - x_k}{d} = \frac{i}{h} \quad \Rightarrow \quad d = (x_c - x_k) \frac{h}{i}$$

- From

$$\frac{d}{D} = \frac{fh}{H'i}$$

# Appendix-3-5

Substitute into equation for  $z/Z$

$$\frac{z}{Z} = \frac{d}{D} \frac{Bh}{H'b_e}$$

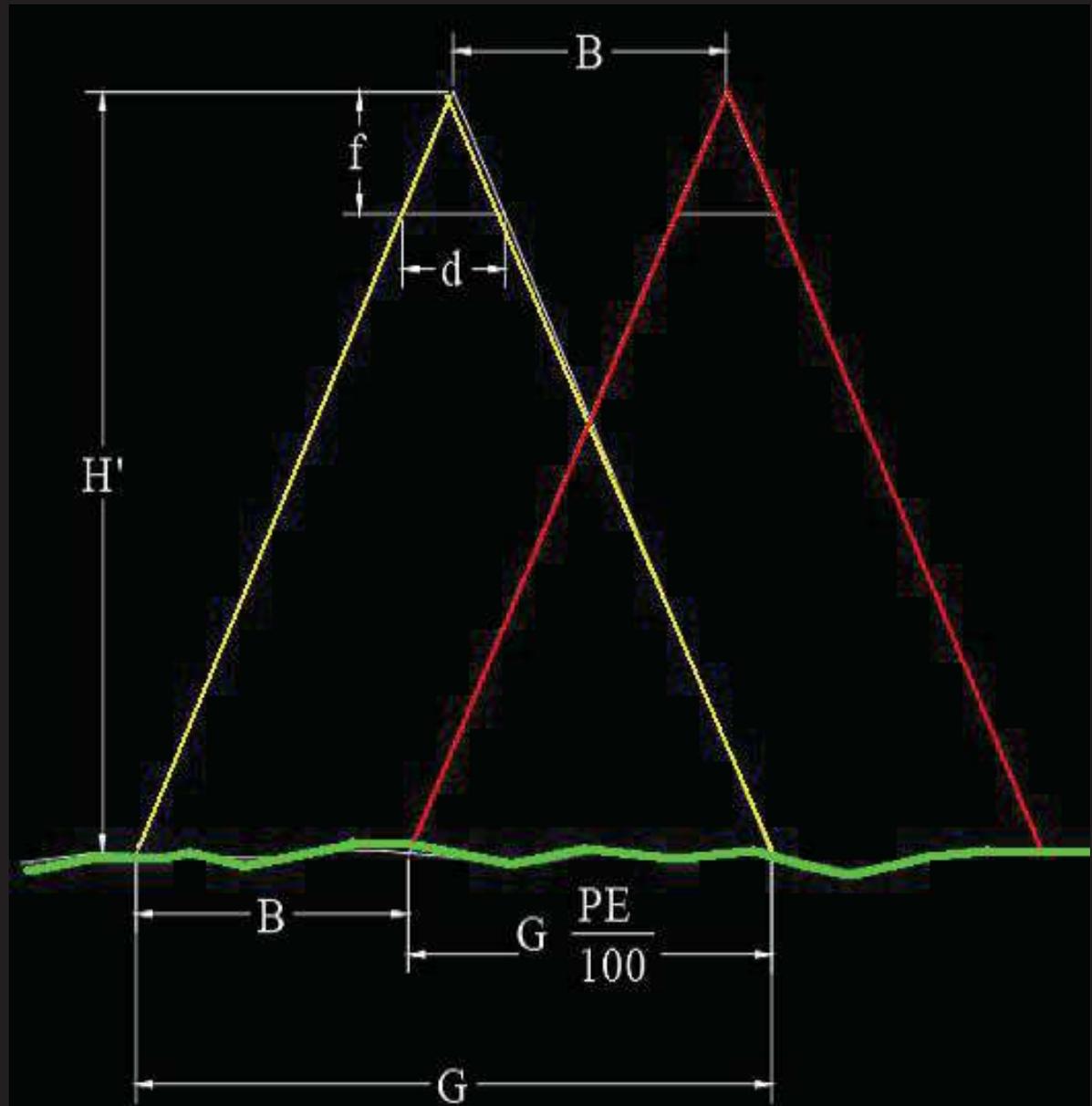
If  $Bh/(H'b_e)$  is 1, there is no vertical exaggeration.

Thus, magnitude of vertical exaggeration,  $V$ , is given by

$$V \approx \frac{B}{H'} \frac{h}{b_e}$$

# Appendix-3-5

$$V \approx \frac{B}{H'} \frac{h}{b_e}$$



# Appendix-3-5

- From the figure

$$B = G - G \frac{PE}{100} = G \left( 1 - \frac{PE}{100} \right)$$

- From which

$$\frac{H'}{G} = \frac{f}{d} \quad \Rightarrow \quad H' = \frac{fG}{d}$$

# Appendix-3-7

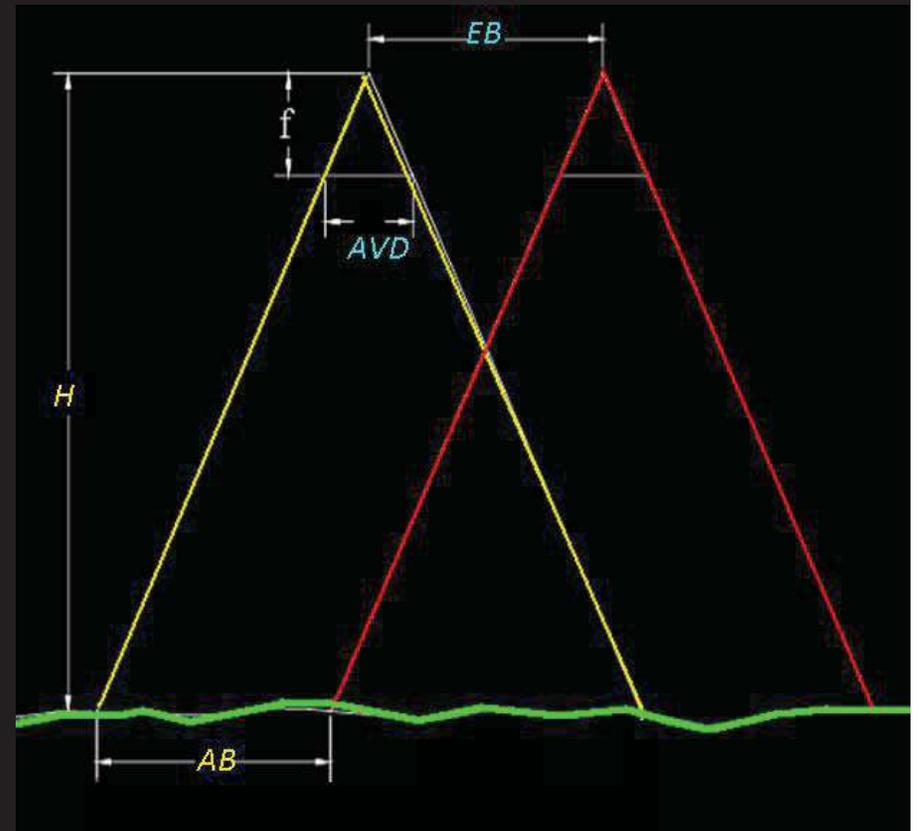
Quiz Solution:

$$VE = \left( \frac{AB}{H} \right) \left( \frac{AVD}{EB} \right)$$

$$\frac{AB}{H} = \left( 1 - \frac{60}{100} \right) \left( \frac{230}{152.4} \right) = 0.6$$

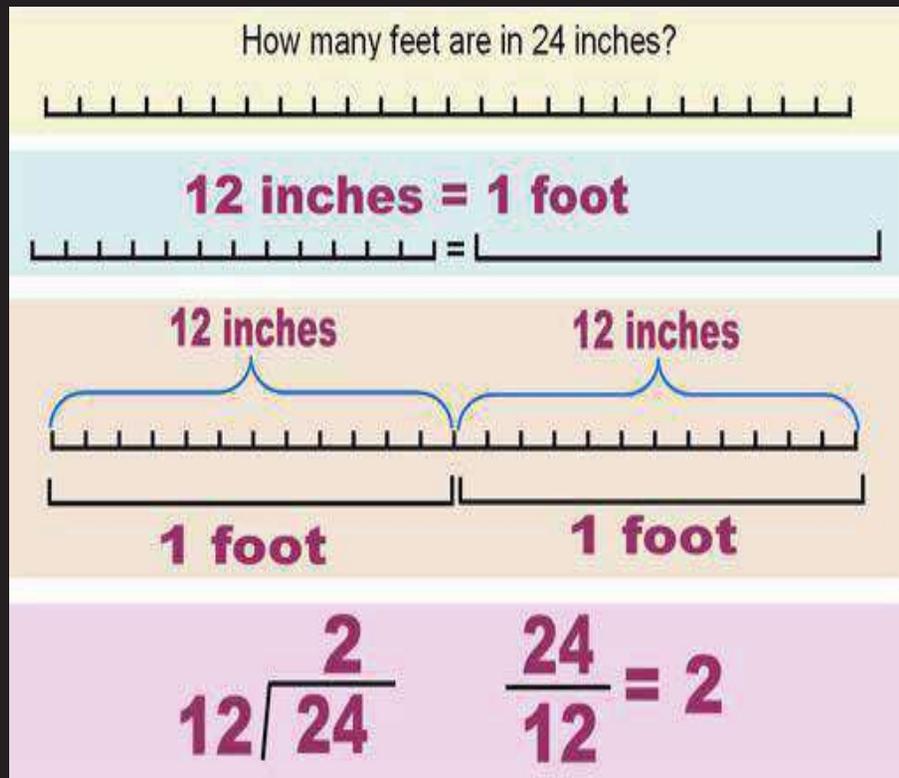
$$(AVD/EB) = 1/(0.15)$$

$$\therefore VE = 0.6 \left( \frac{1}{0.15} \right) = 4$$



# Appendix-3-6

## difference feet and foot



### FEET      VERSUS      FOOT

The plural form of foot

Can either refer to  
- the lower part of the leg  
- measurement of height, length and depth

The singular form of feet

The plural form of foot

Generally used with plural measurements

Used with plural measurements, especially in spoken language

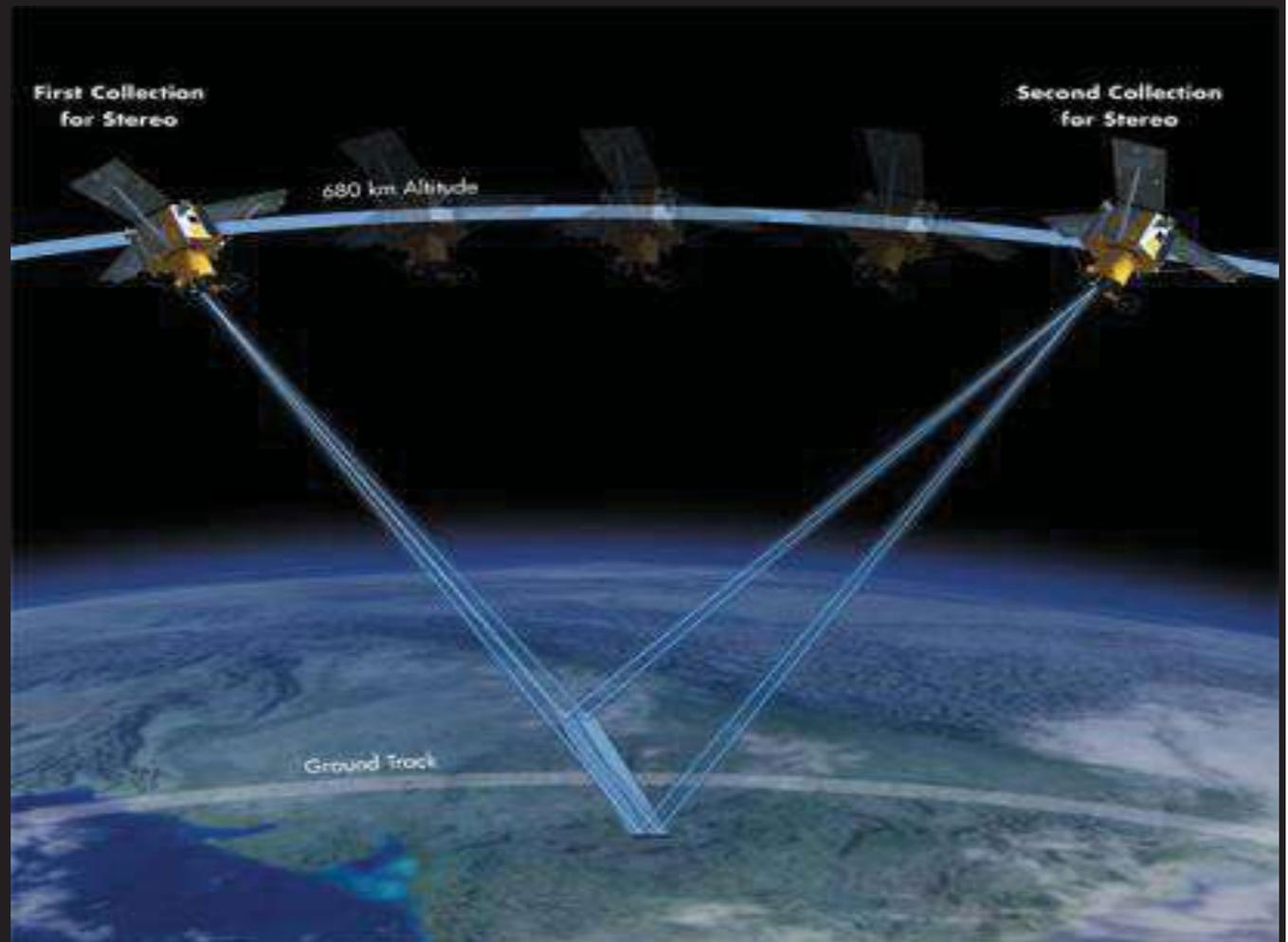
Not used with plural measurements when the measurement acts as a modifier

Used with plural measurements when the measurement acts as a modifier

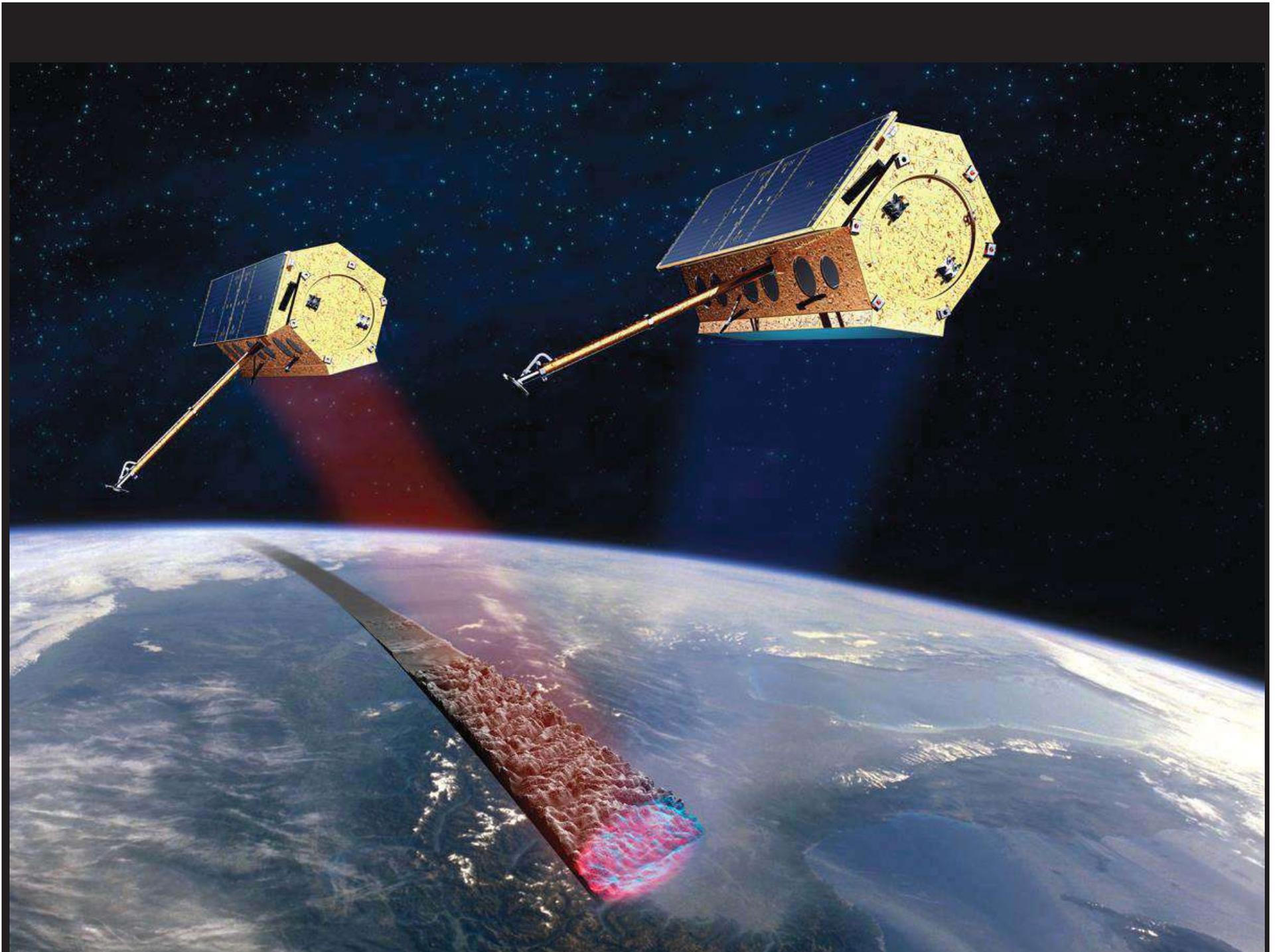


Manual vs. digital stereoscopes.  
Source: Ghent University

<http://www.seos-project.eu/modules/3d-models/3d-models-c02-p04-s01.html>



Stereo IKONOS Satellite Image data collection.  
Source: Satellite Imaging Corporation. All rights reserved.



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

# *Aerial Photography & Photogrammetry*

## Lecture 4-2

# Principles of Stereoscopic Vision

*Lecturer: Faisal Ghazi Mohammed*

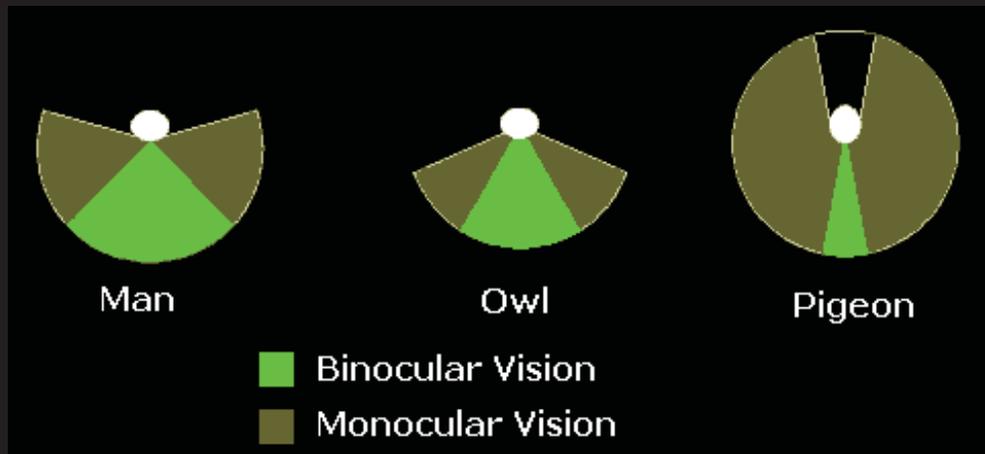
Email: [faisal@scbaghdad.edu.iq](mailto:faisal@scbaghdad.edu.iq)  
[faiselgm73@gmail.com](mailto:faiselgm73@gmail.com)

*2017-2022*

*All rights reserved*

### 3- Theory of Stereoscopy

- Nature provides us with two excellent examples that help us understand why we see things the way we do.
- The **animal** world can be separated into two categories those that are **predators** and those that are **prey**.
- All predators have **binocular** vision and all prey animals have **Monocular** vision.



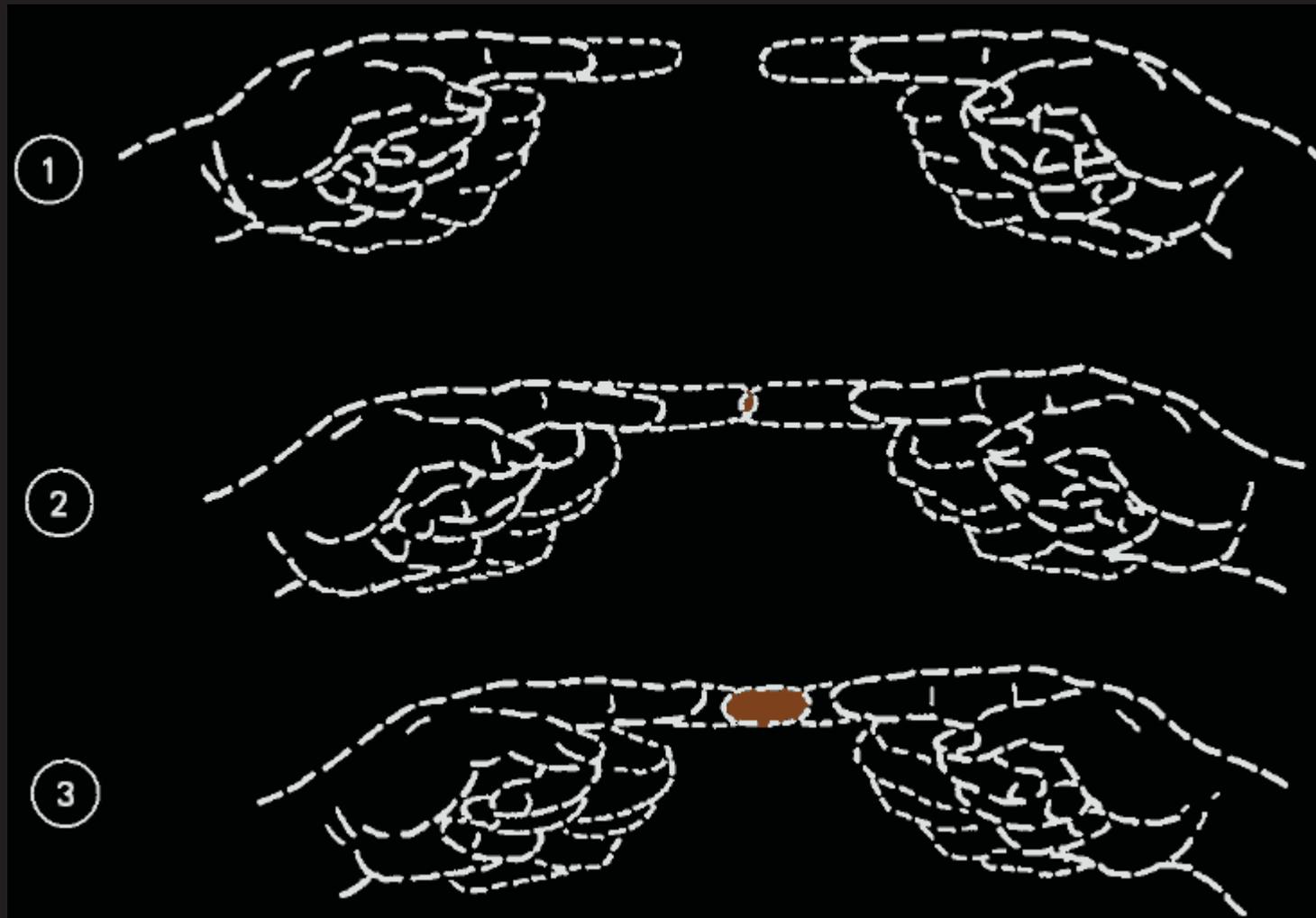
### 3- Theory of Stereoscopy

- **Single** eye **can't perceive** depth (pencil demonstration). Need a second eye to see depth.
- The phenomenon of stereoscopic vision involves both **mechanical and physiological** principles. Our vision is so natural that we seldom stop to analyze it.
- Although a **single** human eye has a wide range of view both horizontally and vertically, it is very limited in its ability to **convey a sense** of depth.

### 3- Theory of Stereoscopy

- Except by inference or association with other objects, a **single** eye **cannot** accurately determine whether one object is nearer or farther away than another object.
- Fortunately, we have two eyes and are thereby able to perceive depth.
- A simple exercise will show you why we need two eyes to visualize depth.
- **Figure 3.10** shows sausage exercise

### 3- Theory of Stereoscopy



**Figure 3.10.** The “sausage exercise.” This exercise helps develop the ability to see stereoscopically without the aid of a stereoscope. (Courtesy of the U.S. Department of the Army.)

## 3- Theory of Stereoscopy

### 3-1 Accommodation and Convergence

- **Accommodation** – the change of focus of the eye for distance. See [Appendix-3-3](#)
- **Convergence** – when eyes focus on a nearby object they converge so that lines of sight join at the object.

Use the sausage exercise to make the lines of sight parallel (focus on infinity) and still see sharply. ([Figure 3.10](#))

## 3- Theory of Stereoscopy

### 3-2 Depth Perception

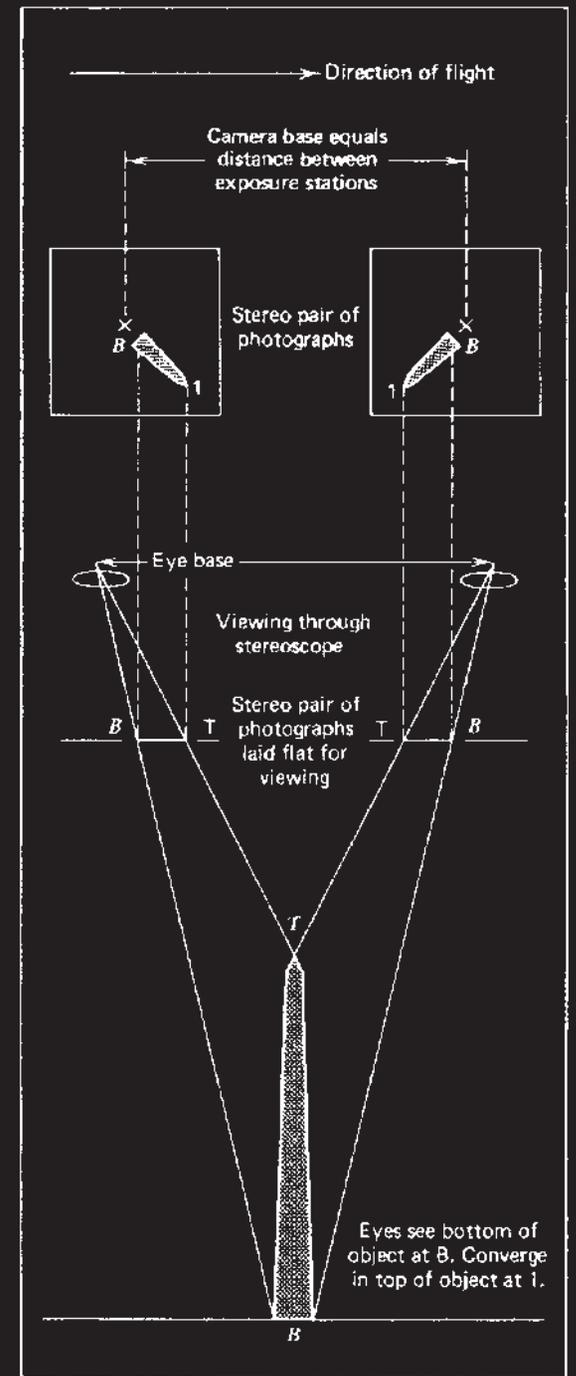
- Look at ([Figure 3.11](#)) If the left eye sees only the left image and the right eye sees only the right image, the object (Washington Monument) will appear in 3-D.

# 3- Theory of Stereoscopy

## 3-2 Depth Perception

- If we orient these photos and view them so that the left eye sees only the image on the left photo, and the right eye sees only the image on the right photo, we have a perception of depth.
- As you can see at the bottom of the illustration, the top of the **monument** appears to be at **T** and the bottom at **B**.

**Figure 3.11.** Mechanics of stereoscopic viewing. (Courtesy of the Department of the Army, Navy, and the Air Force.)



## 3- Theory of Stereoscopy

### 3-3 The Floating Dot Principle

- This principle can be applied in the **transfer** of principal points **PP** from one photo to the next.
- The dots (**Figure 3.12**) are on transparent material that has been **laid on top** of the correctly oriented photos.
- The left dot has been placed on the left photo and the right dot is placed on the right photo. Thus, the left eye sees the left dot and the right eye sees the right dot. The **two dots fuse together in your brain**, at the apparent position **A**. → You will sense that the image

# 3- Theory of Stereoscopy

## 3-3 The Floating Dot Principle

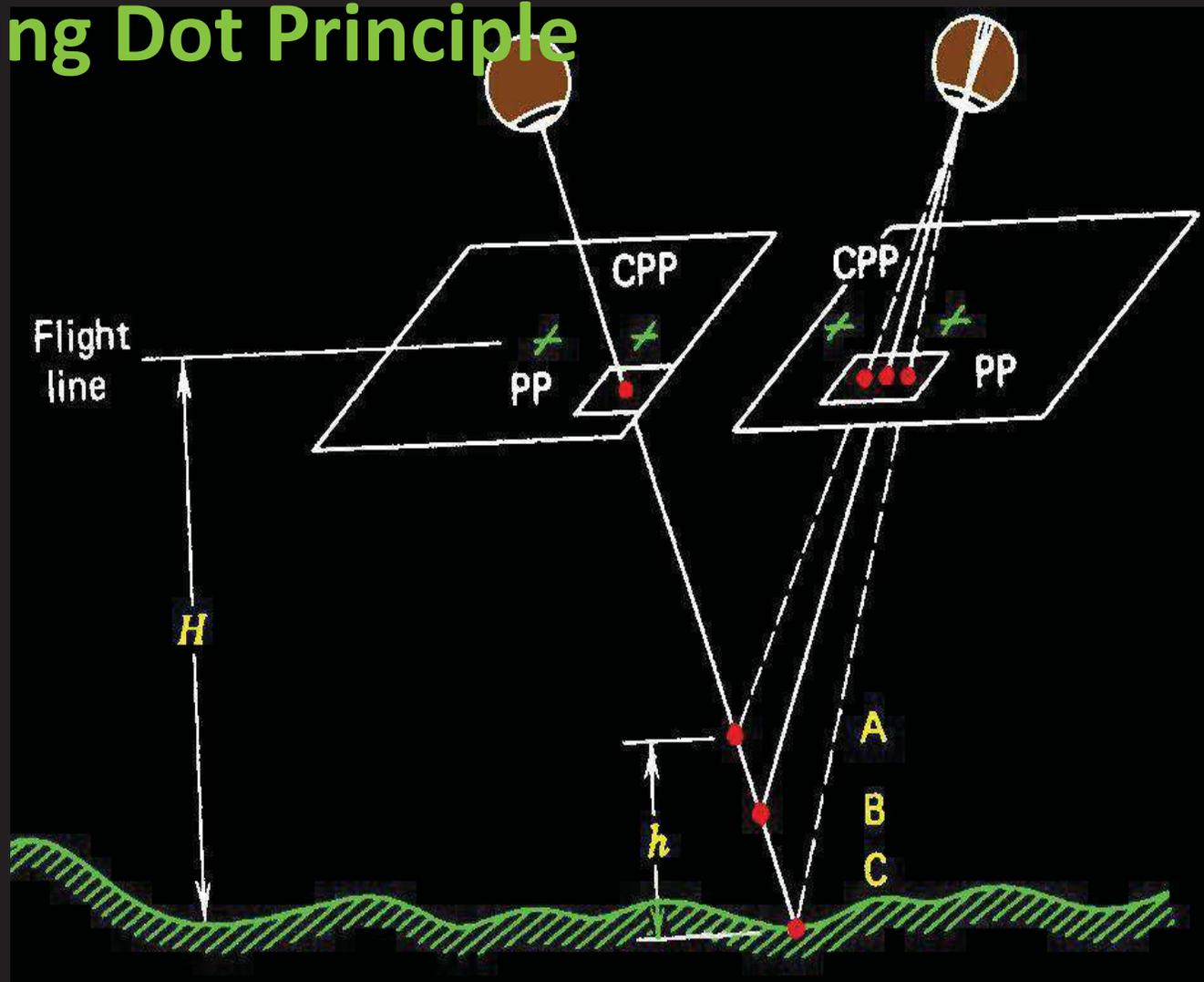


Figure 3.12. The floating-dot principle.

## 3- Theory of Stereoscopy

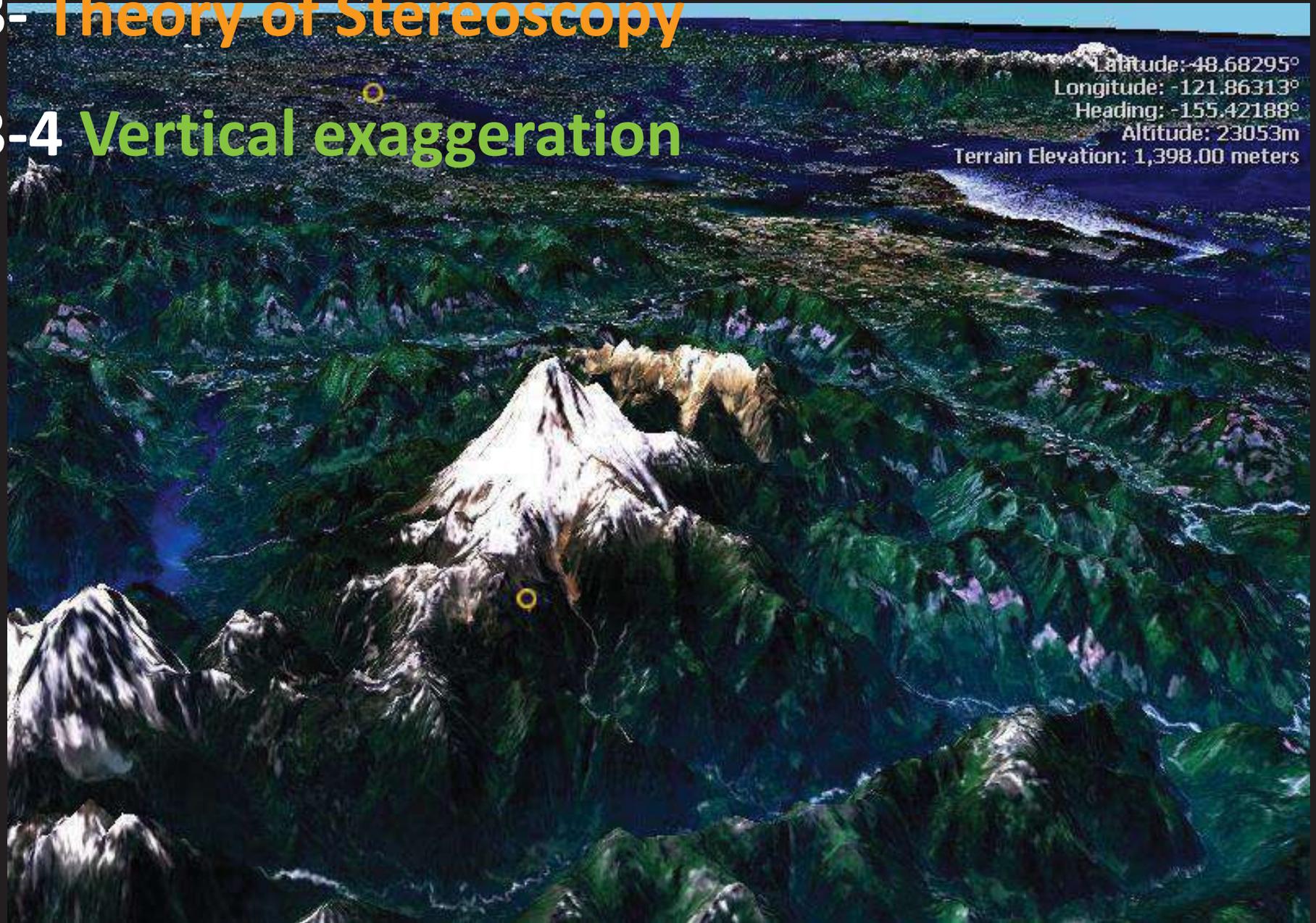
### 3-4 Vertical exaggeration

- Vertical exaggeration (VE) is a scale that is used in raised-relief maps, plans and technical drawings (cross section perspectives), in order to emphasize vertical features, which might be too small to identify relative to the horizontal scale.

$$VE = \frac{\text{vertical scale}}{\text{horizontal scale}}$$

# 3- Theory of Stereoscopy

## 3-4 Vertical exaggeration



A vertically exaggerated mountain. In reality, the terrain would appear much flatter.

## 3- Theory of Stereoscopy

### 3-4 Vertical exaggeration

- Figure 3.13 Shows **stereo triplicate** with higher exaggeration on the right image than the left.
- **Vertical exaggeration** increases with the ratio of the distance between exposures (the **airbase**) over the flying height above the ground.  $VE \sim AB / H$
- *Leave the math for now. Skip the slope estimator for now.*

## 3- Theory of Stereoscopy

### 3-4 Vertical exaggeration



**Figure 3.13.** Vertical exaggeration. The left model of this stereotriplicate has a vertical exaggeration of 2 and the right model has a vertical exaggeration of 4. (Courtesy of Illinois Photographic Service.)

## 3- Theory of Stereoscopy

### 3-4 Vertical exaggeration

- There are other factors involved in vertical exaggeration, but the primary one is the lack of equivalence between this ratio and the corresponding viewing ratio (**Figure 3.14**).
- Vertical exaggeration can be approximately calculated by multiplying the airbase  $AB$  to flying height  $H$  ratio by the inverse of the eyebase  $EB$  to apparent viewing distance  $AVD$  ratio (Miller 1960; LaPrade 1972; Wolf 1974).

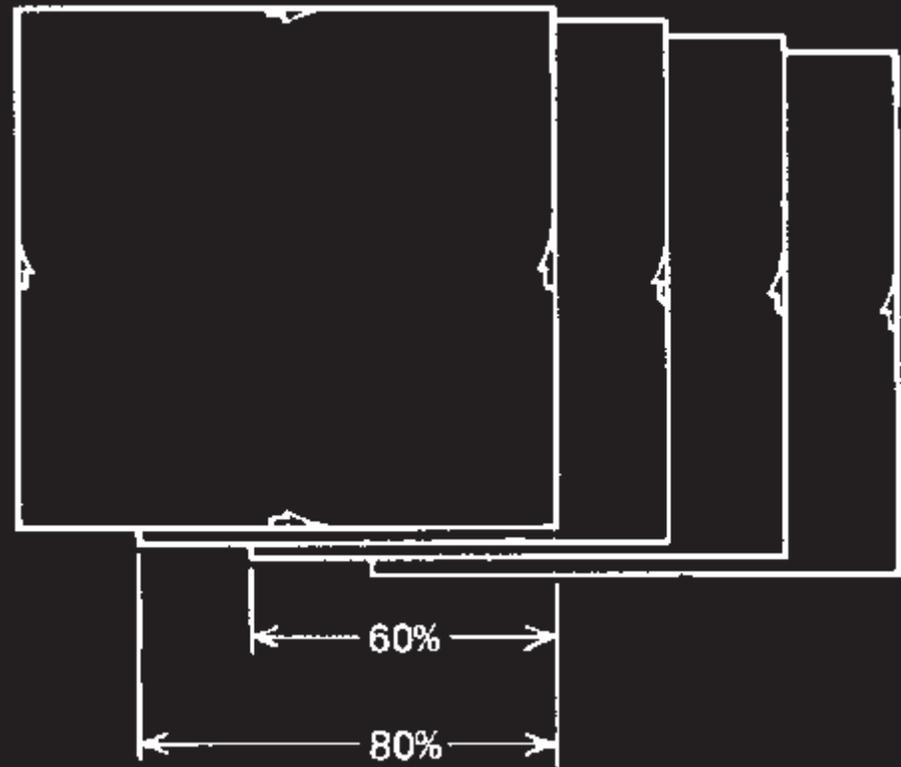
## 3- Theory of Stereoscopy

### 3-4 Vertical exaggeration

Figure 3.14.

Obtaining both 60% and 80% endlap on the same photo mission.

Every other photo is used for 60% endlap.



## 3- Theory of Stereoscopy

### 3-4 Vertical exaggeration

By developing an approximate, but workable, vertical exaggeration equation, we get (see text and [Appendix-3-5](#) for more details):

$$VE = \left( \frac{AB}{H} \right) \left( \frac{AVD}{EB} \right)$$

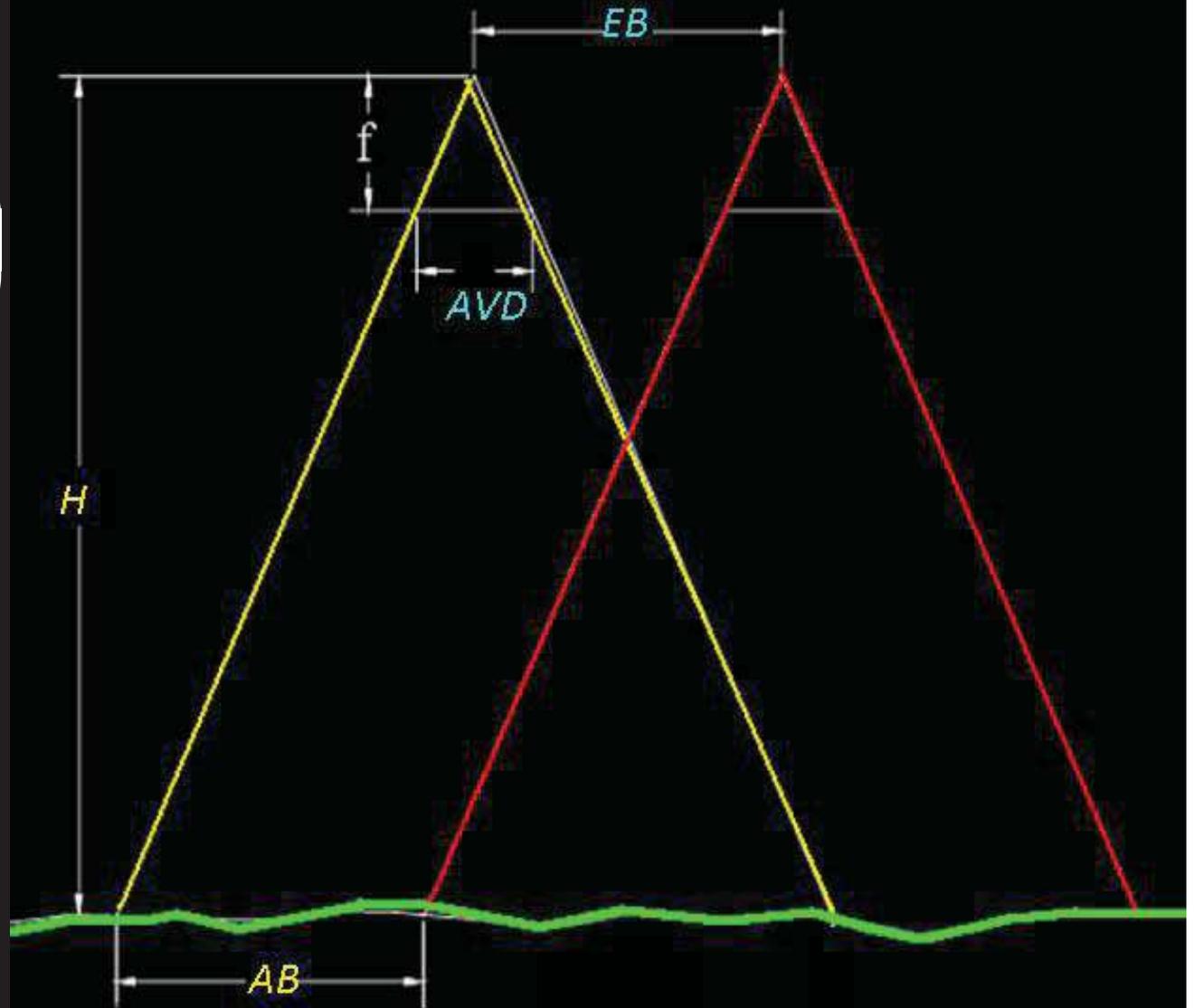
Where:

- $VE$  = Vertical exaggeration
- $AB$  = Airbase (ground distance between exposure stations)
- $H$  = Flying height above the average ground elevation
- $EB$  = Eyebase, or distance between your eyes
- $AVD$  = Apparent stereoscopic viewing distance

# 3- Theory of Stereoscopy

## 3-4 Vertical exaggeration

$$VE = \left( \frac{AB}{H} \right) \left( \frac{AVD}{EB} \right)$$



# 3- Theory of Stereoscopy

## 3-4 Vertical exaggeration

$$VE = \left( \frac{AB}{H} \right) \left( \frac{AVD}{EB} \right)$$

$$= \left( \frac{(1 - \%E)(Fmt \text{ inches})(PSR)}{H \text{ feet}} \right) \left( \frac{1 \text{ foot}}{12 \text{ inches}} \right) \left( \frac{1}{12} \right)$$

Feet is plural of Foot  
See Appendix-3-6  
for more details  
about Foot & Feet  
Diff.

$$= \left( \frac{(1 - \%E)(Fmt \text{ inches})(PSR)}{1.8H} \right) \text{ or } \left( \frac{(1 - \%E)(Fmt \text{ inches})}{1.8f} \right)$$

### Where:

- $\%E$  = Percent endlap (overlap of a stereoscopic pair expressed as a decimal, for example, 60% = 0.6.)
- $1 - \%E$  = Net gain per photo (expressed as a decimal)
- $Fmt$  = Photo format in the direction of the flight in in. or cm
- 1.8 = A constant = 0.15(12 with units of in./ft) or 15 cm/m
- $PSR$  = Photo scale reciprocal = *ground distance / photo distance*
- $H$  = Flying height above the average ground elevation in feet or m

### 3- Theory of Stereoscopy

### 3-4 Vertical exaggeration

$$= \left( \frac{(1 - \% E)(Fmt \text{ inches})(PSR)}{1.8H} \right)$$

#### Example 1:

Determine the vertical exaggeration of a 9×9 in. format photo with 55% endlap taken with a 12 inch lens at 20,000 feet. Suppose photo scale 1:20,000

**Solution:**  $(1 - \% E) = 1 - 0.55 = 0.45$   $Fmt = 9in$

$PSR = \text{ground distance} / \text{photo distance}$   $H = 20000 ft$

$PSR = 20000 / 1$

### 3- Theory of Stereoscopy

#### 3-4 Vertical exaggeration

$$= \left( \frac{(1 - \% E)(Fm \text{ inches})(PSR)}{1.8H} \right)$$

Solution:

$$(1 - \% E) = 1 - 0.55 = 0.45 \quad Fm = 9in$$

$$PSR = 20,000 \quad H = 20000 \text{ ft}$$

$$VE = \left( \frac{(0.45)(9in.)(20,000)}{(1.8 \frac{in.}{ft})(20,000ft)} \right) = 2.25$$

which means that objects in the stereo model appear to be **2.25 times taller** than they really are compared to the horizontal scale.

## 3- Theory of Stereoscopy

### 3-4 Vertical exaggeration

- We can **change** the vertical exaggeration by changing either the **percent endlap** or the flying height, or both, while maintaining the same scale.
- For **example**, let's keep everything the same but decrease the net gain from **45%** to **25%** percent by increasing the percent endlap to **75 %**. This reduces the vertical exaggeration from **2.25** to **1.25**.

## 3- Theory of Stereoscopy

### 3-4 Vertical exaggeration

#### Quiz:

What is the approximate vertical exaggeration for a vertical photo taken with a 152.4 mm focal length camera having a 23 cm square format if the photos were taken with 60% endlap? Suppose photo scale 1:15,000

Appendix-3-7

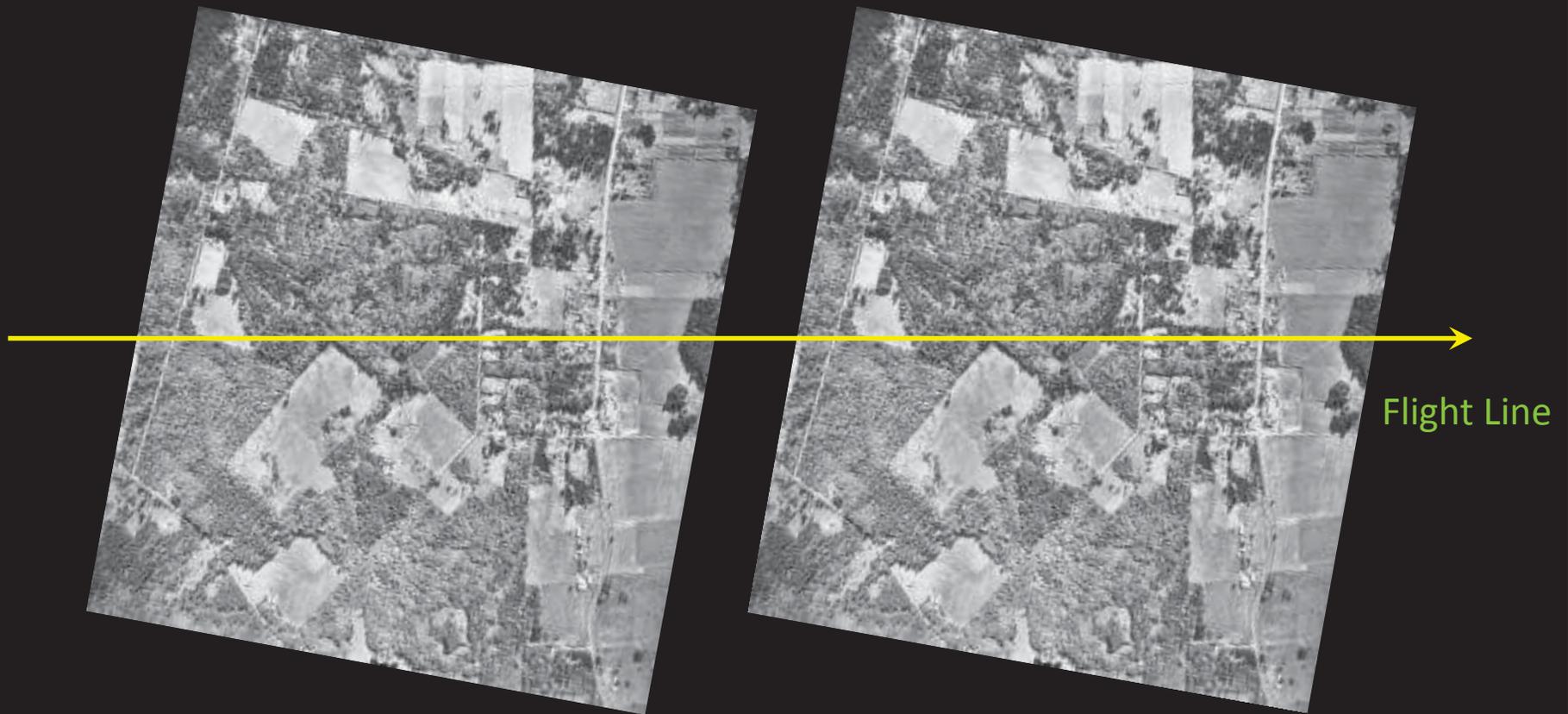
## 3- Theory of Stereoscopy

### 3-5 Proper Orientation of a Stereo Model

1. Obtain 2 photographs consecutively marked on a flight line.
2. Locate and mark the Principal Points on each photograph.
3. Locate and mark the Conjugate Principal Points on each photograph.
4. Line up all 4 points and adjust the distance between photographs to suit your needs.

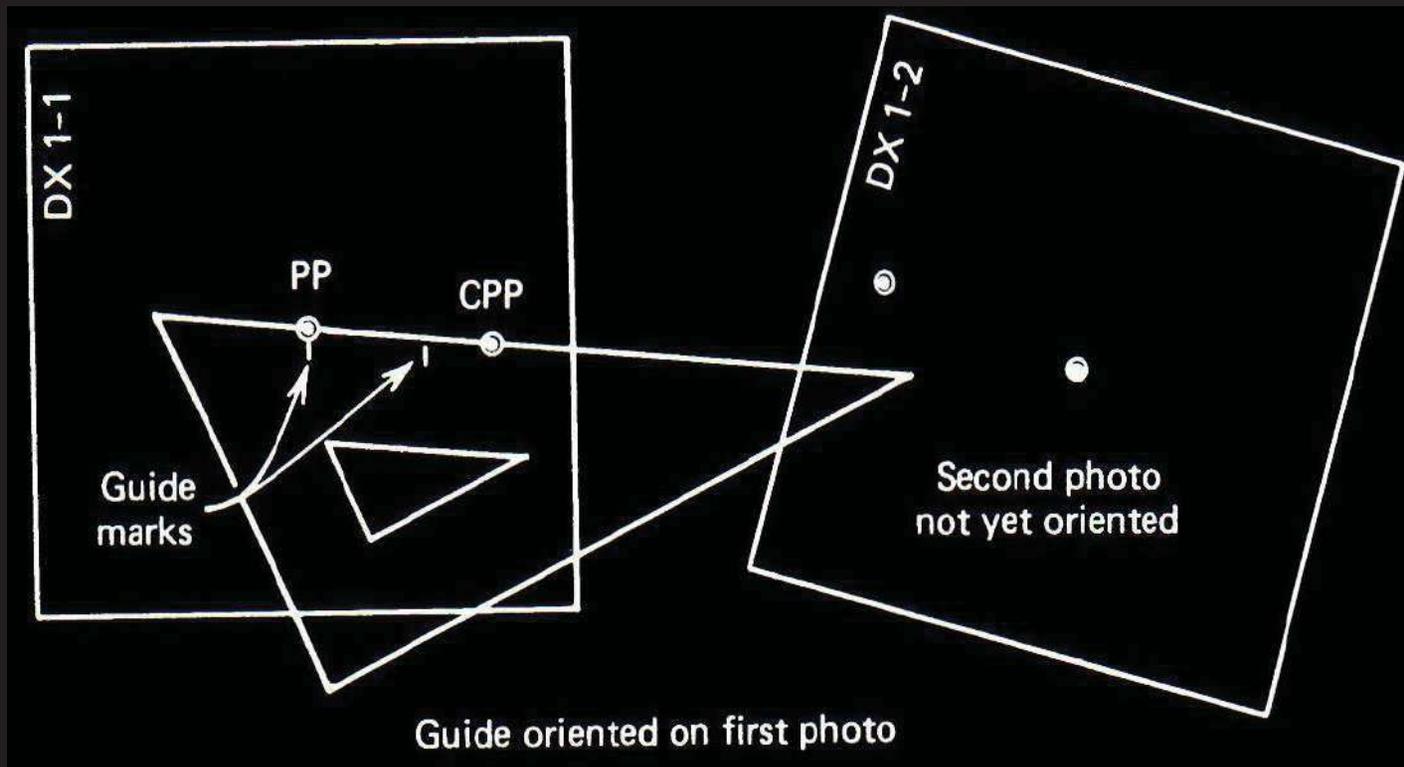
## 3- Theory of Stereoscopy

### 3-5 Proper Orientation of a Stereo Model



## 3- Theory of Stereoscopy

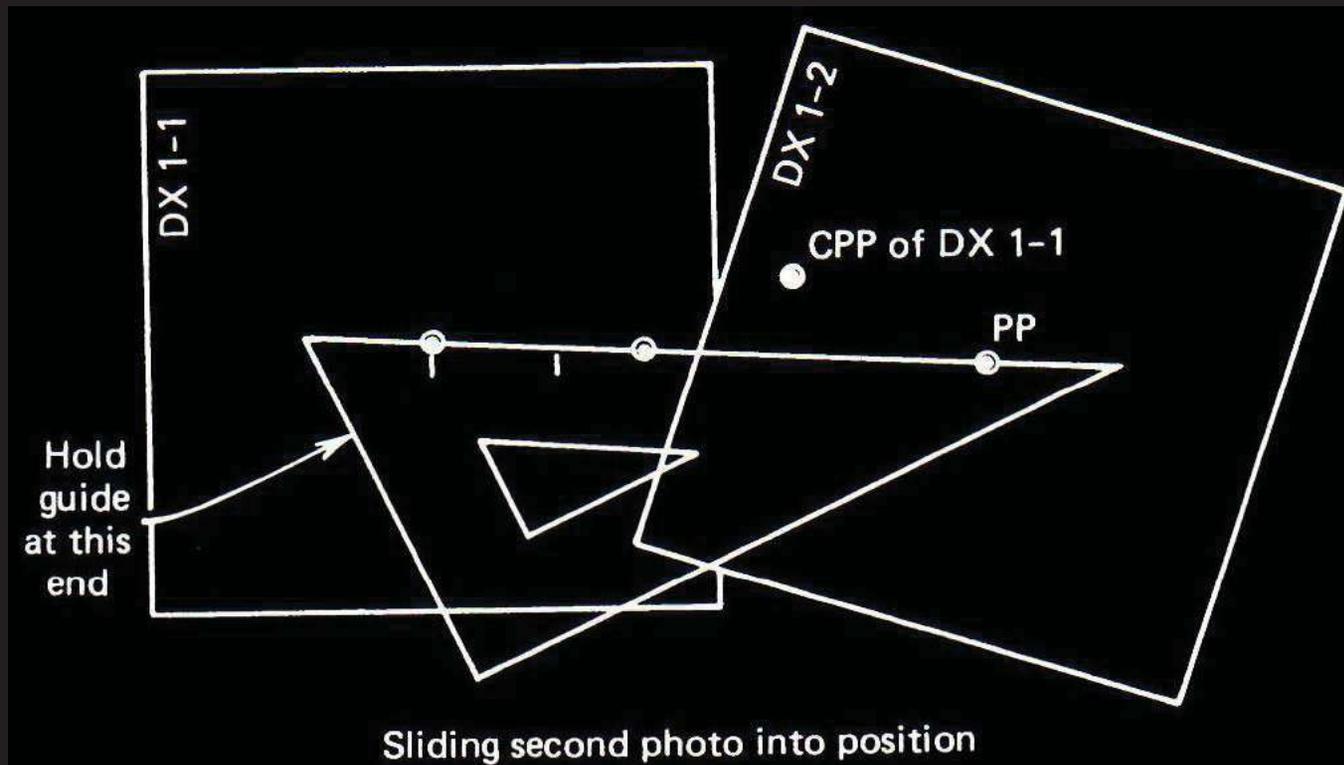
### 3-5 Proper Orientation of a Stereo Model



**Figure 3.17.** Orienting photos for proper stereoscopic viewing. Notice the guide marks on the triangle. (Courtesy of the U.S. Forest Service, Pacific Northwest Forest and Ranger Experiment Station.)

## 3- Theory of Stereoscopy

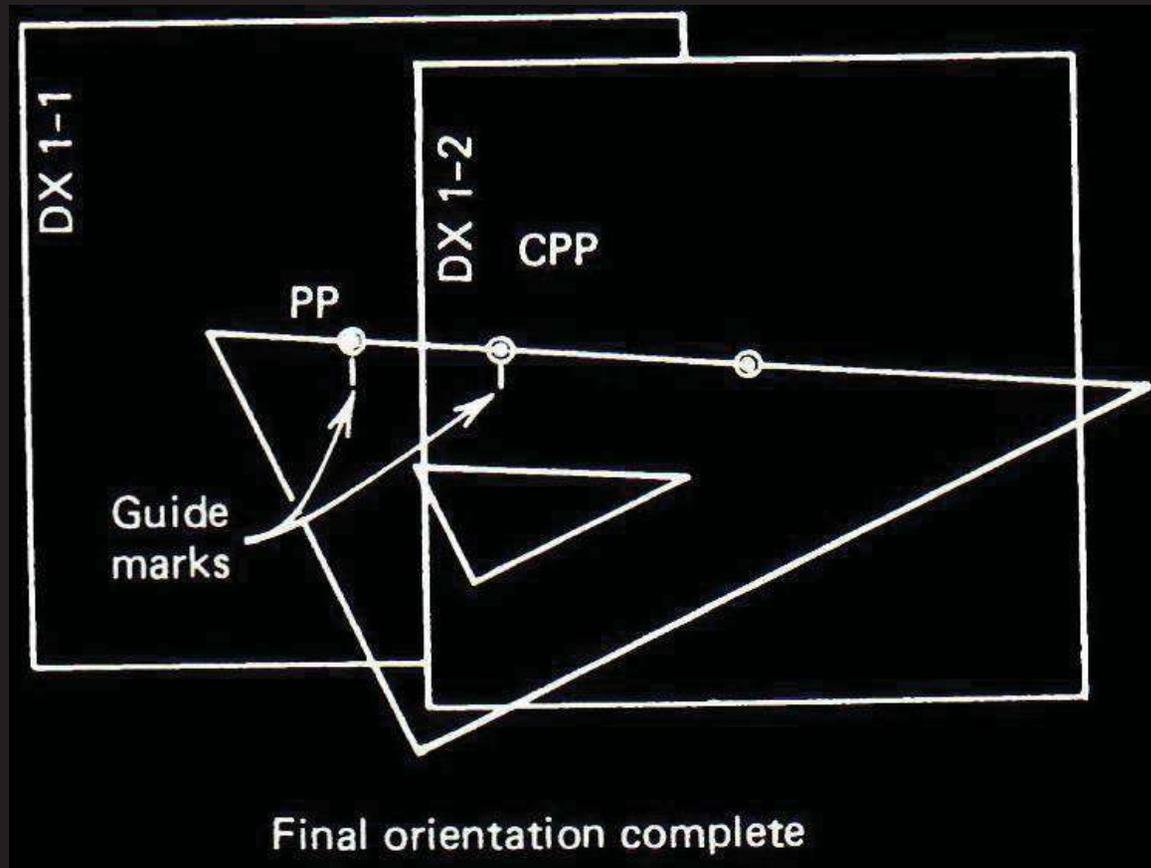
### 3-5 Proper Orientation of a Stereo Model



**Figure 3.17.** Orienting photos for proper stereoscopic viewing. Notice the guide marks on the triangle. (Courtesy of the U.S. Forest Service, Pacific Northwest Forest and Ranger Experiment Station.)

## 3- Theory of Stereoscopy

### 3-5 Proper Orientation of a Stereo Model



**Figure 3.17.** Orienting photos for proper stereoscopic viewing. Notice the guide marks on the triangle. (Courtesy of the U.S. Forest Service, Pacific Northwest Forest and Ranger Experiment Station.)

# Laboratory Exercises 1

- For this laboratory exercise you will need to use aerial photo
- Your instructor will learn how to see stereoscopic image by changing the float point.

**Please refer to the next section in text book, page 65**

## **3.6 A Test for Stereoscopic Perception**

3.6.1 How to Take the Test

3.6.2 How to Grade the Test

# *Homework's*

( all )

## Questions and Problem

In text book (Page 66)

are required



*Thank you*

Any Questions ?



**END**

**of Lecture**

# Appendix-3-1

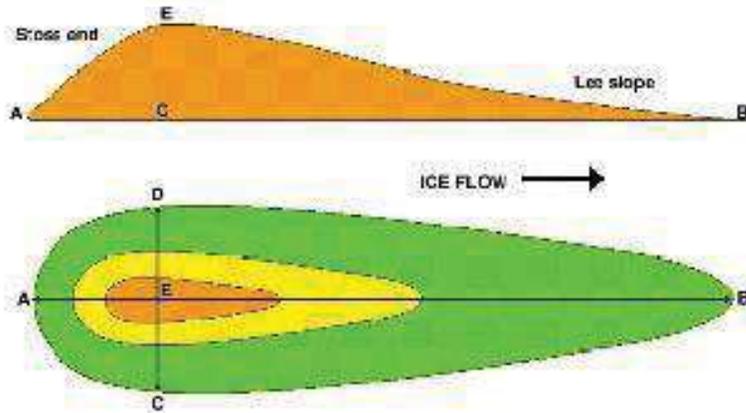
## Anaglyph 3D

**Anaglyph 3D** is the name given to the stereoscopic 3D effect achieved by means of encoding each eye's image using filters of different (usually chromatically opposite) colors, typically red and cyan. Anaglyph 3D images contain two differently filtered colored images, one for each eye. When viewed through the "color-coded" "anaglyph glasses", each of the two images reaches the eye it's intended for, revealing an integrated stereoscopic image. The visual cortex of the brain fuses this into the perception of a three-dimensional scene or composition.

[https://en.wikipedia.org/wiki/Anaglyph\\_3D](https://en.wikipedia.org/wiki/Anaglyph_3D)

# Appendix-3-2

## DRUMLINS



### Characteristics:

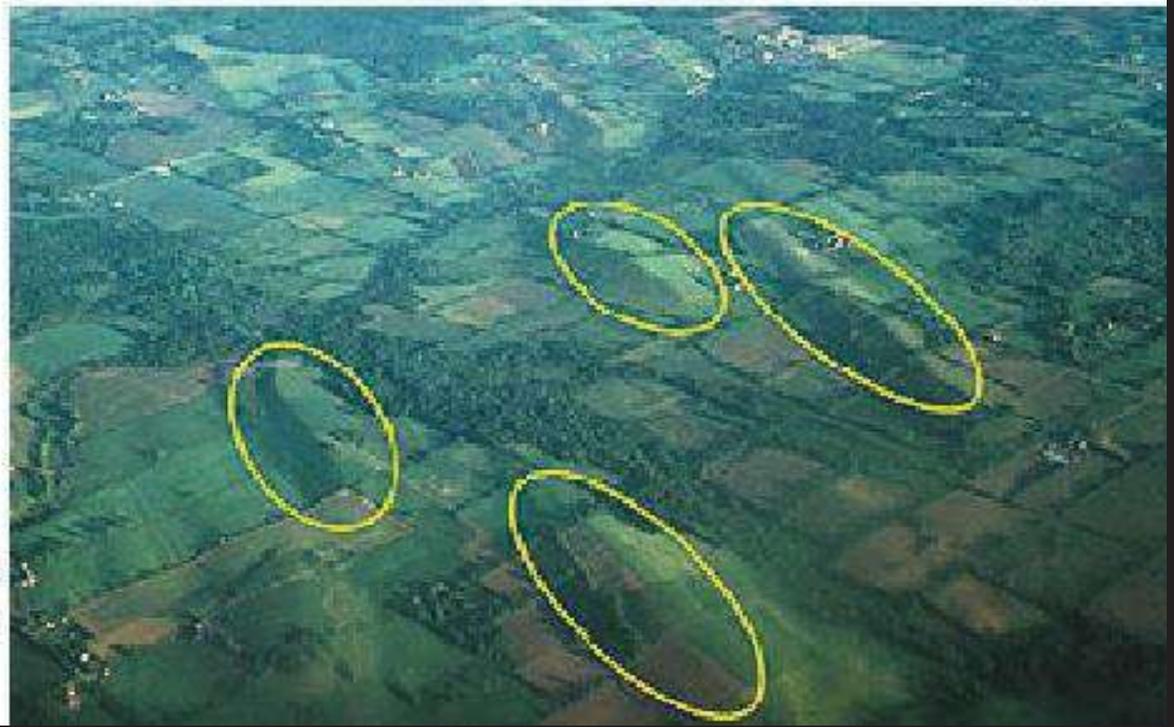
- Smooth elongated mounds of till – long axis parallel to the direction of the ice movement (mounds of glacial debris streamlined into elongated hills)
- Where found in clusters – called “**drumlin swarm**” – classic ‘basket of eggs topography’

- Range in size from small mounds (2m high / 10m long) to huge hills – can be over a km long and 100m in height.

- shape of drumlin measured using elongation ratio =

$$\frac{\text{Length of drumlin}}{\text{max width}}$$

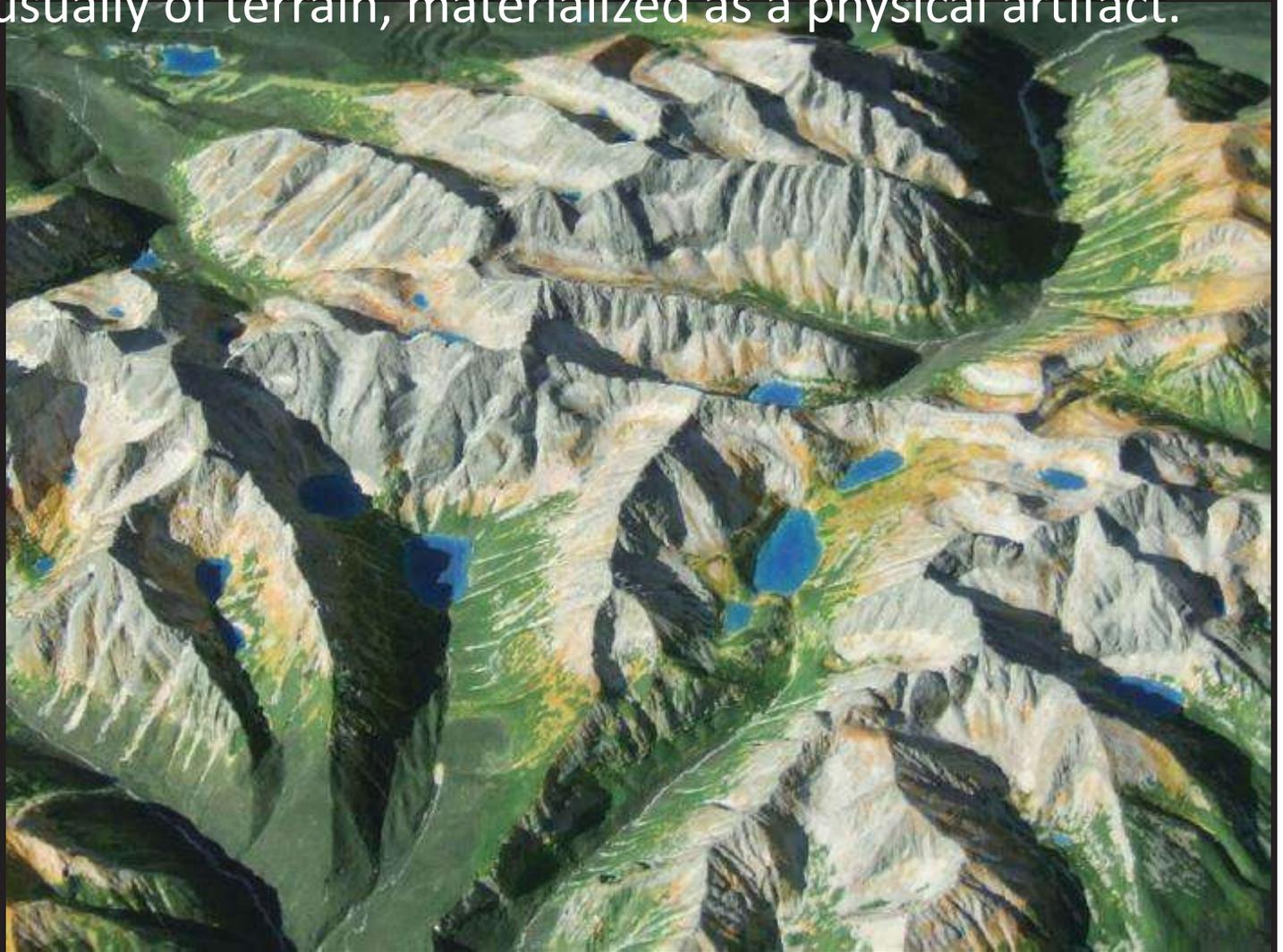
- Usually between 25:1 and 4:1  
– greater elongation suggests more powerful ice flow.



# Appendix-3-2

A **raised-relief map** or **terrain model** is a three dimensional representation, usually of terrain, materialized as a physical artifact.

When representing terrain, the vertical dimension is usually exaggerated by a factor between five and ten; this facilitates the visual recognition of terrain features.

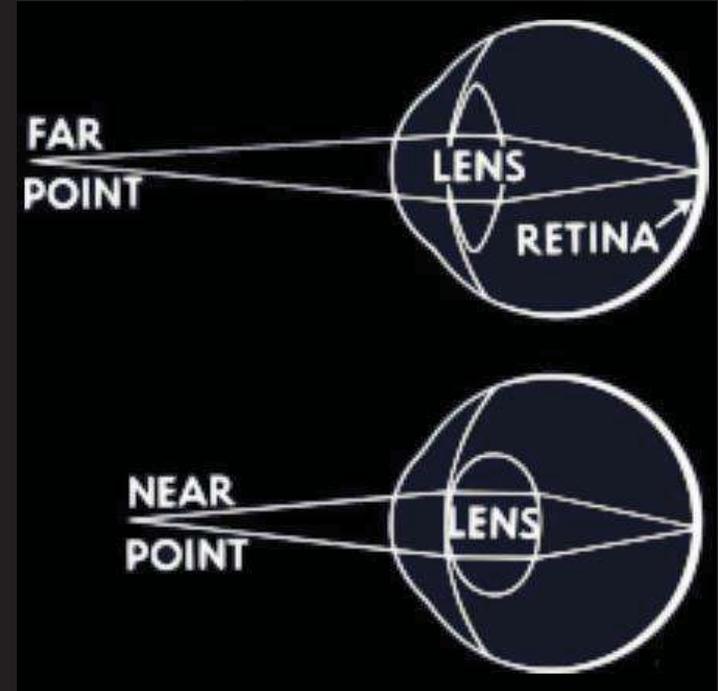
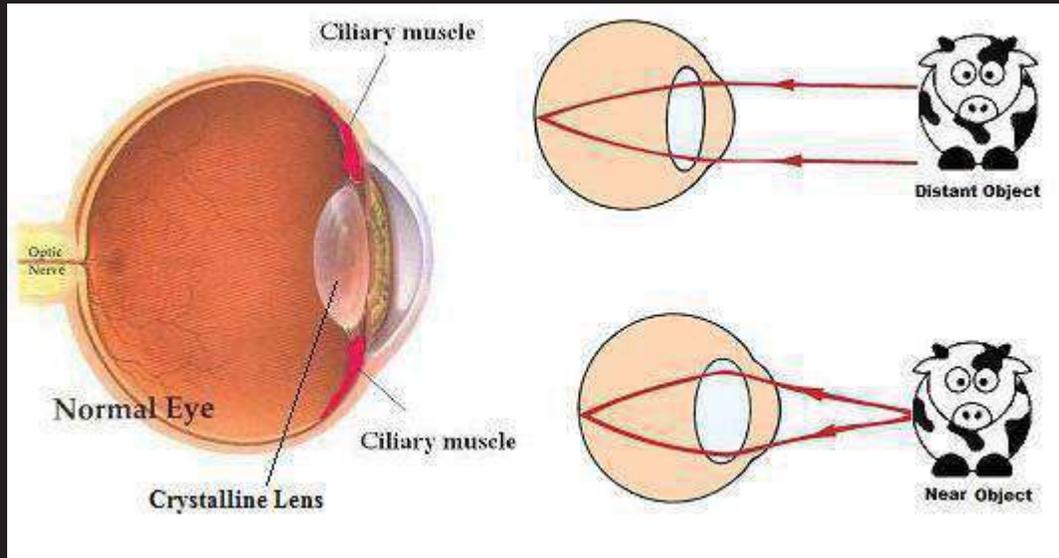


Hand-made **raised-relief map** of the High Tetras in scale 1: 50 000

# Appendix-3-3

Moore Vision Skills saved to Accommodation/Eye Focusing

An eye focusing problems is an inability to easily refocus eyes or maintain clear focus.

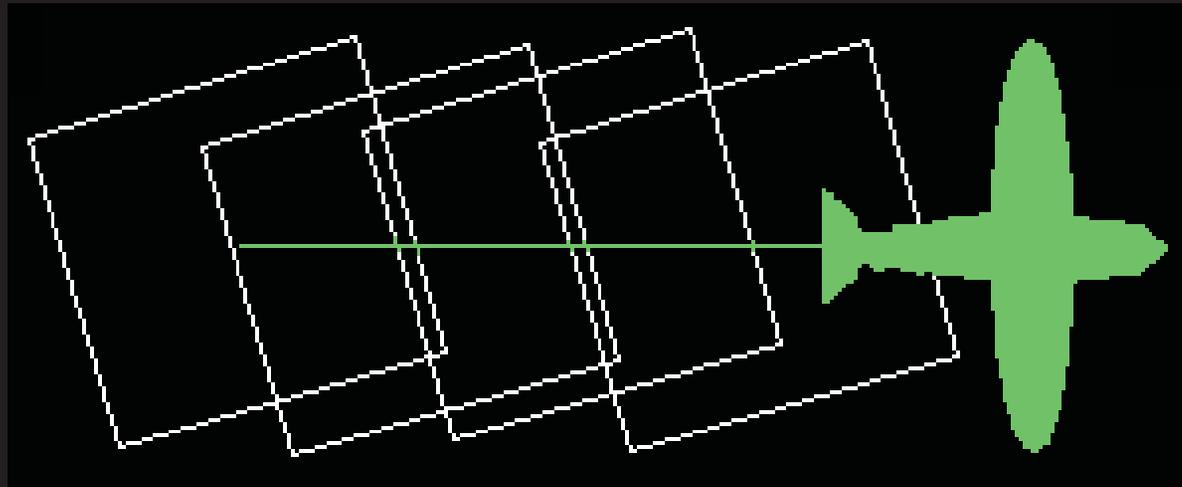


ضوء من نقطة واحدة من كائن بعيد وضوء من نقطة واحدة لكائن قريب تم نقلها للبؤرة عن طريق تغيير تقوس العدسة .

# Appendix-3-4

## Crab

- Crab is the condition caused by the failure to orient the camera with respect to the planned flight line.
- Crab should not affect more than 5% of the photograph.



# Appendix-3-5

## 3-4 Vertical exaggeration

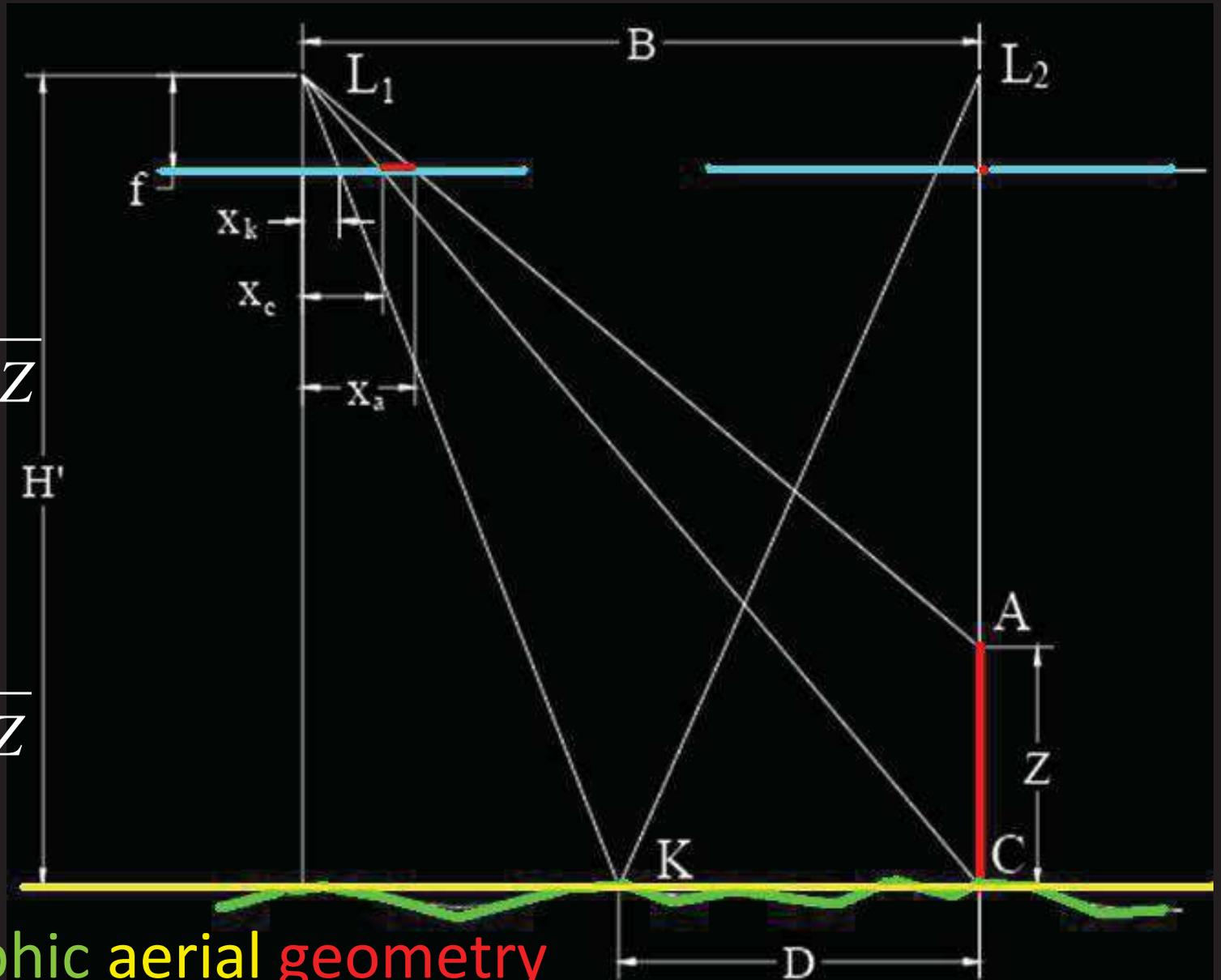
- **Apparent scale disparity** between horizontal and vertical scales
- **Primary cause:** lack of equivalence between **photographic** base-height ratio (  $B/H'$  ) and **stereo-viewing** base-height ratio (  $b_e/h$  )

# Appendix-3-5

$$\frac{X_a}{B} = \frac{f}{H' - Z}$$



$$X_a = \frac{Bf}{H' - Z}$$



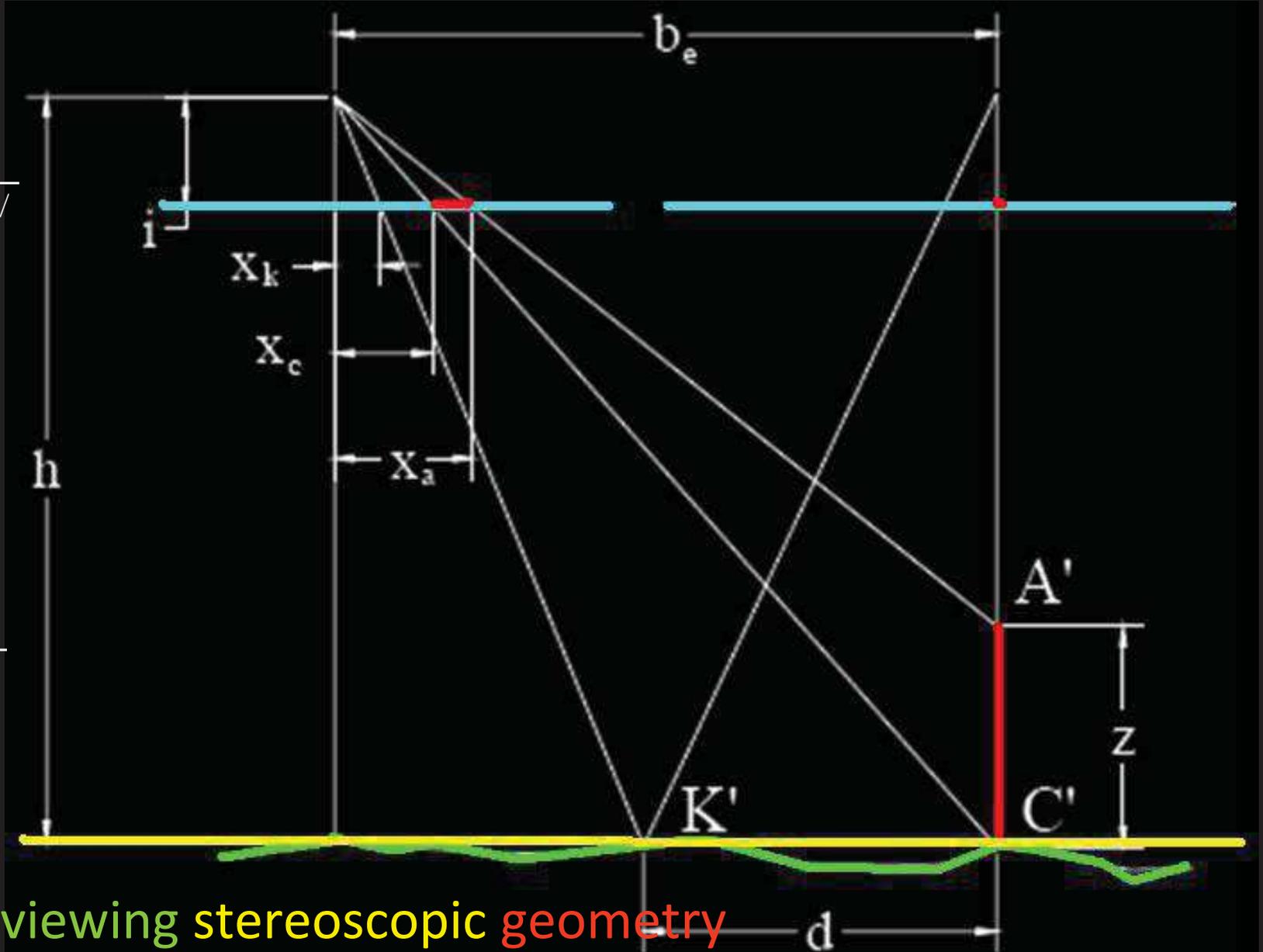
photographic aerial geometry

# Appendix-3-5

$$\frac{X_c}{B} = \frac{f}{H'}$$



$$X_c = \frac{Bf}{H'}$$



stereo-viewing stereoscopic geometry

# Appendix-3-5

From aerial geometry:

$$\frac{X_a}{B} = \frac{f}{H' - Z} \implies X_a = \frac{Bf}{H' - Z}$$

From stereoscopic geometry:

$$\frac{X_c}{B} = \frac{f}{H'} \implies X_c = \frac{Bf}{H'}$$

Subtracting:

$$X_a - X_c = Bf \frac{Z}{(H')^2 - H'Z}$$

# Appendix-3-5

- From similar triangles:

$$\frac{X_a}{b_e} = \frac{i}{h-z} \Rightarrow X_a = \frac{b_e i}{h-z}$$

$$\frac{X_c}{b_e} = \frac{i}{h} \Rightarrow X_c = \frac{b_e i}{h}$$

- Subtracting,

$$X_a - X_c = b_e i \frac{z}{h^2 - hz}$$

# Appendix-3-5

Equating the 2 equations for  $X_a - X_c$

$$Bf \frac{Z}{(H')^2 - H'Z} = b_e i \frac{z}{h^2 - hz}$$

• But  $Z$  and  $z$  are considerably smaller than  $H'$  and  $h$ , thus

$$\frac{BfZ}{(H')^2} \approx \frac{b_e iz}{h^2} \quad \Rightarrow \quad \frac{z}{Z} = \frac{fh}{H'i} \frac{Bh}{H'b_e}$$

# Appendix-3-5

- From similar triangles in 2 diagrams

$$\frac{x_c - x_k}{D} = \frac{f}{H'} \quad \Rightarrow \quad D = (x_c - x_k) \frac{H'}{f}$$

$$\frac{x_c - x_k}{d} = \frac{i}{h} \quad \Rightarrow \quad d = (x_c - x_k) \frac{h}{i}$$

- From

$$\frac{d}{D} = \frac{fh}{H'i}$$

# Appendix-3-5

Substitute into equation for  $z/Z$

$$\frac{z}{Z} = \frac{d}{D} \frac{Bh}{H'b_e}$$

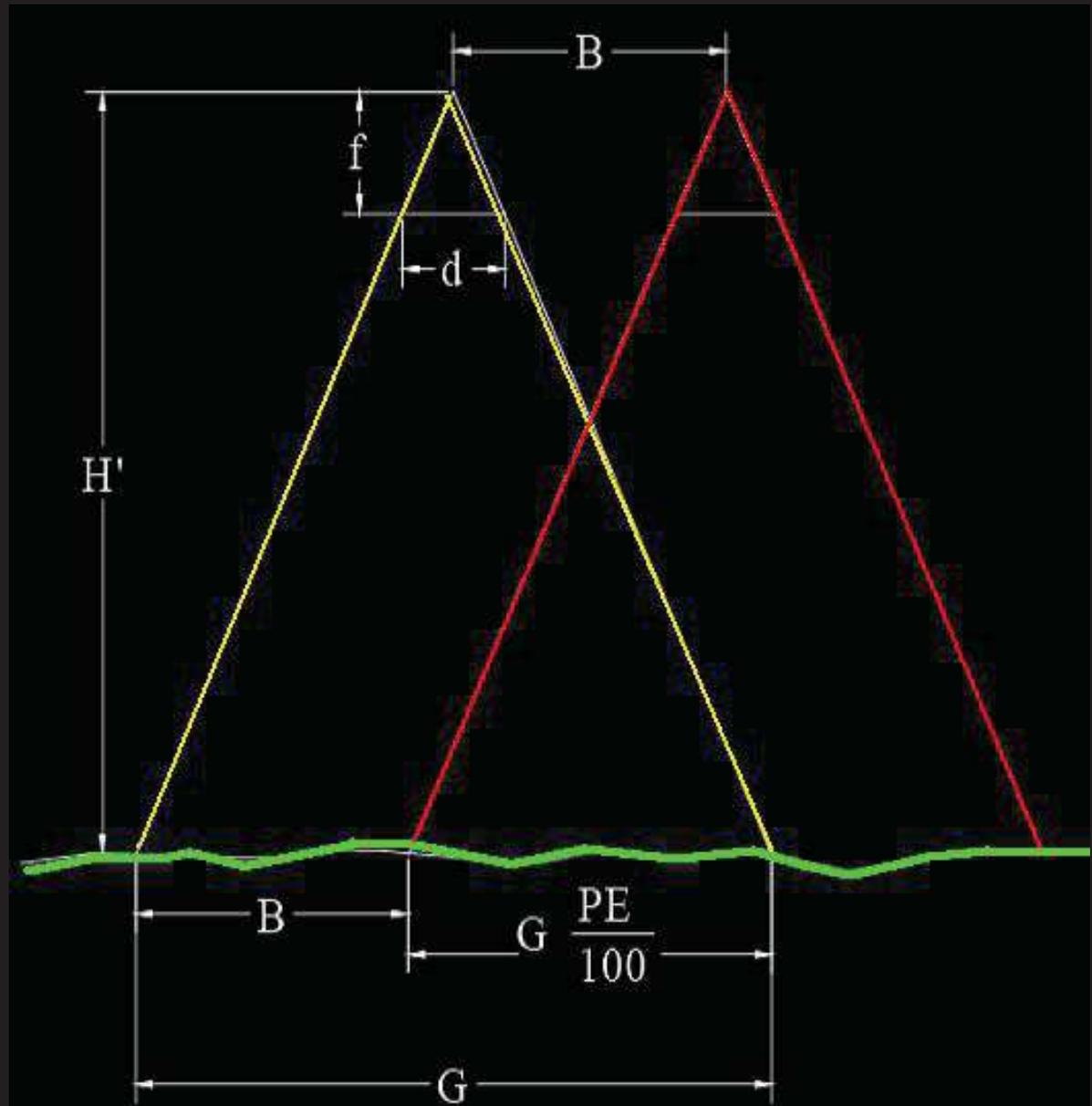
If  $Bh/(H'b_e)$  is 1, there is no vertical exaggeration.

Thus, magnitude of vertical exaggeration,  $V$ , is given by

$$V \approx \frac{B}{H'} \frac{h}{b_e}$$

# Appendix-3-5

$$V \approx \frac{B}{H'} \frac{h}{b_e}$$



# Appendix-3-5

- From the figure

$$B = G - G \frac{PE}{100} = G \left( 1 - \frac{PE}{100} \right)$$

- From which

$$\frac{H'}{G} = \frac{f}{d} \quad \Rightarrow \quad H' = \frac{fG}{d}$$

# Appendix-3-7

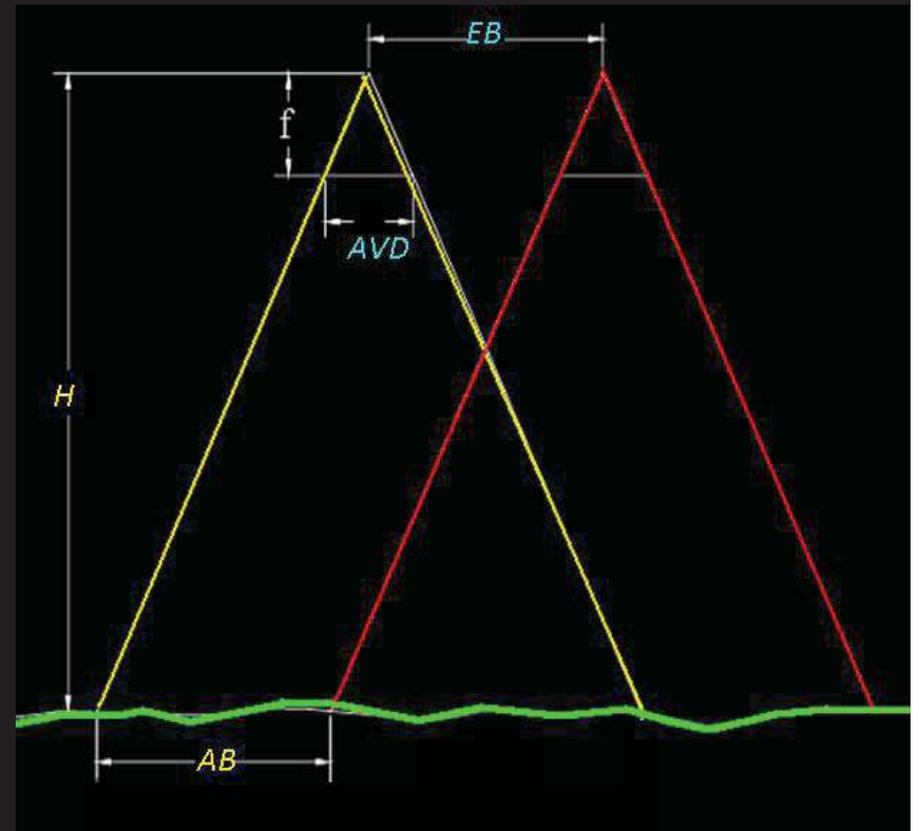
Quiz Solution:

$$\frac{AB}{H} = \left(1 - \frac{60}{100}\right) \left(\frac{230}{152.4}\right) = 0.6$$

$$(AVD/EB) = 1/(0.15)$$

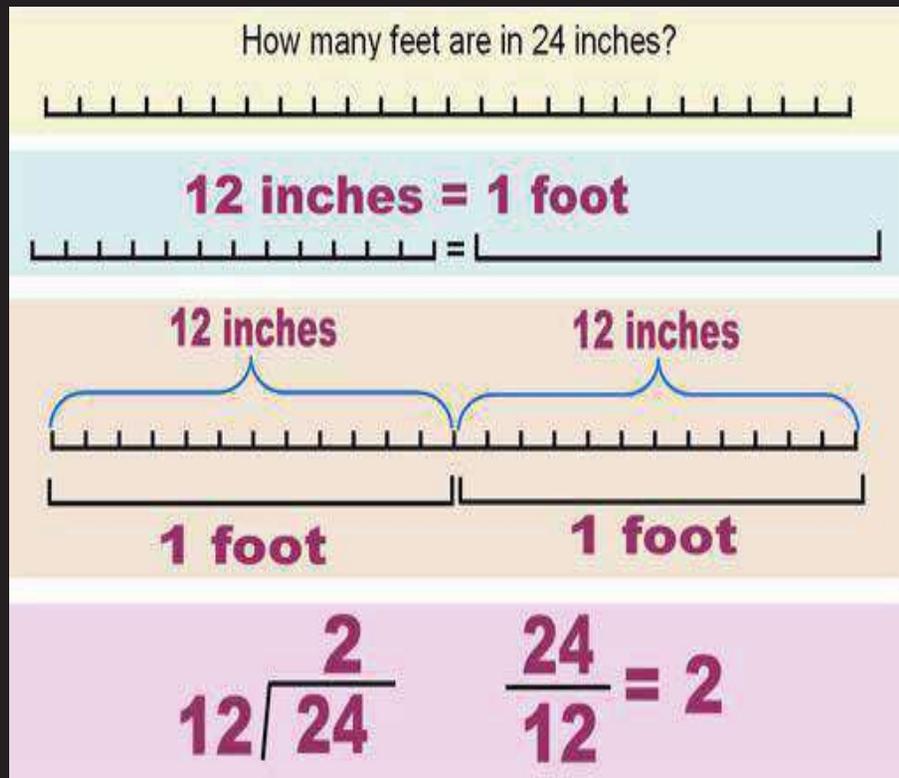
$$\therefore VE = 0.6 \left(\frac{1}{0.15}\right) = 4$$

$$VE = \left(\frac{AB}{H}\right) \left(\frac{AVD}{EB}\right)$$



# Appendix-3-6

## difference feet and foot



### FEET      VERSUS      FOOT

The plural form of foot

Can either refer to  
- the lower part of the leg  
- measurement of height, length and depth

The singular form of feet

The plural form of foot

Generally used with plural measurements

Used with plural measurements, especially in spoken language

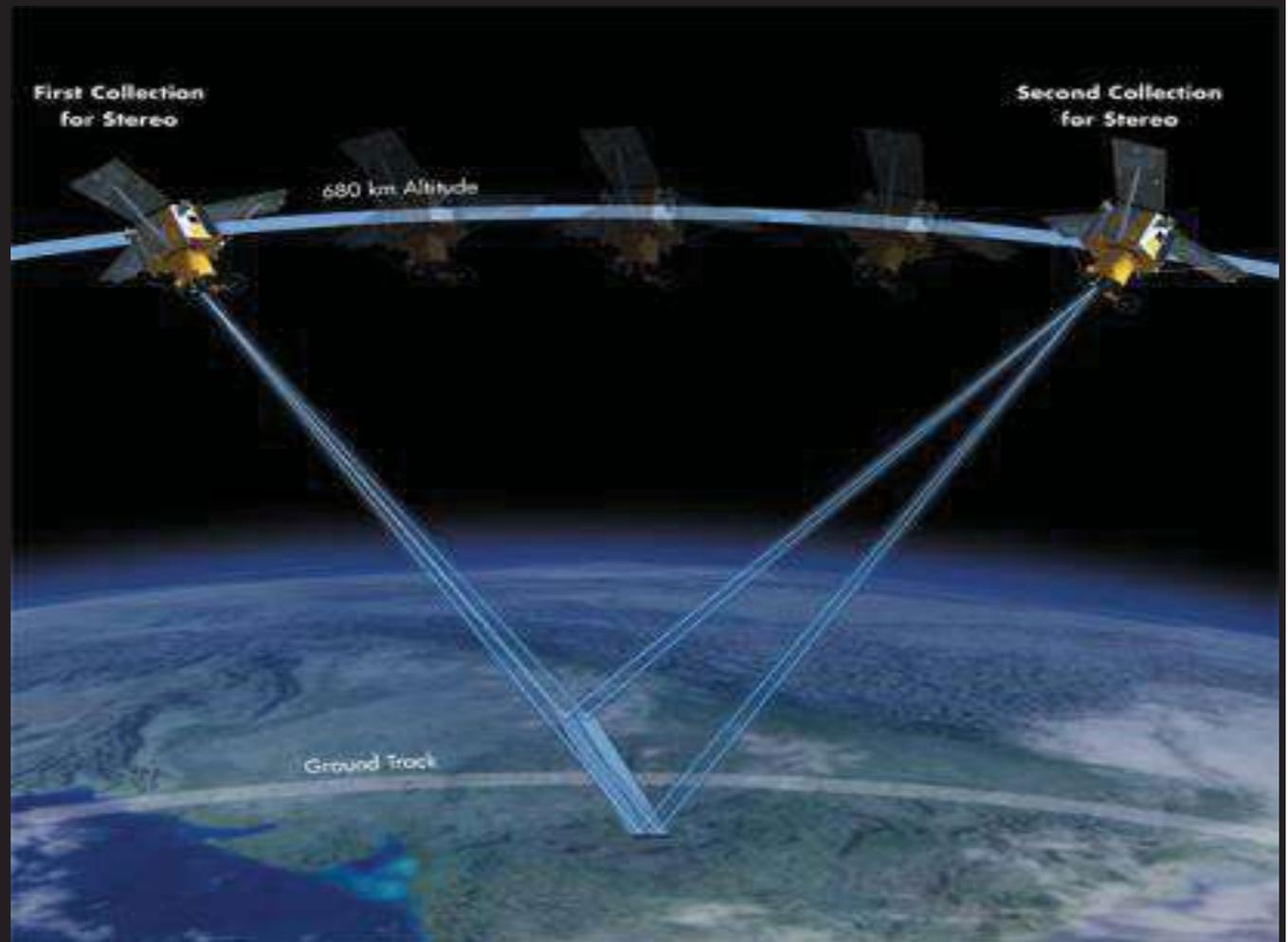
Not used with plural measurements when the measurement acts as a modifier

Used with plural measurements when the measurement acts as a modifier

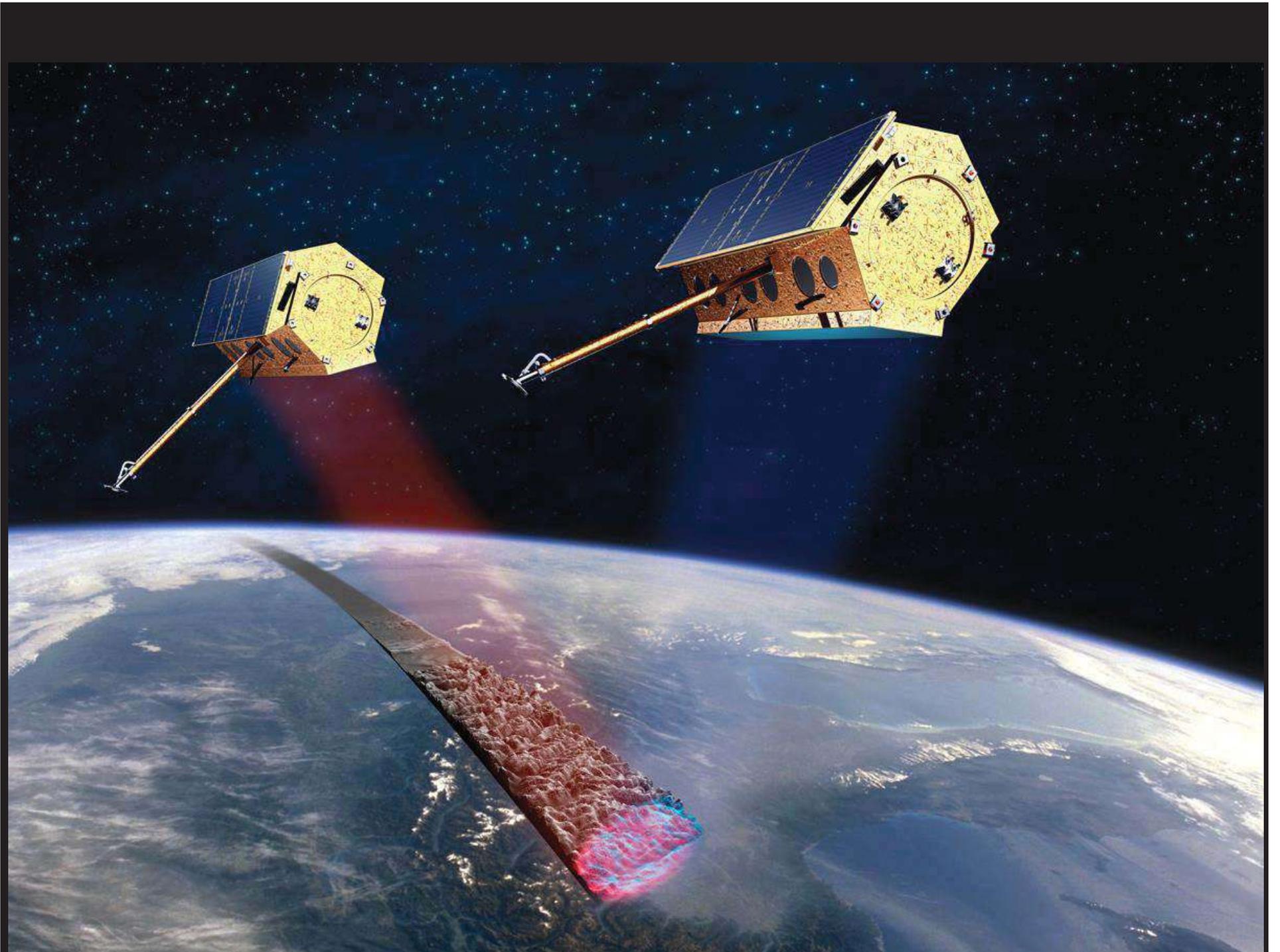


Manual vs. digital stereoscopes.  
Source: Ghent University

<http://www.seos-project.eu/modules/3d-models/3d-models-c02-p04-s01.html>



Stereo IKONOS Satellite Image data collection.  
Source: Satellite Imaging Corporation. All rights reserved.



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

**Lecture**

# **Colour image processing**

*Lecturer:*

***Dr. Faisal Ghazi Mohammed***

Email: [faisal@scbaghdad.edu.iq](mailto:faisal@scbaghdad.edu.iq)  
[faiselgm73@gmail.com](mailto:faiselgm73@gmail.com)

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*All rights reserved*



# Contents

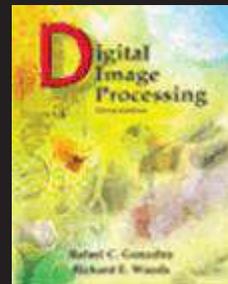
**Today we'll look at color image, covering:**

## **1. Color Fundamentals**

- *Electromagnetic Radiation*
- *Illumination - Reflection - Detection*
- *The Human Eye*
- *Digital Cameras*

## **2. Color Models**

- *Color spaces: **RGB+HIS+YUV***
- *Pseudo coloring*
- *Multispectral data*



Chapter 2.4 in  
*Image Processing  
Analysis, and  
Machine Vision*

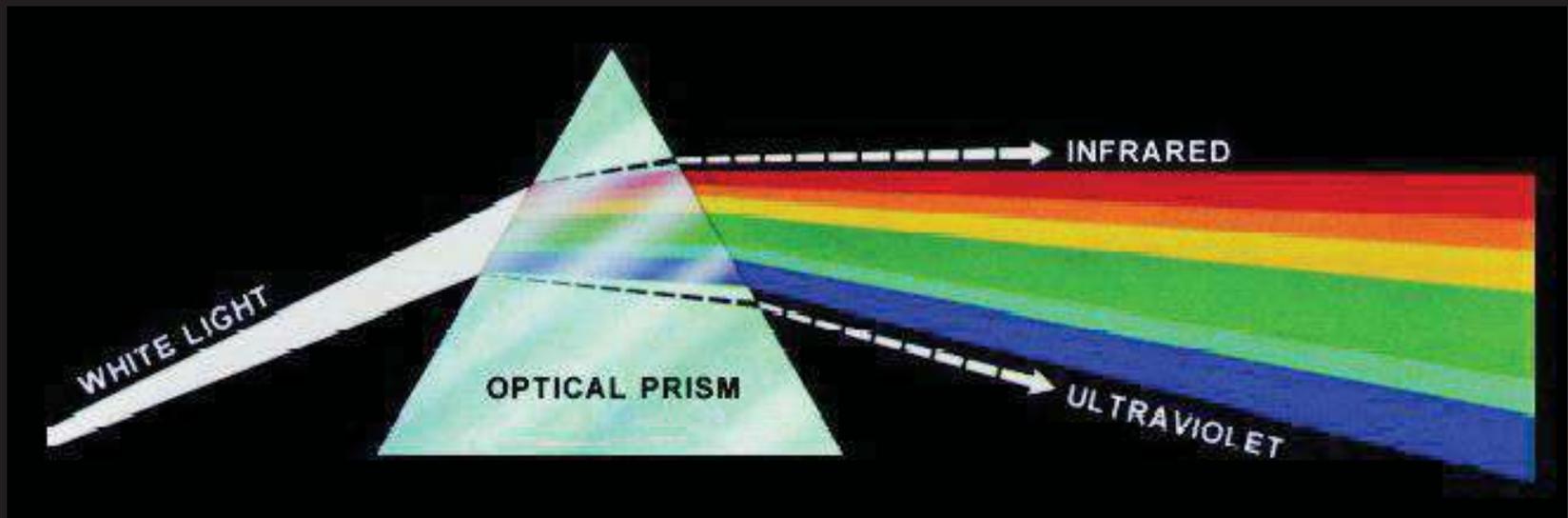
Chapter 6 in  
*Gonzalez & Woods:  
Digital Image  
Processing, 3rd ed.,  
2008.*

# 1- Color Fundamentals



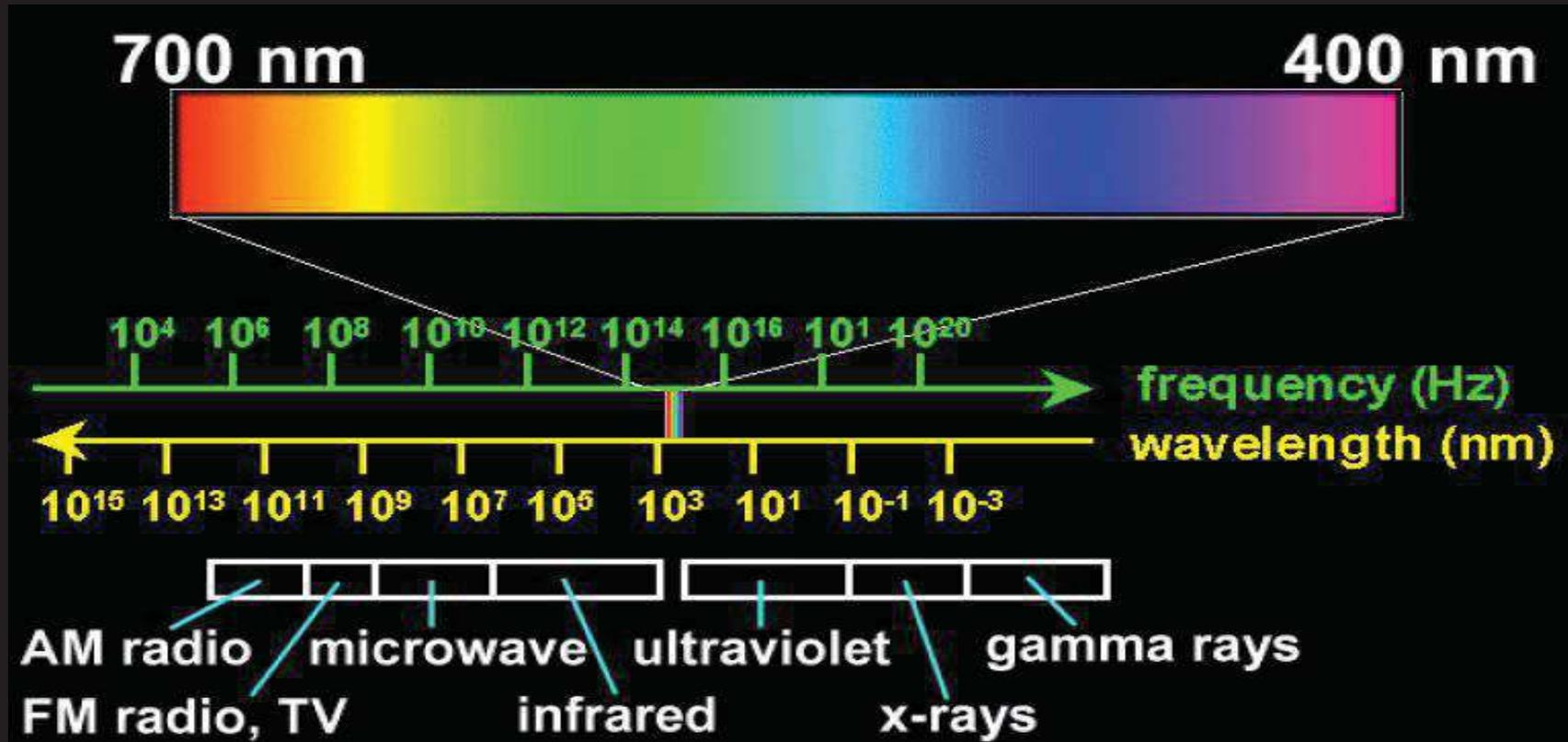
## Electromagnetic Radiation:

In 1666 Sir Isaac Newton discovered that when a beam of sunlight passes through a glass prism, the emerging beam is split into a *spectrum* of colors



# 1- Color Fundamentals

## Electromagnetic Radiation:



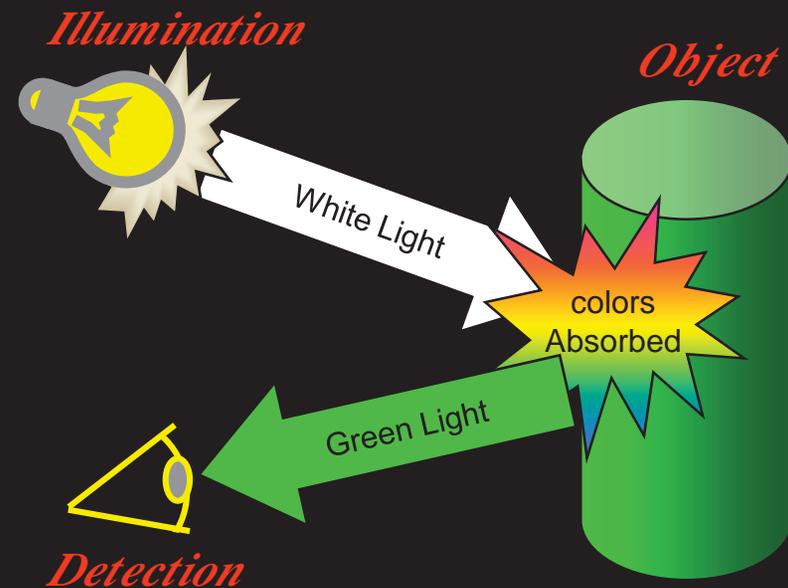
1. Every imaging system selects *one* or *several* spectral windows.
2. For *gray scale* image we have a *single* window, typically covering most of the visible spectrum.

# 1- Color Fundamentals



## *The Image Formation:*

The registered image depends on the spectral properties of:



1. *Illumination* - light from, e.g., the sun or a lamp
2. *Object or Scene* - light can be reflected, absorbed or transmitted
3. *Detection* - can be, e.g., a camera or the human eye



# *1- Color Fundamentals*

## *Light Properties:*

### *Illumination:*

- 1. Achromatic light* - White or uncolored light that contains all visual wavelengths in a *complete mix*.
- 2. Chromatic light* - Colored light.
- 3. Monochromatic light* - Light with a single wavelength, e.g., a laser.

### *Reflection:*

- Colors we see are typically a mix of wavelengths.
- The dominant wavelength reflected by an object decides the “**color tone**” or **hue**.
- If many wavelengths are reflected in equal amounts, an object appears to be gray.

# *1- Color Fundamentals*

## *Light Properties: What is Brightness?*

3 basic qualities are used to describe the **quality** of a chromatic light source:

**1. Radiance**, watts [W] The total amount of energy that flows from the light source.

**2. Luminance**, lumens [lm] A measure of the amount of energy an observer perceives from the light source (measured in lumens).

Note we can have high radiance, but low luminance.

**3. Brightness**, A subjective descriptor that is practically impossible to measure.  
notion that embodies the intensity of light

# *1- Color Fundamentals*



## Luminance/Contrast

Hello, here is some text. Can you read what it says?

Hello, here is some text. Can you read what it says?

Hello, here is some text. Can you read what it says?

Hello, here is some text. Can you read what it says?

Hello, here is some text. Can you read what it says?

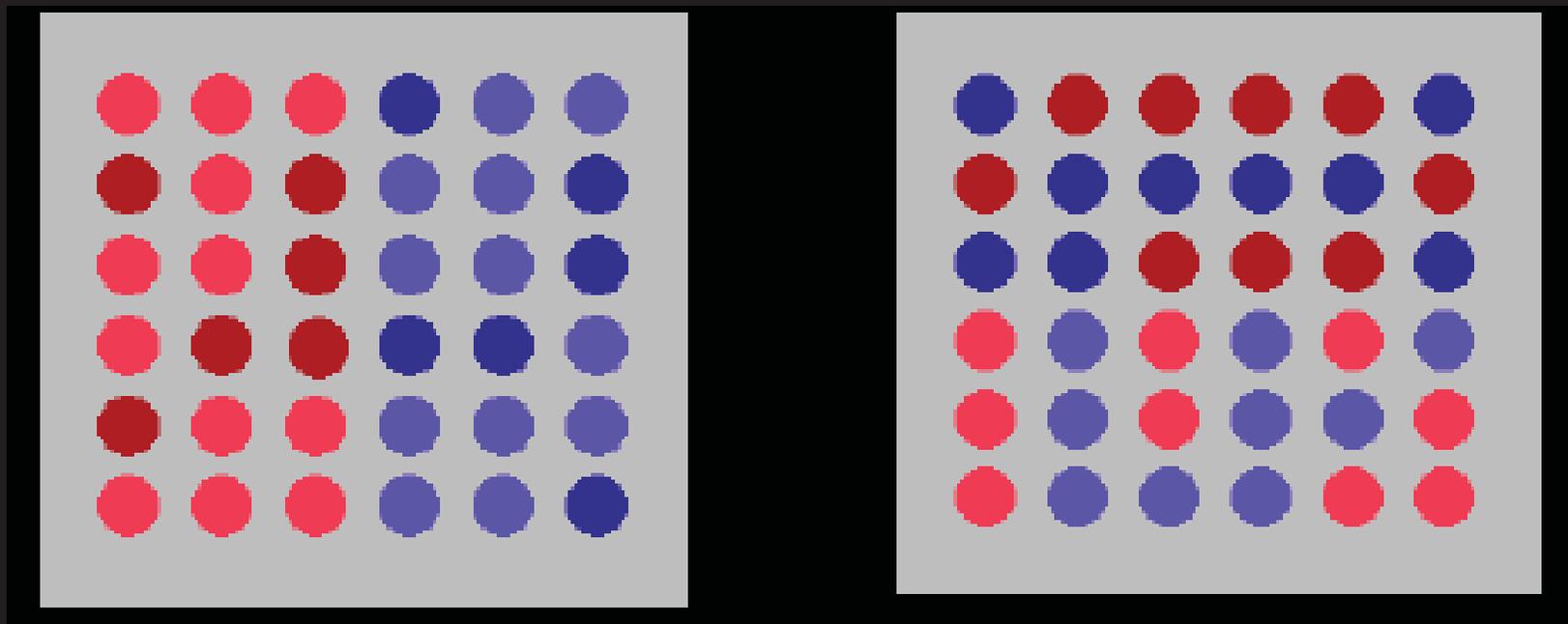
Hello, here is some text. Can you read what it says?

Hello, here is some text. Can you read what it says?

# 1- Color Fundamentals



## Brightness



# *1- Color Fundamentals*



Color has Three Dimensions or Qualities:

1.Hue

2.Value

3.Intensity

# *1- Color Fundamentals*

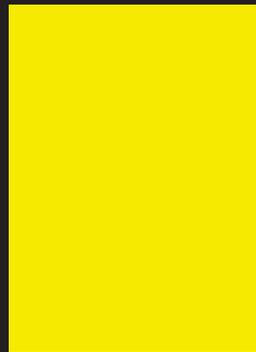


## **1. Hue**

The name given to a color.



*RED*



*YELLOW*



*VIOLET*

# *1- Color Fundamentals*



## **2. INTENSITY (Saturation)**

The brightness or dullness of a color.

***FUSCHIA - HIGH INTENSITY***

***OLIVE - LOW INTENSITY***

# *1- Color Fundamentals*



## **3. VALUE**

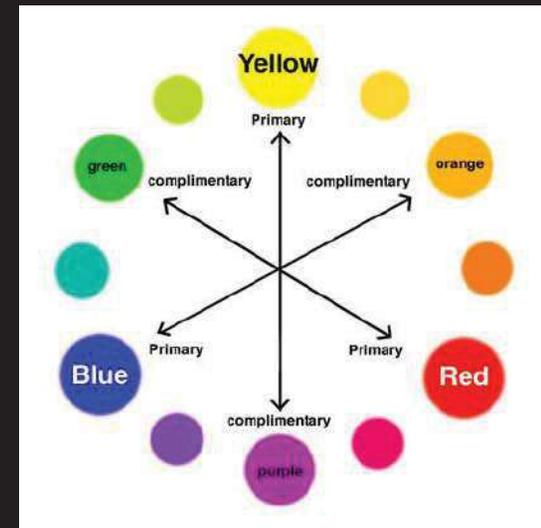
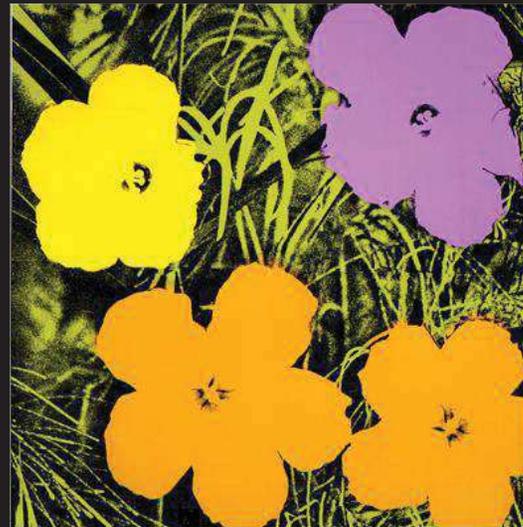
*The lightness or darkness  
of a color*

# 1- Color Fundamentals



## Complementary

Two colors that are the direct opposite of each other, such as red and green and blue-purple and yellow-orange. Complementary colors create the most contrast and balance in design.



Andy Warhol (American, 1928-1987) *Flowers*, 1970, screen print on paper, 36 x 36 in. The Andy Warhol Museum, Pittsburgh Founding Collection, Contribution The Andy Warhol Foundation for the Visual Arts, Inc. © AWF

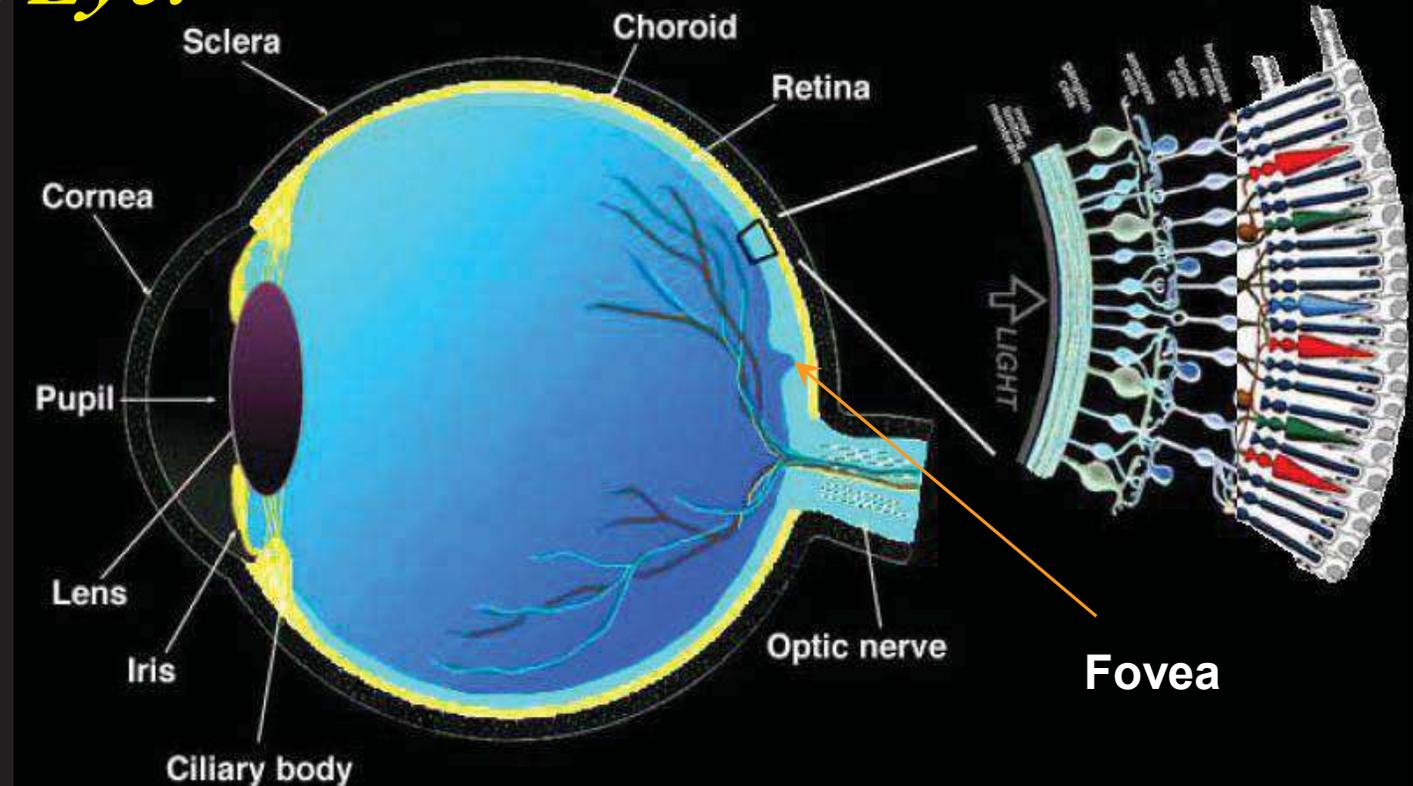
*Space Fruit: Still Lifes (Pears)*, 1979 screen print on Lenox Museum Board 30 x 40 in. The Andy Warhol Museum, Pittsburgh Founding Collection, Contribution Dia Center for the Arts © AWF

# 1- Color Fundamentals



## The Human Eye:

Human color vision is **achieved** through 6 to 7 million **cones** in each eye  
Most localized in the **fovea**.



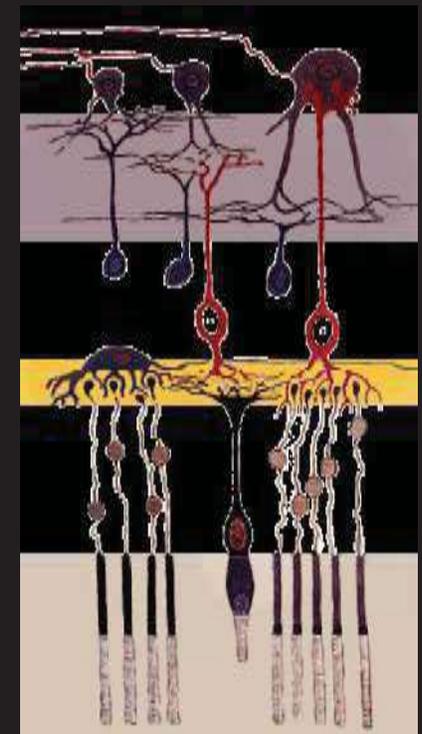
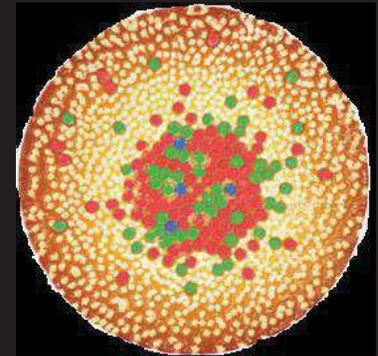
1. The **fovea** is the region of the eye where you see sharp.
2. Photoreceptor cells: **Rods** are responsible for the perception of lightness and **Cones** for color vision.

# 1- Color Fundamentals



## *The Human Eye:* The Rods

1. **Rods** are sensitive to light in the middle of the visual spectra, peak at 498 nm.
2. There are about **100** million rod cells per eye.
3. Slower response **time** than cones but about 100 times more sensitive to light.  
**Night** vision depends on the rods.
4. Dominates the retina **outside** the *fovea*, this means that peripheral vision depends on the rods.



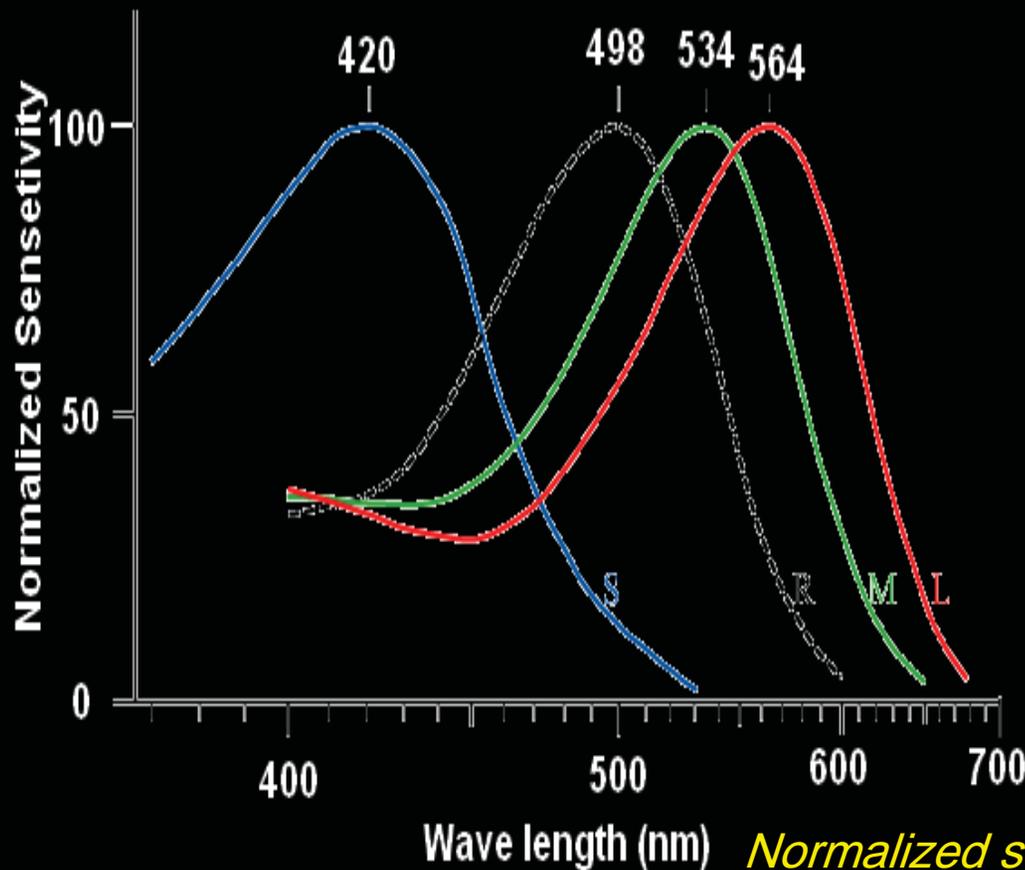
**In the dark all cats are gray**

# 1- Color Fundamentals



## The Human Eye: The Cones

- About 6 million cone cells per eye. Most localized in the *fovea*.
- Three types of cones:



S (blue), peak at 420 nm, (2%)

M (green), peak at 534 nm, (33%)

L (red), peak at 564 nm, (65%)

Dotted line - rods

S - cones for short wavelengths

M - cones for medium wavelengths

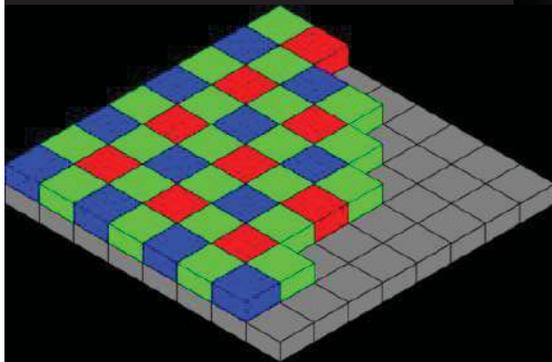
L - cones for long wavelengths

Wave length (nm) *Normalized sensitivity of rods and cones*

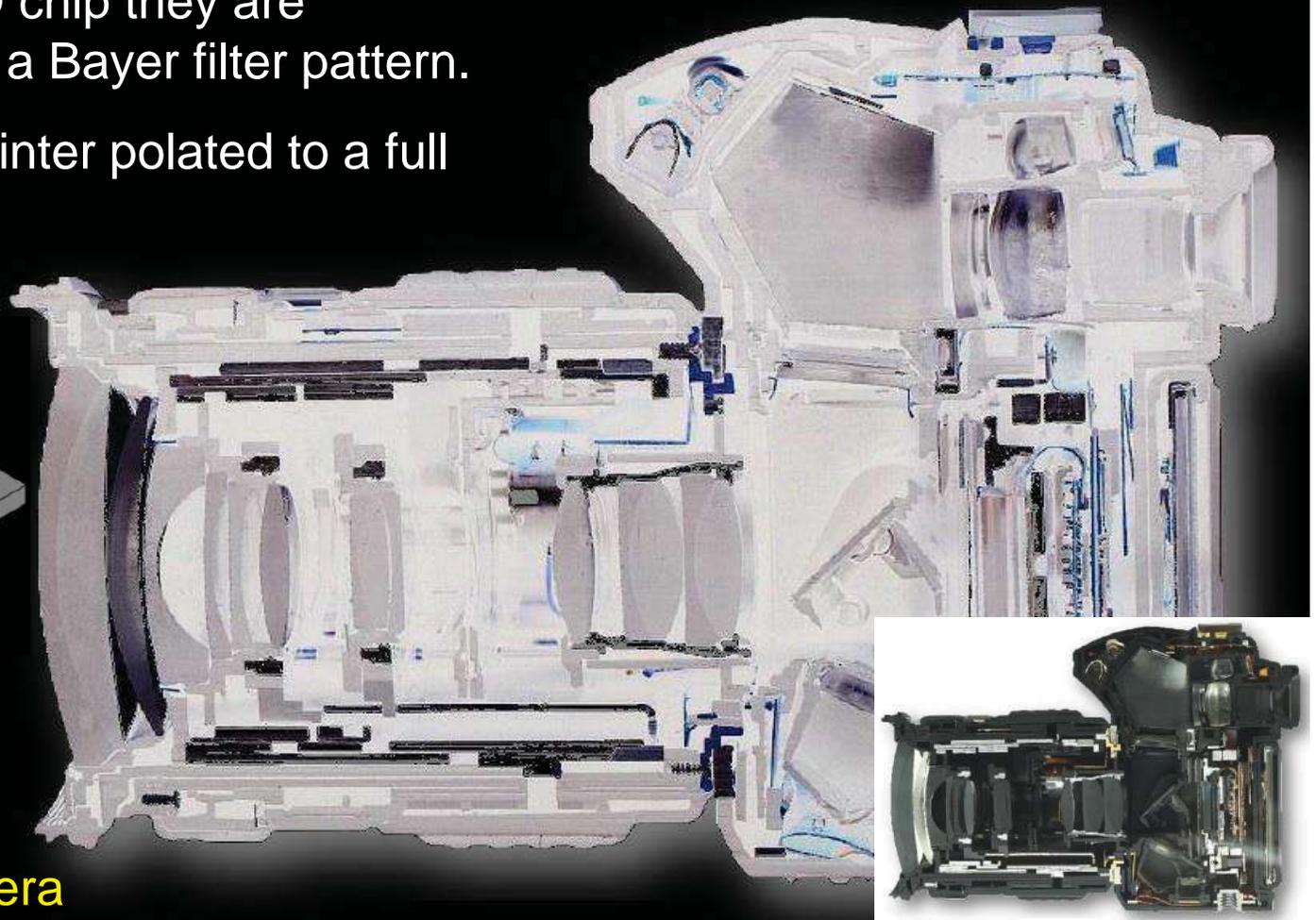
# *1- Color Fundamentals*

## *Digital cameras as detectors*

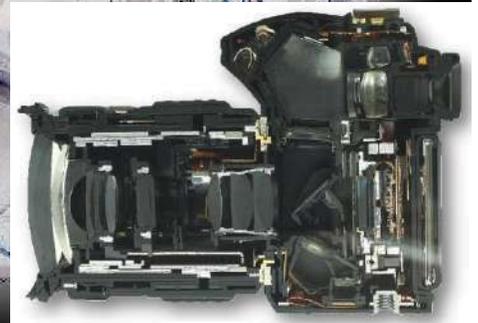
1. Much like the human eye a digital camera has sensors sensitive to three colors. In a CCD chip they are differentiated by a Bayer filter pattern.
2. Values are then interpolated to a full RGB image.



Bayer mosaic



Camera



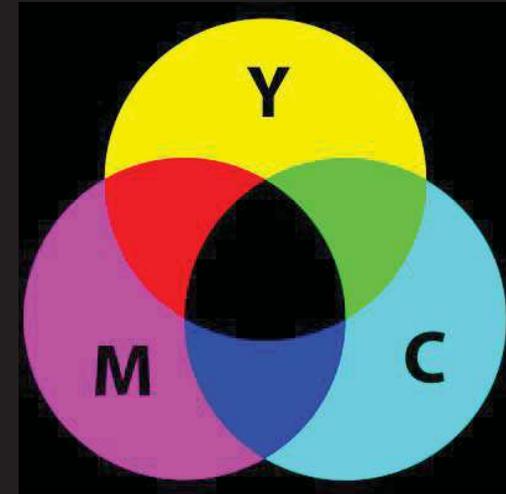
# 1- Color Fundamentals



## Types of Color Theories

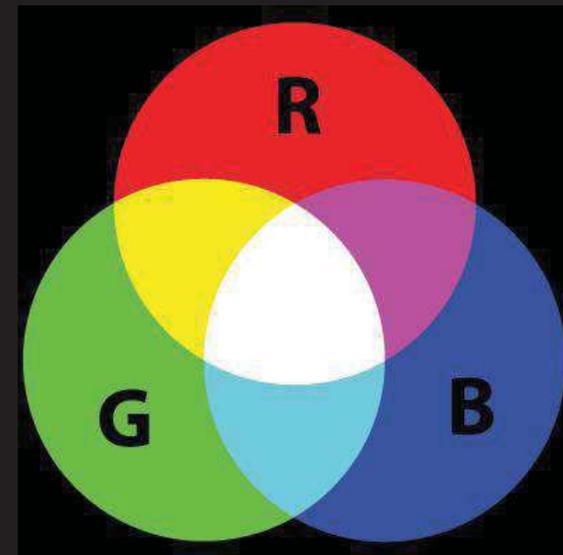
### 1. Subtractive Theory

The **subtractive**, or **pigment** theory deals with how white light is absorbed and reflected off of colored surfaces.



### 2. Additive Theory

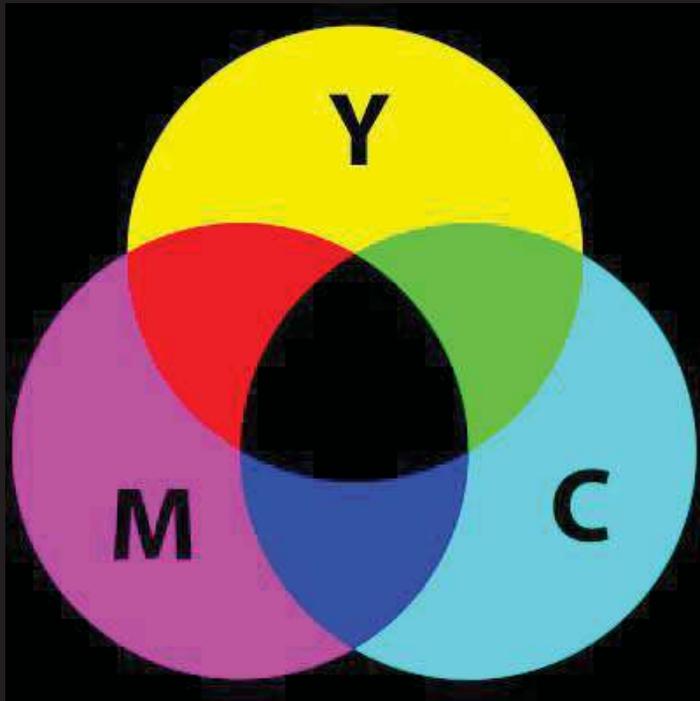
The **Additive**, or **light** theory deals with radiated and filtered **light**.



# 1- Color Fundamentals



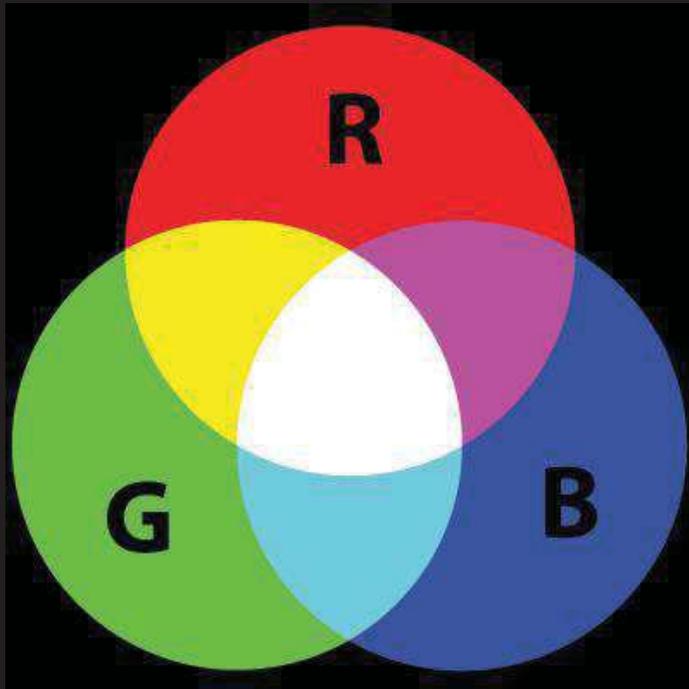
## Subtractive Theory



1. Black **absorbs** most light
2. White **reflects** most light
3. Colored **Pigments absorb** light and reflect only the frequency of the pigment color.
4. All colors other than the pigment colors are absorbed so this is called **subtractive** color theory.
5. The primary colors in Subtractive Theory are:
  - Cyan (C)
  - Magenta (M)
  - Yellow (Y)
  - Black (K)
6. Subtractive or Pigment Theory is used in **printing** and **painting**.

# 1- Color Fundamentals

## Additive Theory

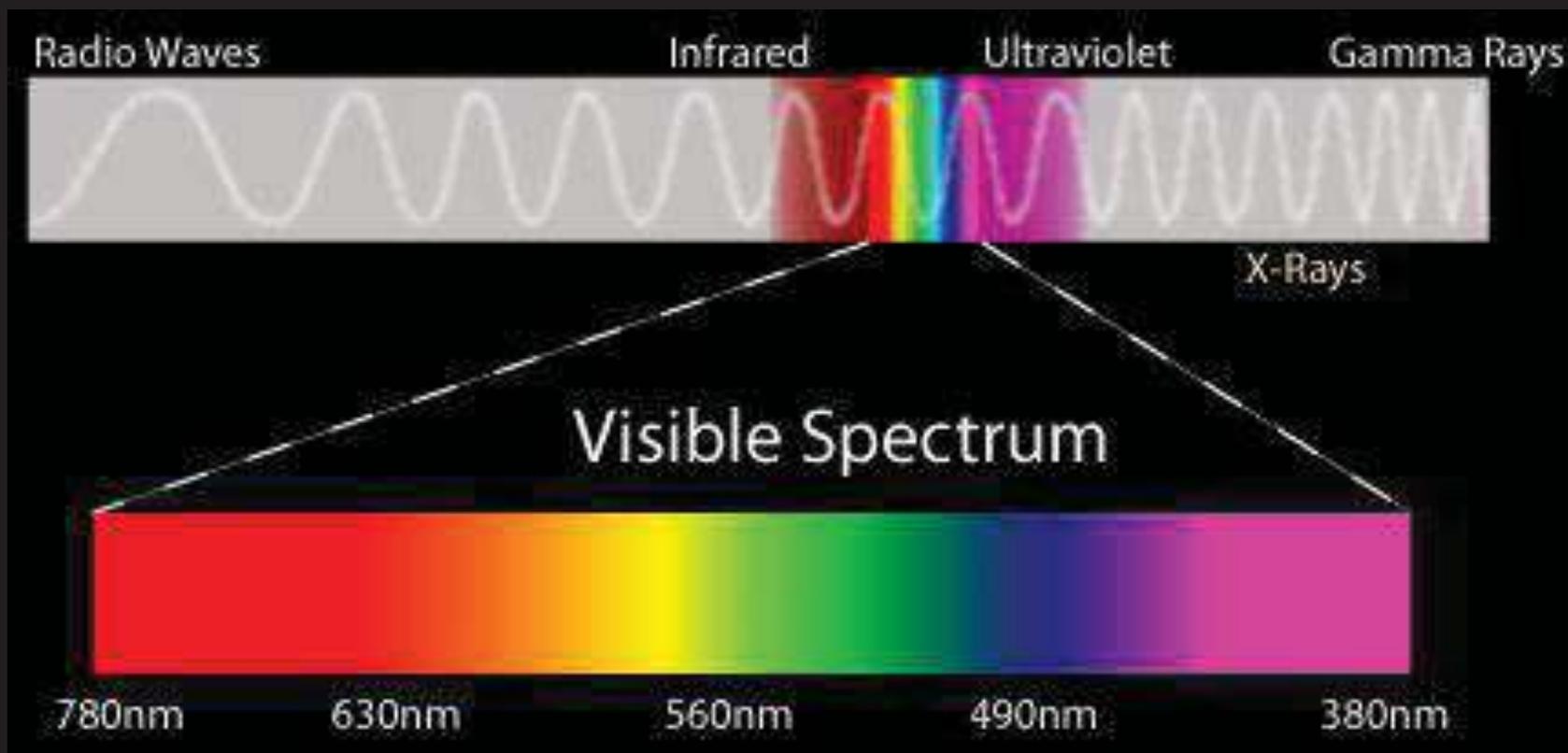


1. Black radiates **no light**
2. White (sun) radiates **all light**
3. Video is the process of capturing and radiating light, therefore it uses Additive (**Light**) Theory not Subtractive (**Pigment**) Theory.
4. The primary colors in Additive Theory are:
  - **Red** ( R )
  - **Green** ( G )
  - **Blue** ( B )
5. The primary colors add together to make white
6. Light Theory is also called Additive Theory.
7. Light Theory is used in **Television**, **theater lighting**, computer **monitors**, and **video** production.

# *1- Color Fundamentals*



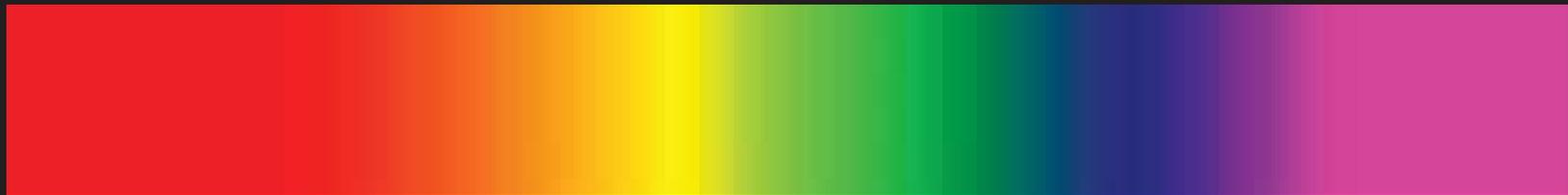
## *The Visible Spectrum*



# *1- Color Fundamentals*



## *The Color Wheel*

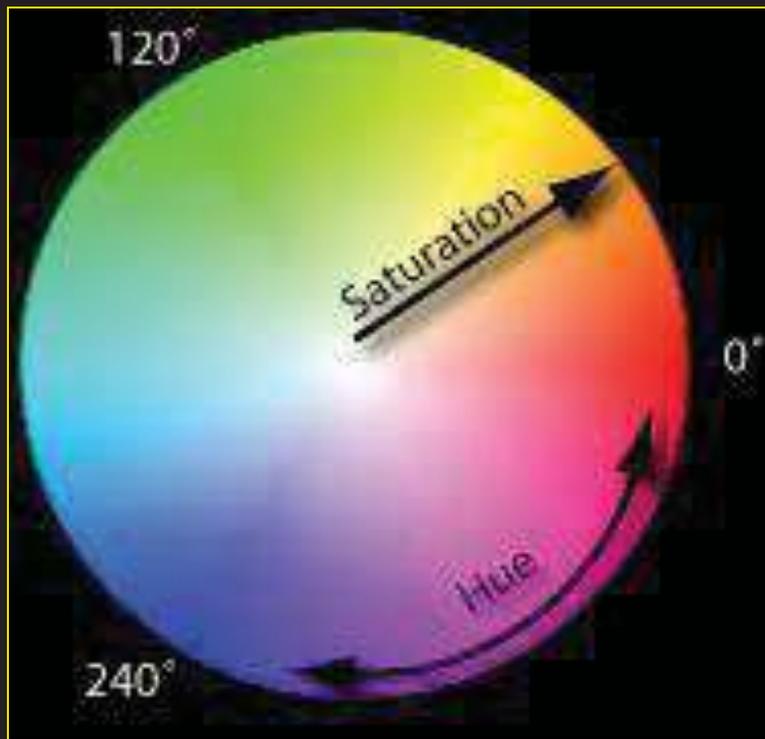


If the ends of the spectrum are bent around a color wheel is formed:



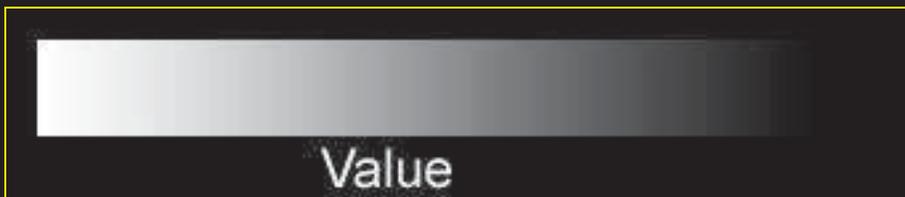
# 1- Color Fundamentals

## The Color Wheel



Colors on the wheel can be described using three parameters:

1. **Hue**: degrees from  $0^\circ$  to  $360^\circ$
2. **Saturation**: brightness or dullness
3. **Value**: lightness or darkness



As suggested by Henry Albert Munsell in *A Colour Notation*, 1905)

# 1- Color Fundamentals



## The Color Wheel: Hue



Hue or Spectral Color is represented as an angle.

### 1. Primary Colors:

- $0^\circ$  = Red
- $120^\circ$  = Green
- $240^\circ$  = Blue

### 2. Secondary Colors:

- $60^\circ$  = Yellow
- $180^\circ$  = Cyan
- $300^\circ$  = Magenta

# 1- Color Fundamentals



## The Color Wheel: Saturation



1. Saturation or Chroma is the intensity of a color.
2. A **highly** saturated color is bright and appears closer to the edge of the wheel.
3. A more **unsaturated** color is dull.
4. A color with **no saturation** is achromatic or in the **grey scale**.



# 1- Color Fundamentals

## The Color Wheel: Value

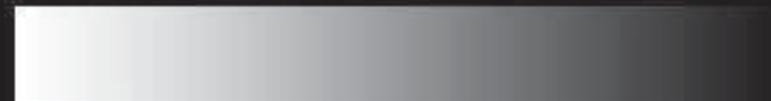


"the quality by which we distinguish a light color from a dark one."

- Albert Henry Munsell  
*A Colour Notation* 1905

### Value

represents the luminescent contrast value between black and white



Value





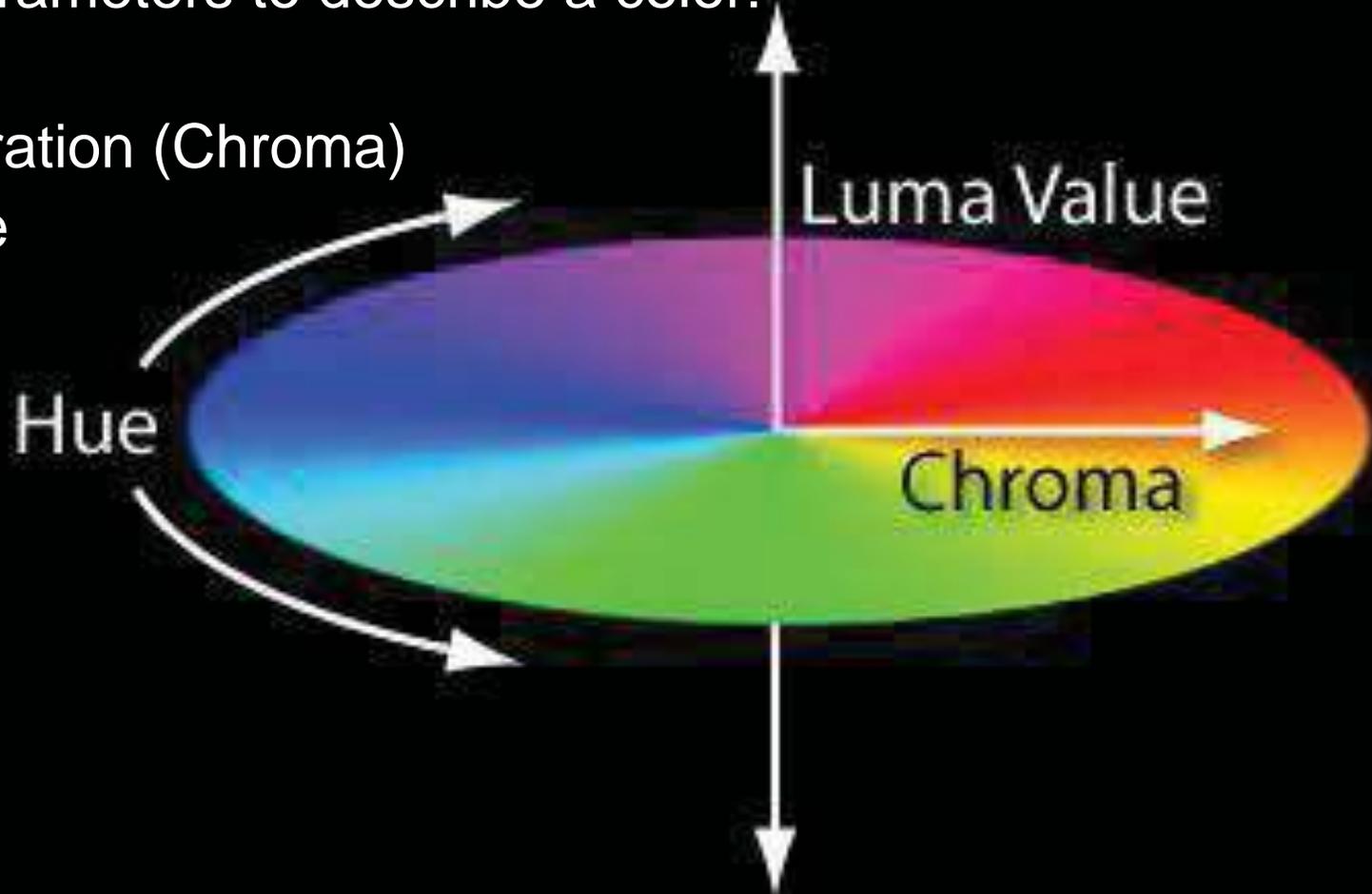
# 1- Color Fundamentals



## The Color Wheel 3d

Three parameters to describe a color:

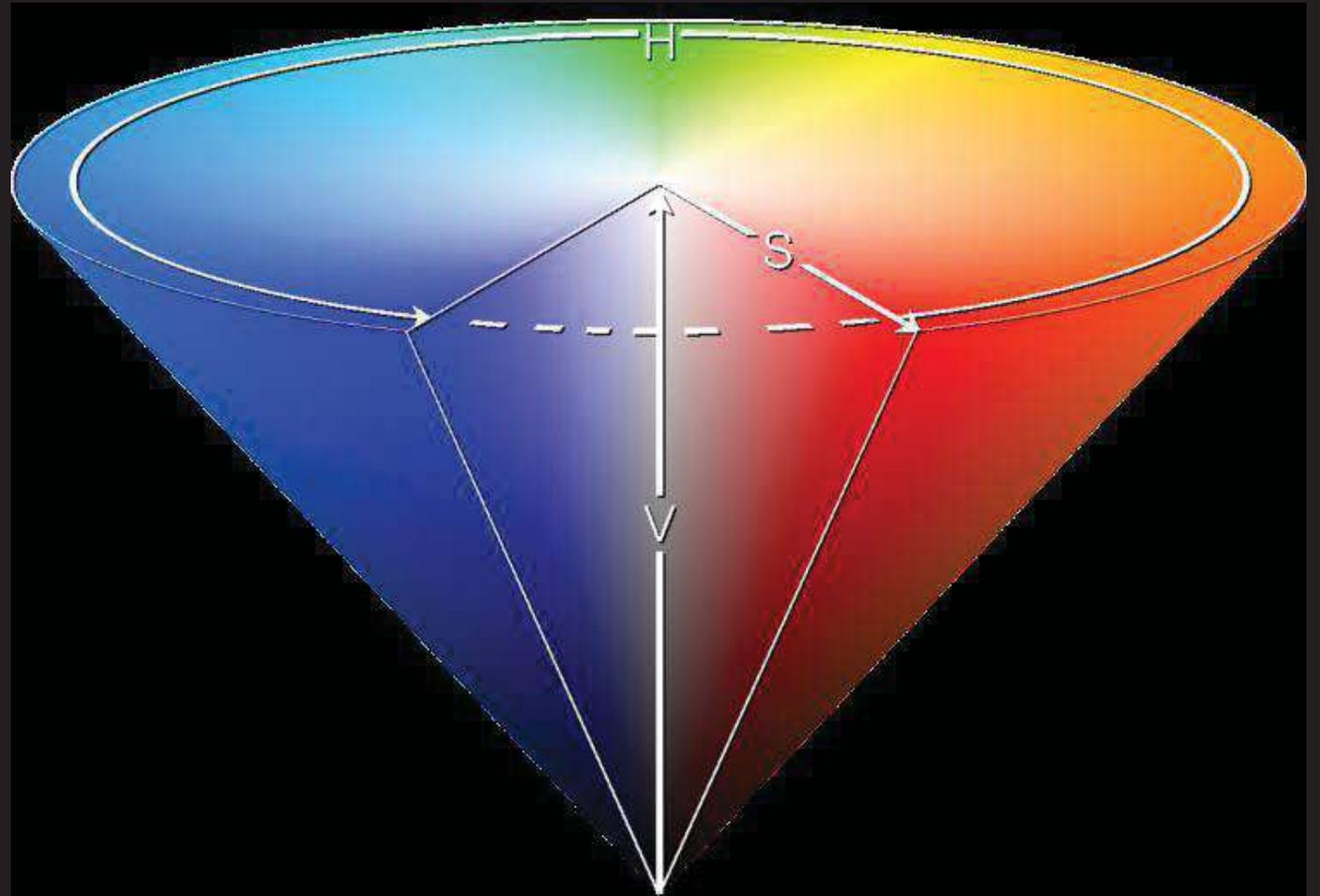
1. Hue
2. Saturation (Chroma)
3. Value



# *1- Color Fundamentals*



## *The Color Wheel 3d*



## 2- Color Models

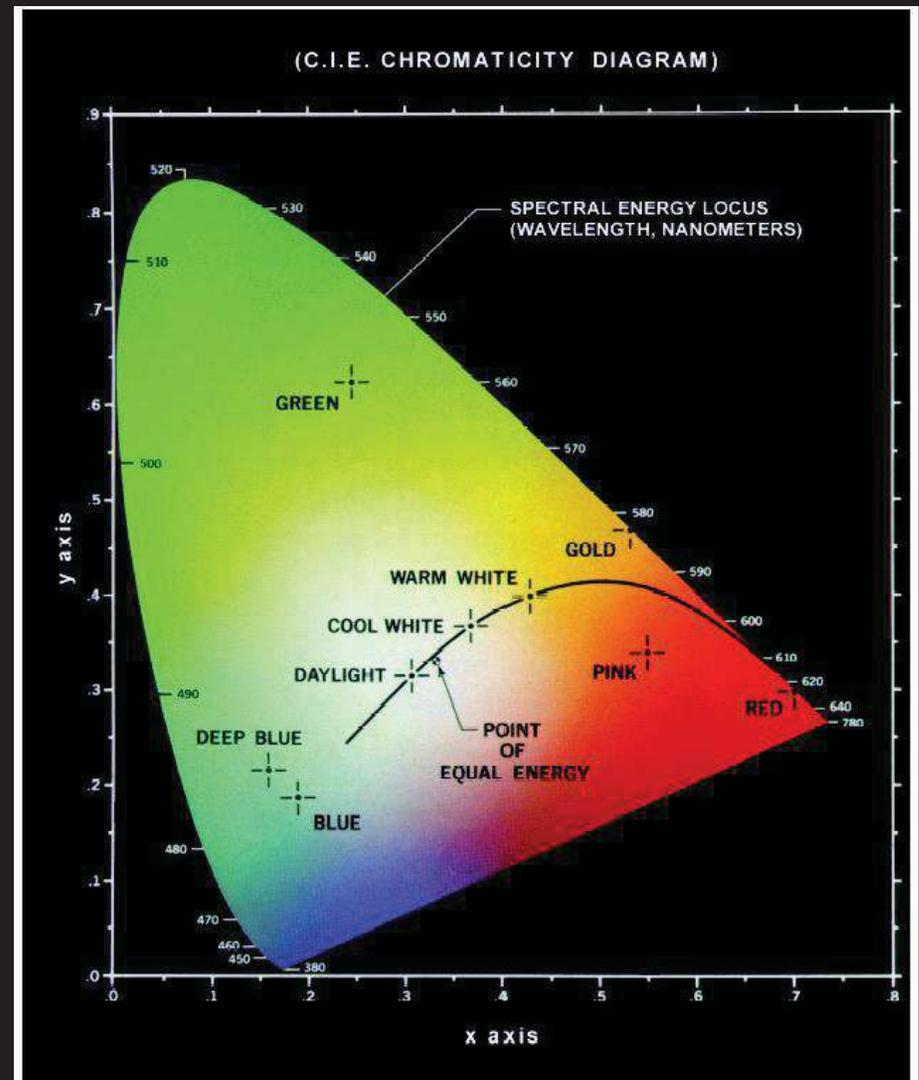
### CIE 1931 xyz color space

Based on a large scale study with volunteers evaluate color samples.

Representing all of the colors visible to the average person.

A 2D projection of the XYZ color space is the CIE chromaticity diagram (to the right).

Standard white when  $X=Y=Z$



CIE 1931 xy chromaticity diagram

## *2- Color Models*



### *CIE 1931 xyz color space*

- In 1931, the CIE (**C**ommission **I**nternational de L'**E**clairage, or **I**nternational **C**ommission on Illumination) defined three standard primaries (**X**, **Y**, **Z**).
- The **Y** primary was intentionally chosen to be identical to the **luminous-efficiency** function of human eyes.

## 2- Color Models



### *CIE 1931 xyz color space*

Specifying colors systematically can be achieved using the CIE **chromacity diagram**

On this diagram the **x**-axis represents the proportion of **red** and the **y**-axis represents the proportion of **red used**

The proportion of **blue** used in a color is calculated as:

$$z = 1 - (x + y)$$

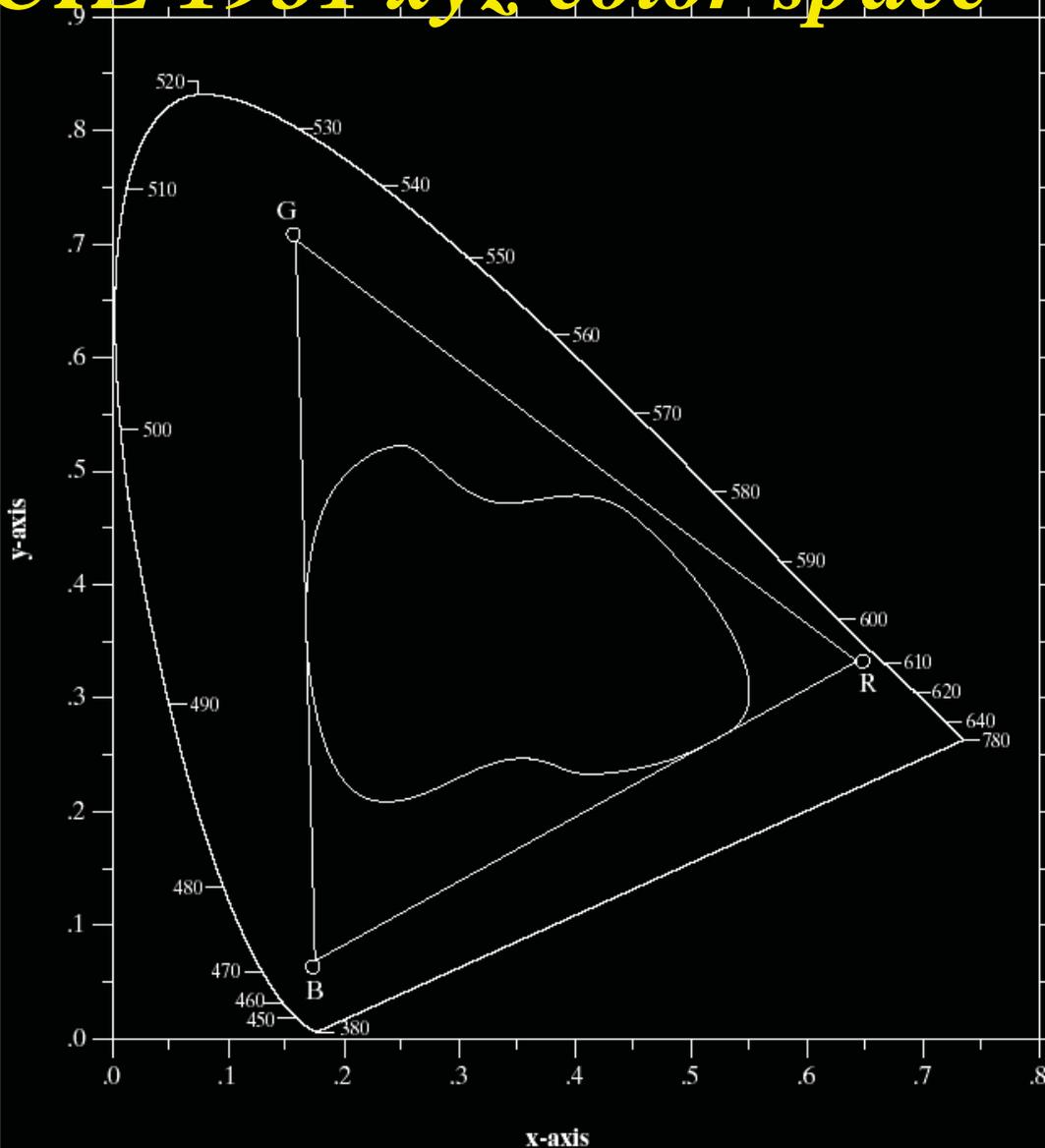
## *2- Color Models*

### *CIE 1931 xyz color space*

1. Any color located on the boundary of the chromaticity chart is fully saturated
2. The point of equal energy has equal amounts of each color and is the CIE standard for pure white
3. Any straight line joining two points in the diagram defines all of the different colors that can be obtained by combining these two colors additively
4. This can be easily extended to three points

## 2- Color Models

### CIE 1931 xyz color space



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

This means the entire color range cannot be displayed based on any three colors

The triangle shows the typical color gamut produced by RGB monitors

The strange shape is the gamut achieved by high quality color printers

## *2- Color Models*

### *Color Models*

From the previous discussion it should be obvious that there are different ways to model color

We will consider two very popular models used in color image processing:

1. **RGB** (Red Green Blue)
2. CMY
3. **HSL** (Hue Saturation Intensity/Luminosity)
4. HSV (Hue Saturation Value)
5. YUV

## 2- Color Models

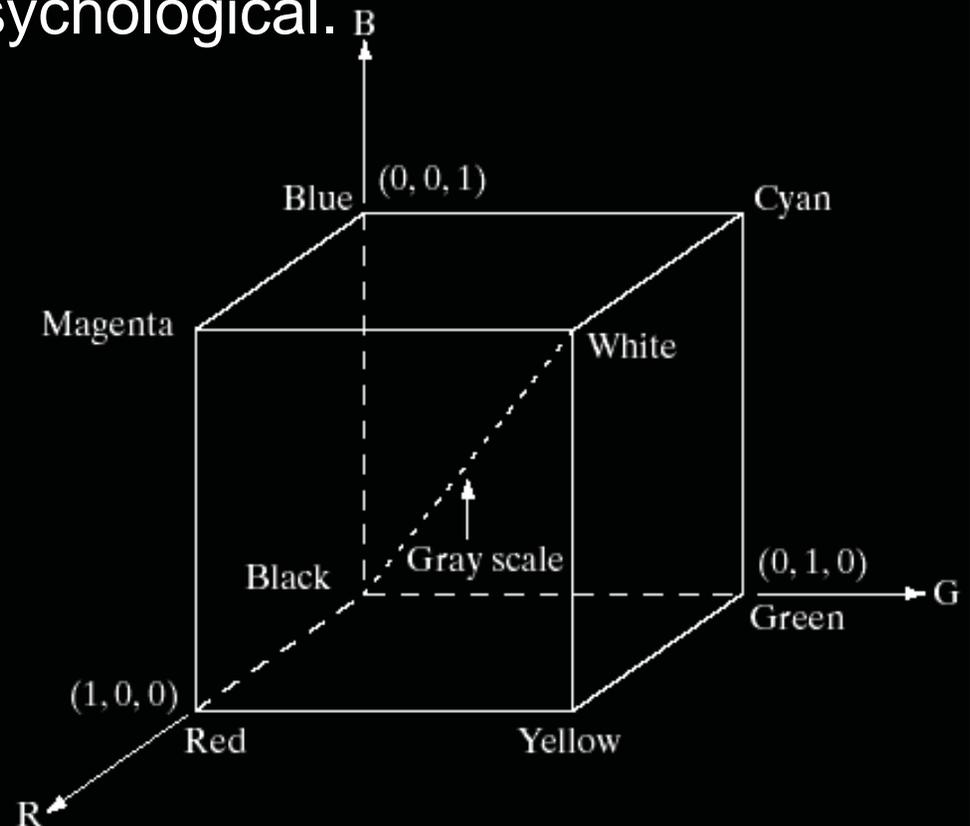


### Color Models: RGB/CMY

Red Green Blue / Cyan Magenta Yellow

RGB is closer to the physiological side of our vision (the three cone types) rather than the psychological.

$$[C M Y] = [1 1 1] - [R G B]$$





## 2- Color Models

**Color Models:** Even more color spaces

**CMY:** is the CMY color space with a black component added, used for printing where a black pigment is used along with Cyan, Magenta and Yellow.

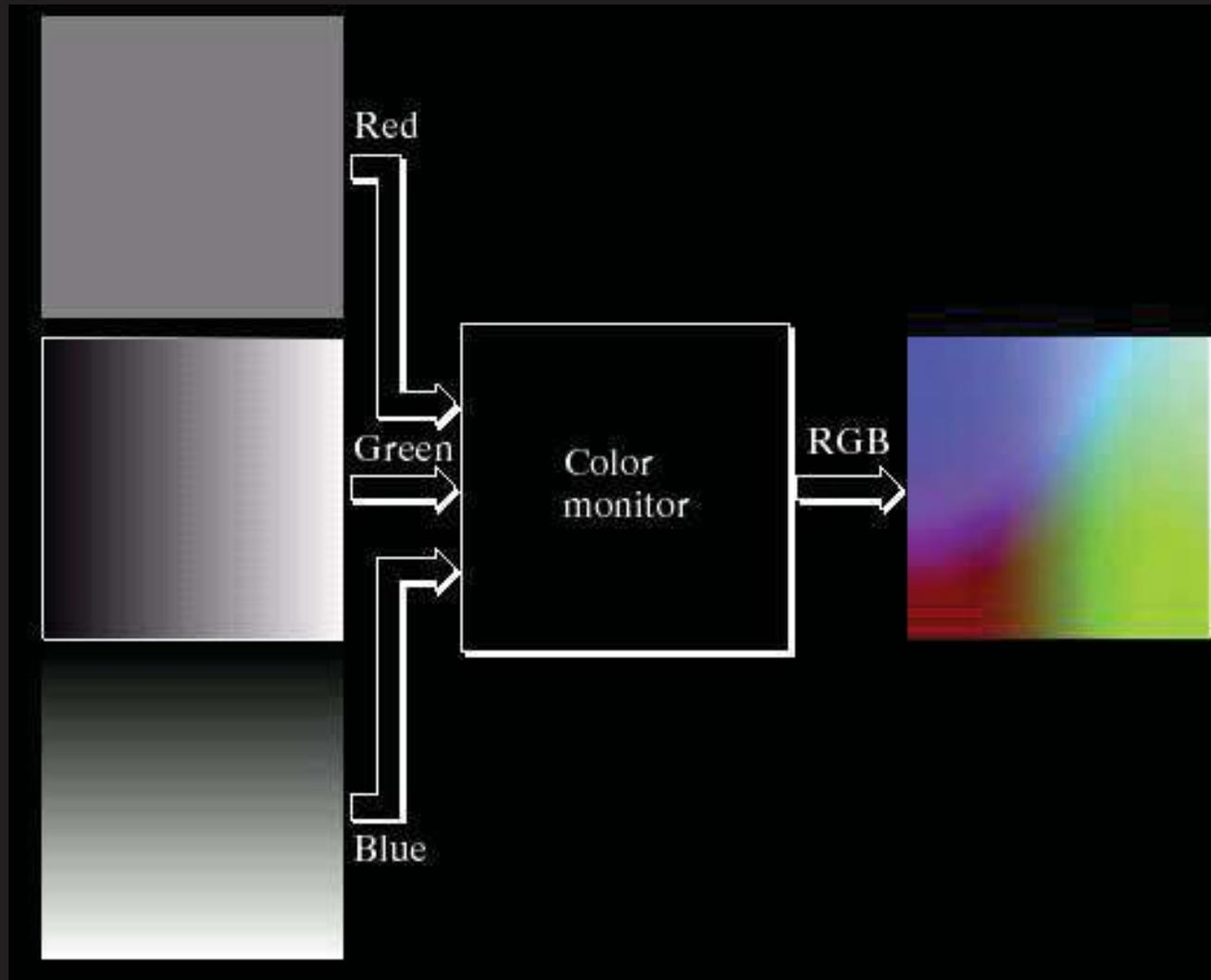


# 2- Color Models

## Color Models: RGB/CMY



Images taken from Gonzalez & Woods, Digital Image Processing (2002)



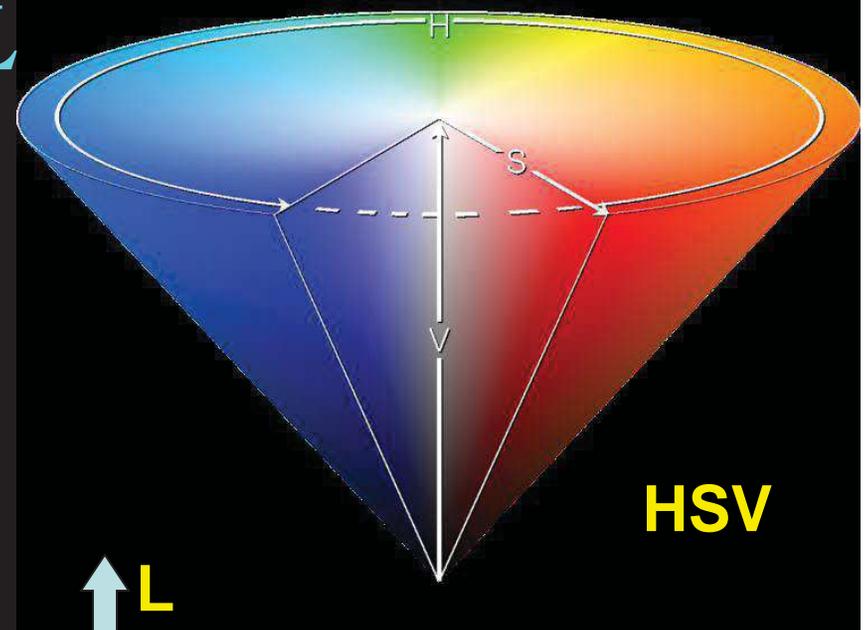
# 2- Color Models



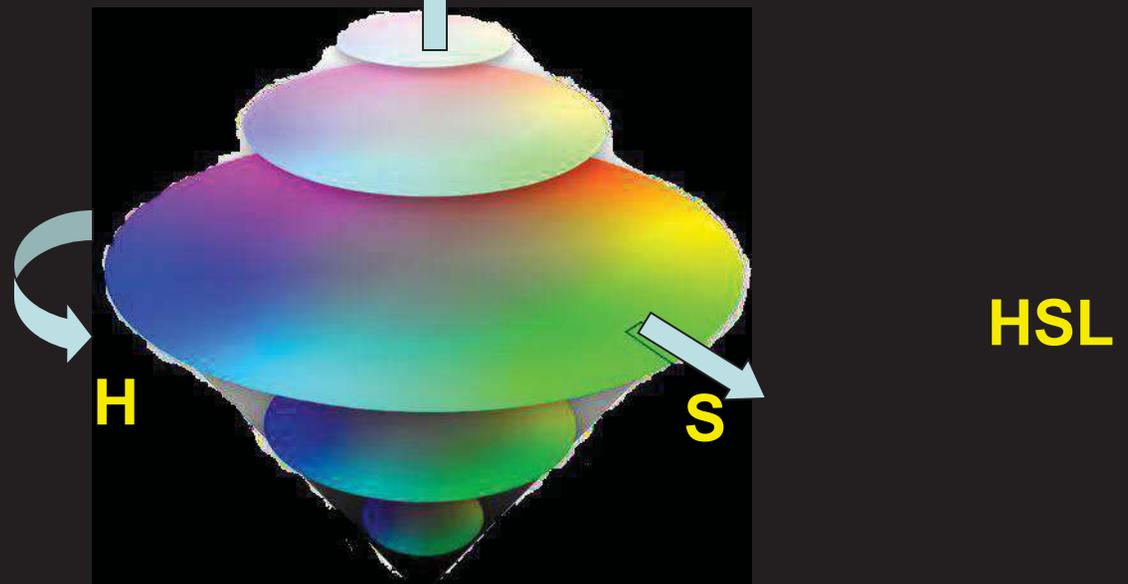
## Color Models: HSV/HSL

Hue, angle  
Saturation, radius  
Value, Lightness, height

The HSV and HSL color spaces has intensity decoupled from color information.



Example for HSV color picker

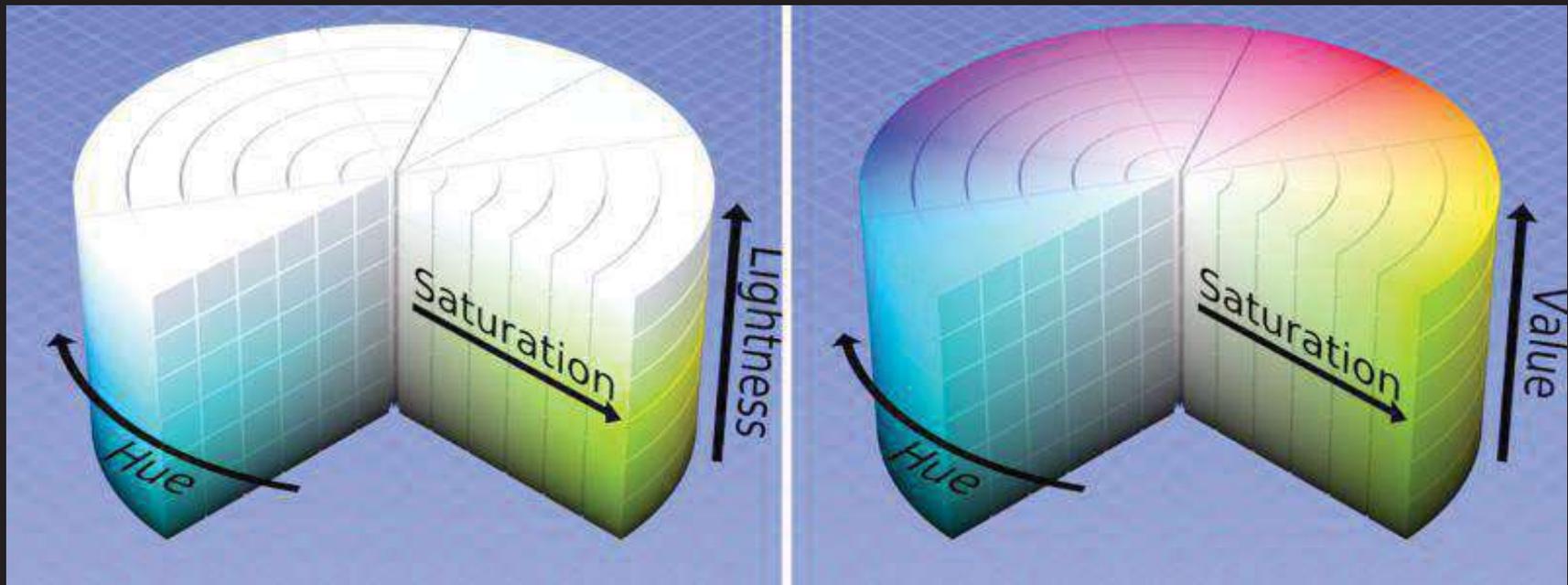


## 2- Color Models



### Color Models: Difference between HSL/HSV

1. The difference is how saturation and lightness/value hang together.
2. For some people it is non **intuitive** to be have almost white colors that are fully saturated while others likes the lightness property in HSL.

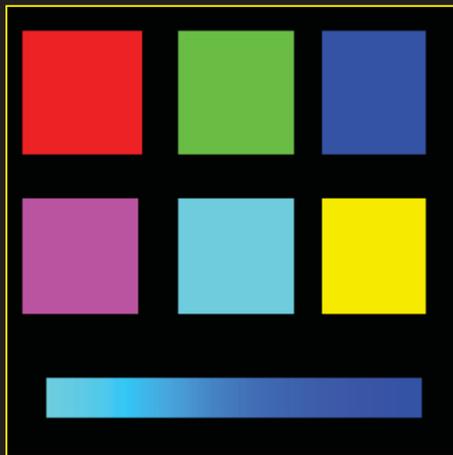
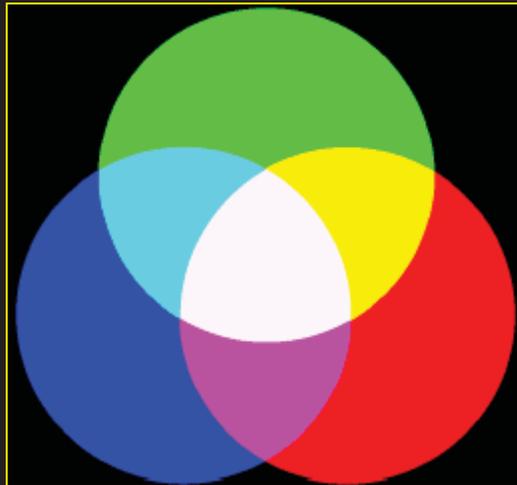


# 2- Color Models

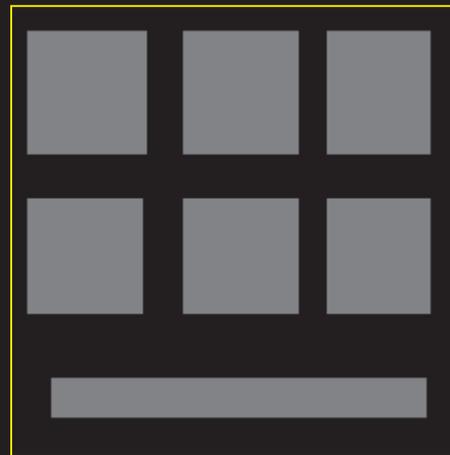


## Color Models: Difference between HSL/HSV

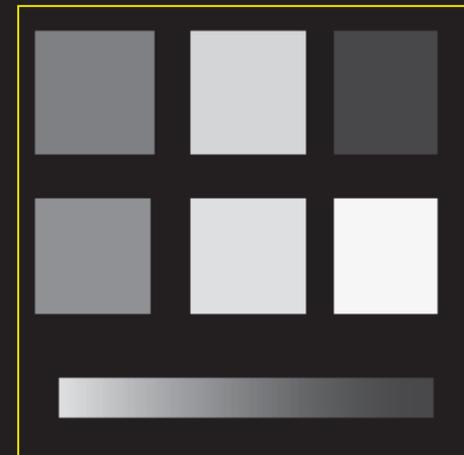
RGB



HSL



HSV



## 2- Color Models

### Converting From RGB To HSL

Given a color as R, G, and B its H, S, and I values are calculated as follows:

$$H = \begin{cases} \theta & \text{if } B \leq G \\ 360 - \theta & \text{if } B > G \end{cases}$$

$$\theta = \cos^{-1} \left\{ \frac{\frac{1}{2} [(R-G) + (R-B)]}{\left[ (R-G)^2 + (R-B)(G-B) \right]^{\frac{1}{2}}} \right\}$$

$$I = \frac{1}{3}(R + G + B)$$

$$S = 1 - \frac{3}{(R + G + B)} [\min(R, G, B)]$$

## 2- Color Models



### Converting From HSL To RGB

Given a color as H, S, and I it's R, G, and B values are calculated as follows:

– For RG sector ( $0 \leq H < 120^\circ$ )

$$R = I \left[ 1 + \frac{S \cos H}{\cos(60 - H)} \right]$$

$$G = 3I - (R + B)$$

$$B = I(1 - S)$$

– For GB sector ( $120^\circ \leq H < 240^\circ$ )

$$G = I \left[ 1 + \frac{S \cos(H - 120)}{\cos(H - 60)} \right]$$

$$B = 3I - (R + G)$$

## 2- Color Models



### Converting From HSL To RGB

– For BR sector ( $240^\circ \leq H \leq 360^\circ$ )

$$R = 3I - (G + B)$$

$$G = I(1 - S)$$

$$B = I \left[ 1 + \frac{S \cos(H - 240)}{\cos(H - 180)} \right]$$

## 2- Color Models

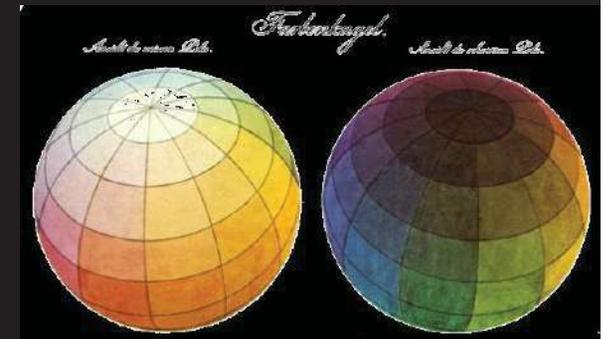
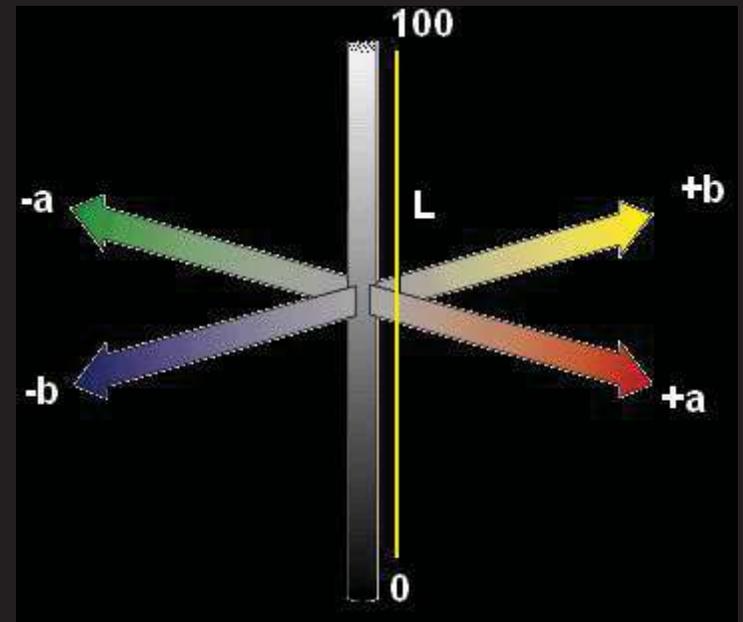


### Color Models: CIE L\*a\*b\* or CIELAB

The most complete color space specified by the International Commission on Illumination, CIE in 1976.

Created to:

1. represent all colors visible to the **human** eye.
2. serve as a **device independent** model to be used as a reference.
3. be perceptually uniform - equal distance should have equal perceptual difference.
4. Remember the 200 years old color model:





## 2- Color Models

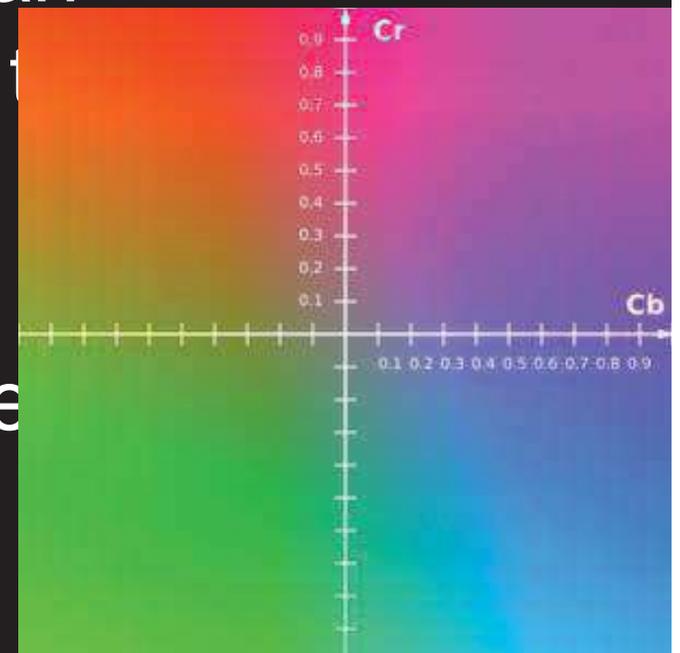
**Color Models:** Even more color spaces

**YCbCr:** is similar to CIE  $L^*a^*b^*$

1. Used in the JPEG file format and for digital TV.

2. Uses the fact that the human eye is more sensitive to variation in lightness than hue and saturation.

**YUV:** is similar to YCbCr, used for analogue TV.



# 2- Color Models



## YUV Color Transform

- Initially, for PAL analog video, it is now also used in CCIR 601 standard for digital video

- Y (luminance) is the CIE Y primary.

$$Y = 0.299R + 0.587G + 0.114B \dots\dots\dots (1)$$

- Chrominance* is defined as the difference between a color and a reference white at the same luminance. It can be represented by **U** and **V** -- the *color differences*.

$$\begin{aligned} U &= B - Y && \dots\dots\dots (2) \\ V &= R - Y && \dots\dots\dots (3) \end{aligned}$$

- If b/w image, then  $U = V = 0$ .  $\rightarrow$  No chrominance!
- \*\* In actual PAL implementation:

$$\begin{aligned} U &= 0.492 (B - Y) \\ V &= 0.877 (R - Y) \end{aligned}$$

*Homework:* Find Backward transform.

$$\begin{aligned} R &= Y + V \\ G &= Y - (0.195 \times U) - (0.509 \times V) \\ B &= Y + U \end{aligned}$$

## 2- Color Models



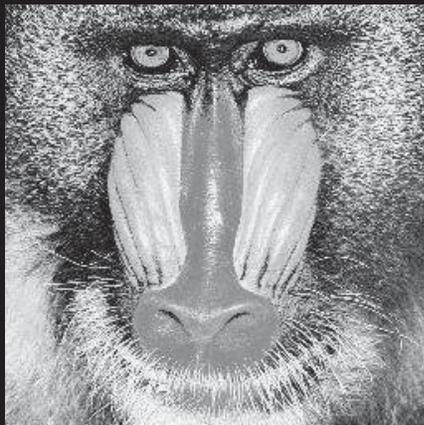
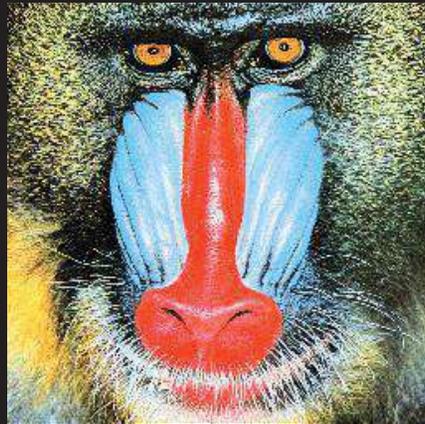
### YUV Transform Matrix

$$\begin{pmatrix} 0.299 & 0.587 & 0.114 \\ -0.147 & -0.289 & 0.436 \\ 0.615 & -0.515 & -0.100 \end{pmatrix}$$

$$\begin{pmatrix} 1.0 & 0.0 & 1.140 \\ 1.0 & -0.394 & -0.581 \\ 1.0 & 2.028 & 0.0 \end{pmatrix}$$

# Color Models In Video

## YUV Color Transform



Y



U



V

Eye is most sensitive to Y. In PAL, 5.5 MHz is allocated to Y, 1.8 MHz each to U and V

## *2- Color Models*



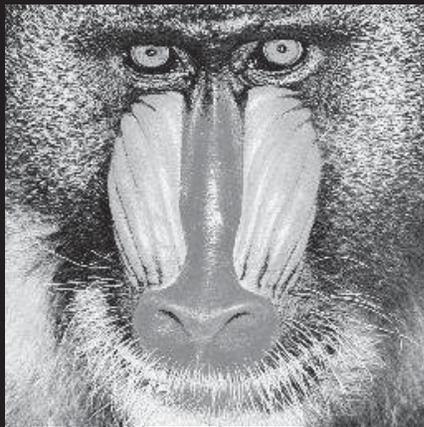
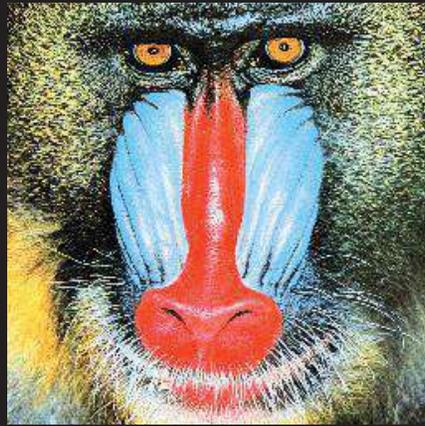
### YIQ Transform Matrix

$$\begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.212 & -0.523 & 0.311 \end{pmatrix}$$

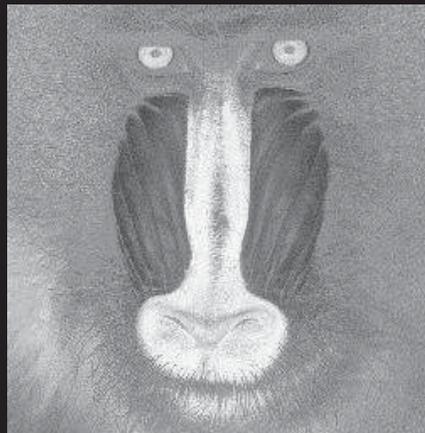
$$\begin{pmatrix} 1 & 0.956 & 0.621 \\ 1 & -0.272 & -0.647 \\ 1 & -1.106 & 1.703 \end{pmatrix}$$

# Color Models In Video

## YIQ Color Transform



Y



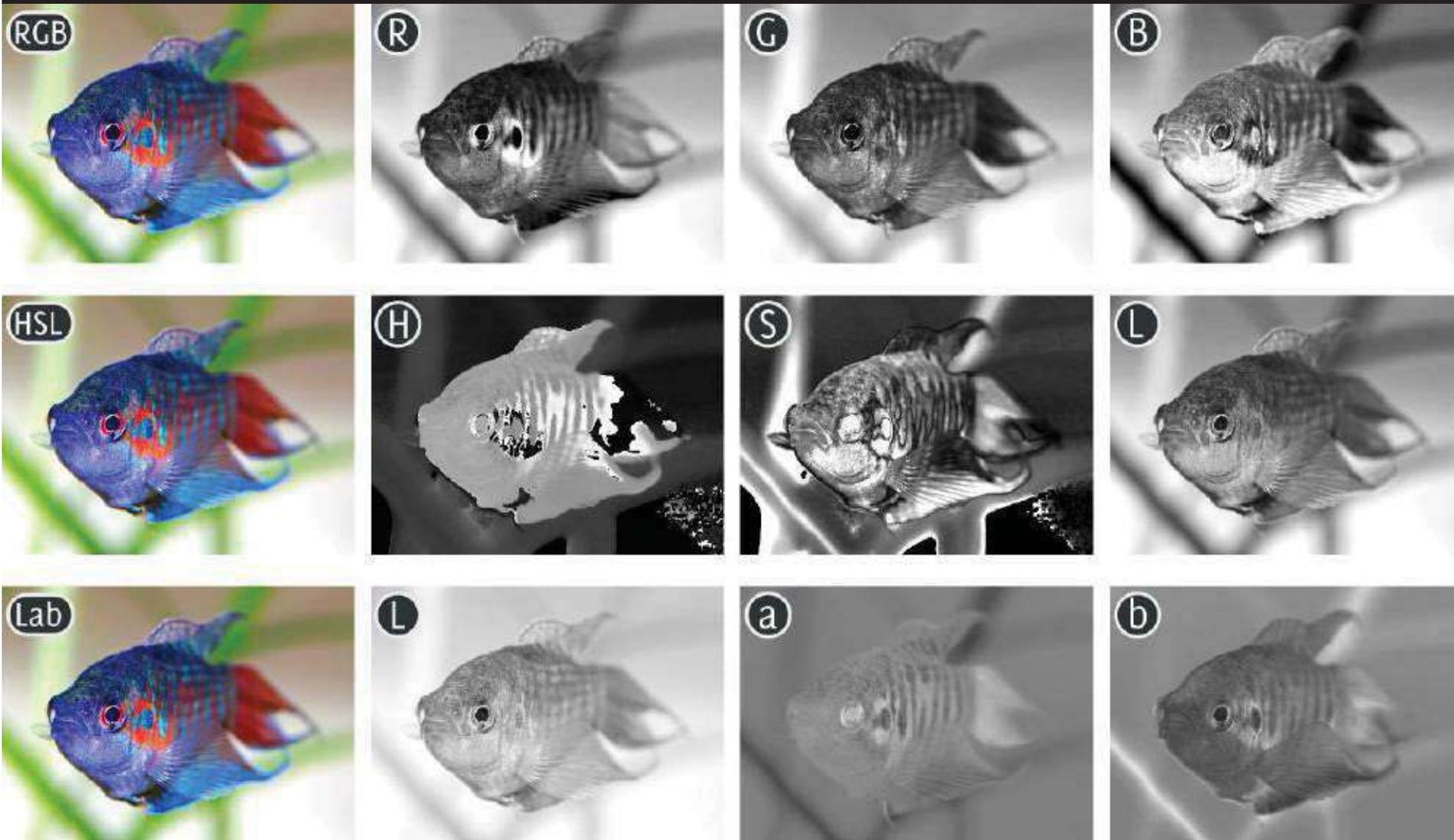
I



Q

Eye is most sensitive to Y. In PAL, 5.5 MHz is allocated to Y, 1.8 MHz each to U and V

# 2- Color Models



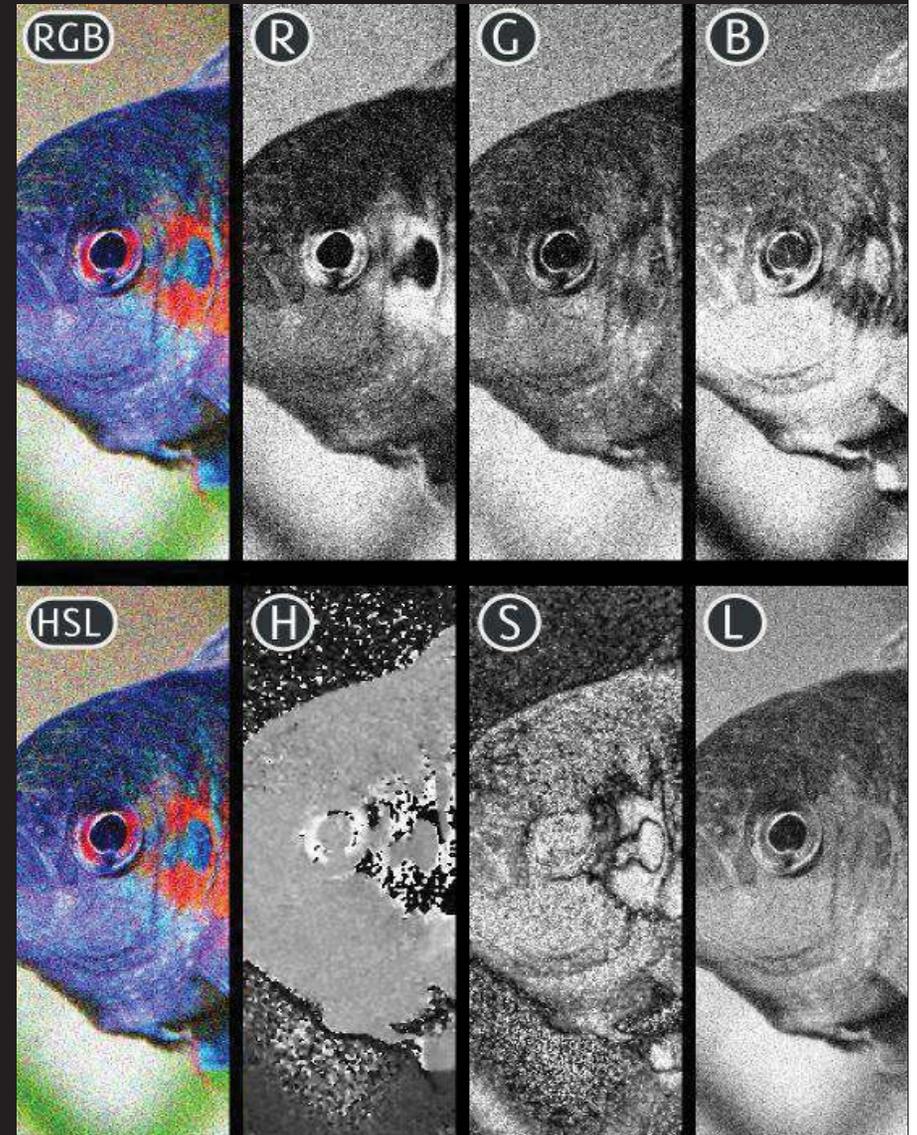
## 2- Color Models



### Noise in color images

Gaussian noise in all color channels

In a HSL representation the noise is most **apparent** in the **H** and **S** channel.



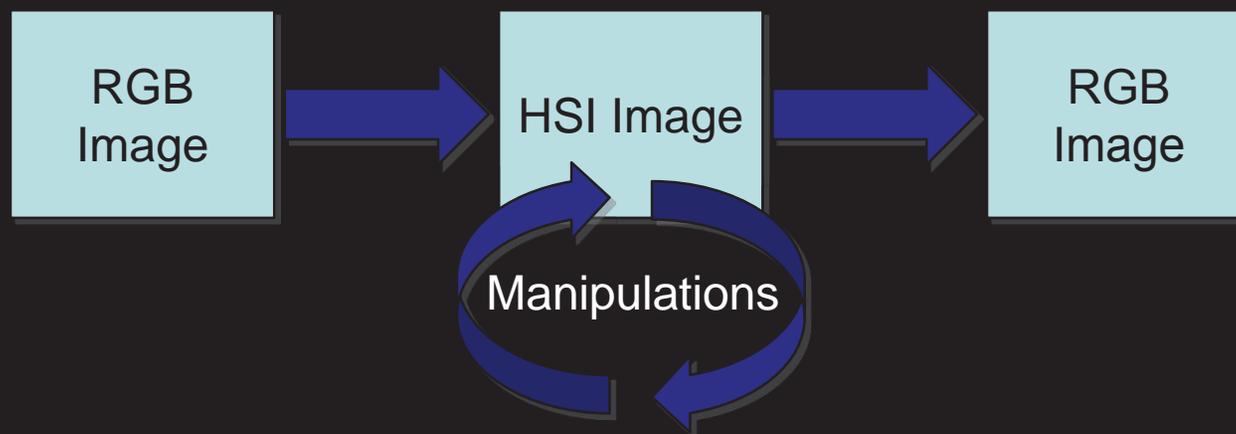
## 2- Color Models



### Manipulating Images In The HSI Model

In order to manipulate an image under the HIS model we:

- First convert it from RGB to HIS
- Perform our manipulations under HSI
- Finally convert the image back from HSI to RGB



## *2- Color Models*

### Gray Level methods on color images

1. In general all image processing methods used for grey level images can be used for color images.
2. They can be carried out on each of the color channels or for example on the intensity only.
3. There is no right or wrong, but the result differ.

### Example for segmentation

Use the intensity to find edges between areas and color values as similarity measure.

## 2- Color Models



### Look out for the H channel

1. You might end up with color artifacts if the H-channel is filtered.
2. Remember that the Hue channel is in degrees **0 to 360** (both red).



Gaussian filtered hue channel

## *2- Color Models*



### More gray level methods on color images

Histogram equalization on all channels in HSV can make things strange.

Only used on the **V** channel the expected result is acquired.



Original



Histogram equalization in  
all channels



Histogram equalization in  
V channels

## 2- Color Models

### Segmentation based on Hue

- Using an intensity decoupled color space segmentation based on color can be relative intuitive.
- Setting an interval for the **hue** around the hue value for **red** in HSV space the red part of the fish is segmented.



Original



Segmented part of hue shown on the value channel.

## 2- Color Models



### Pseudo Coloring

For visualization

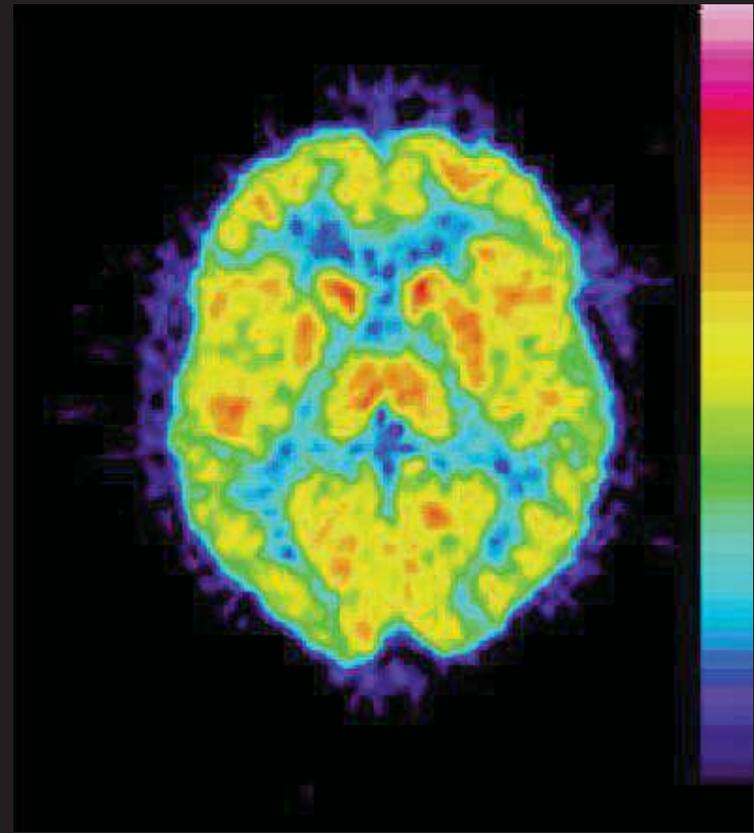
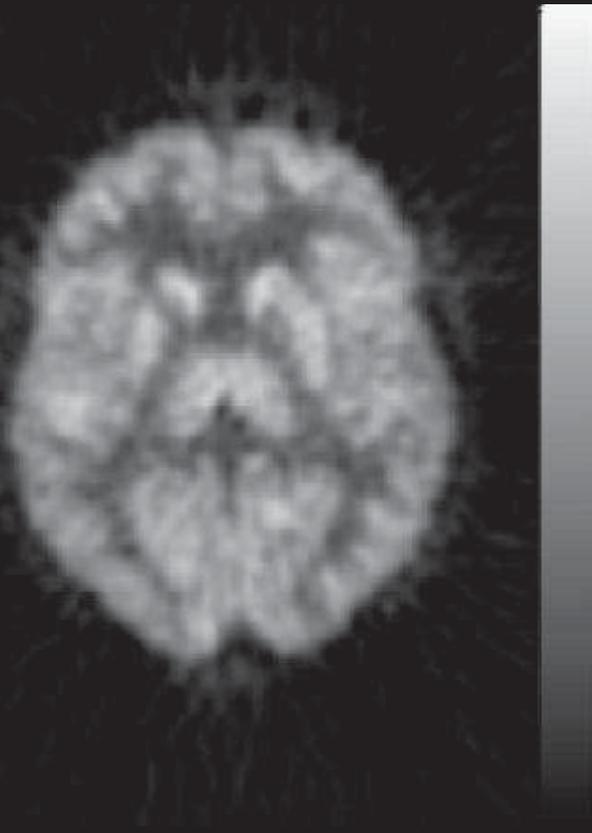
1. The eye can distinguish between ~30 different gray levels and about 350 000 different colors.
2. A gray level image can be displayed as a color image for easier visual inspection. Note: the information content is not changed.
3. This is often used in **modalities** when the samples does not correspond to the visual spectrum.



## *2- Color Models*

### Pseudo Coloring

PET



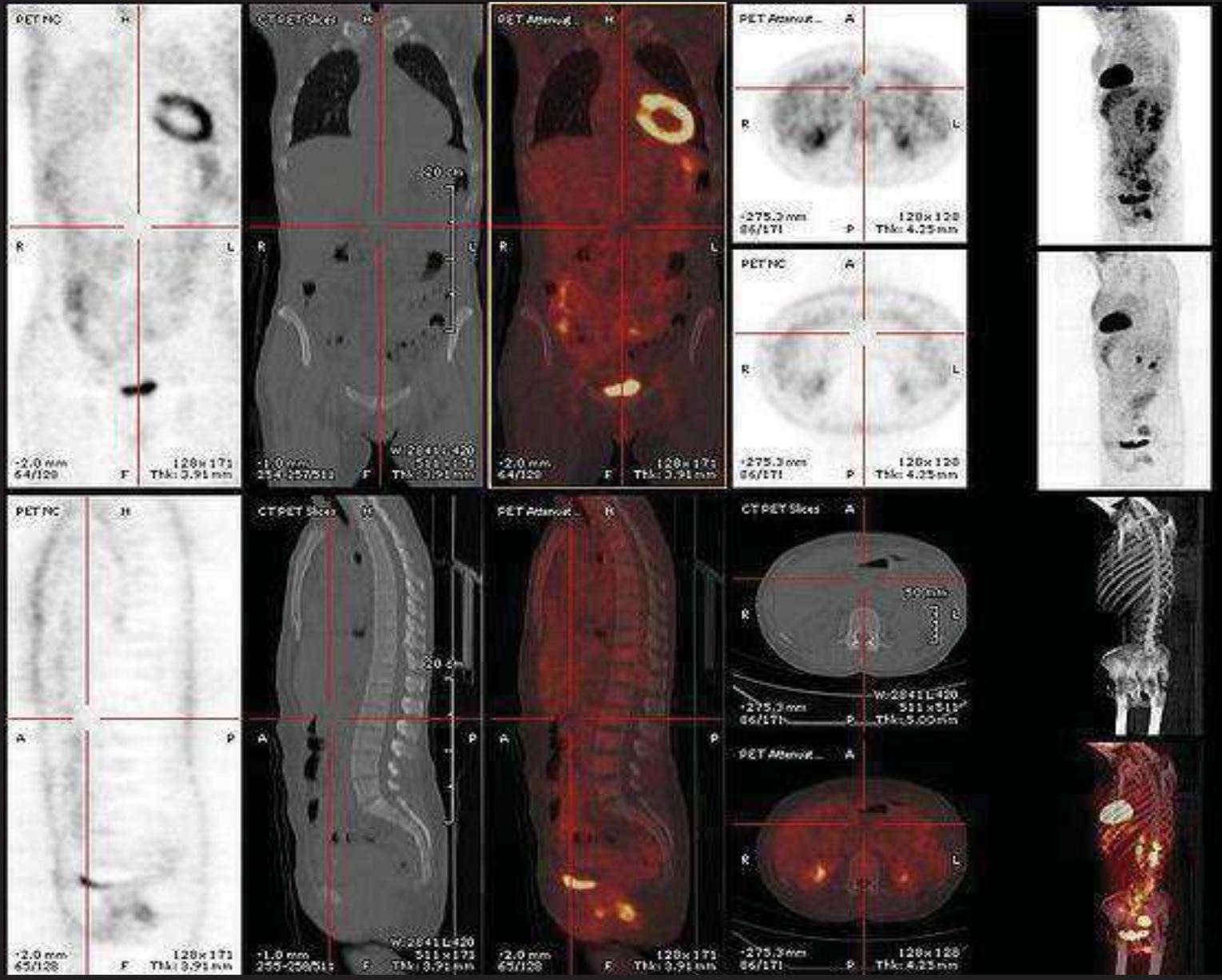
FDG tracer is used, high intensity related to high brain activity.



# 2- Color Models Pseudo Coloring



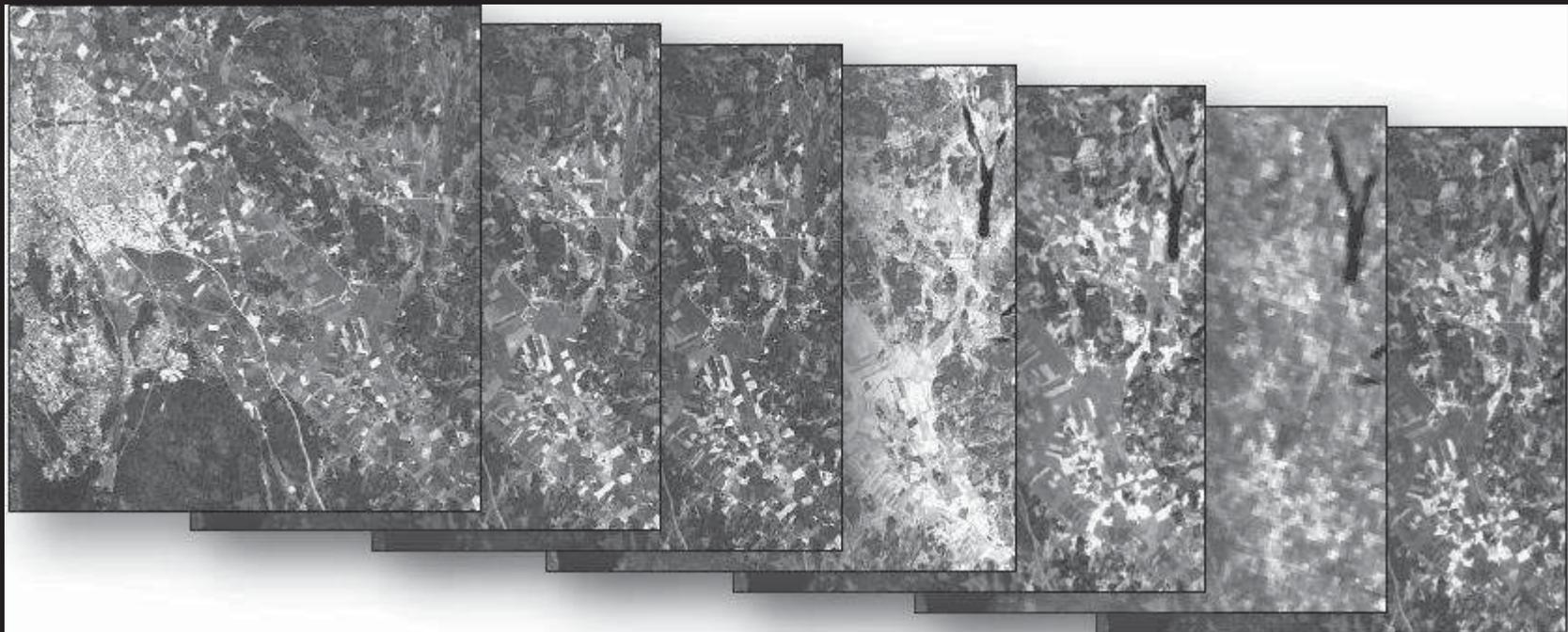
PET + CT



### *3- More than 3 spectral windows*



Why restrict our imaging system to the borders of human vision? Multi spectral data is common in remote sensing, astronomy, thermography, etc.



Multispectral satellite data with 7 spectral bands of the Uppsala urban area.

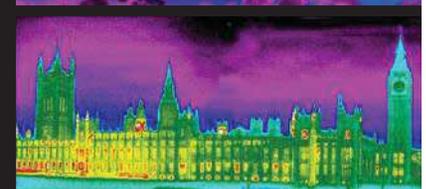
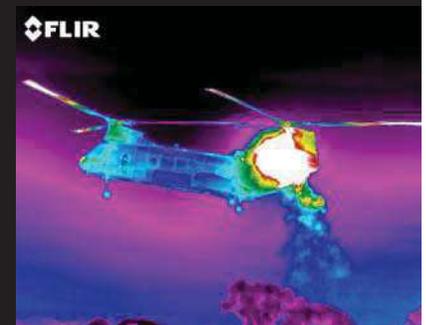
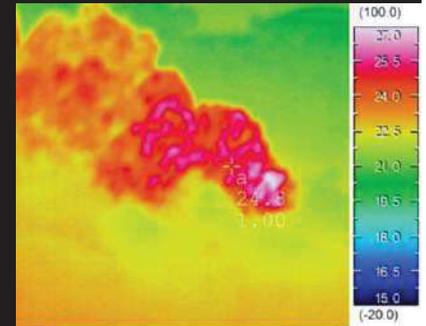
# Summary



1. Every imaging system selects one or several spectral windows. Hence, the spectral dimension is unavoidable.
2. There are several ways to represent the color information in the visible spectra. (Color spaces)
3. The RGB color space is usually unavoidable since viewing devices (LCD/CRT screens) use it.
4. Choose color space based on the specific application and situation.

# Pseudocolor Image Processing

- Pseudocolor (also called **false color**) image processing consists of assigning colors to grey values based on a specific criterion
- The principle use of pseudo color image processing is for human visualization
  - Humans can discern between thousands of color shades and intensities, compared to only about two dozen or so shades of grey



# Pseudo color Image Processing

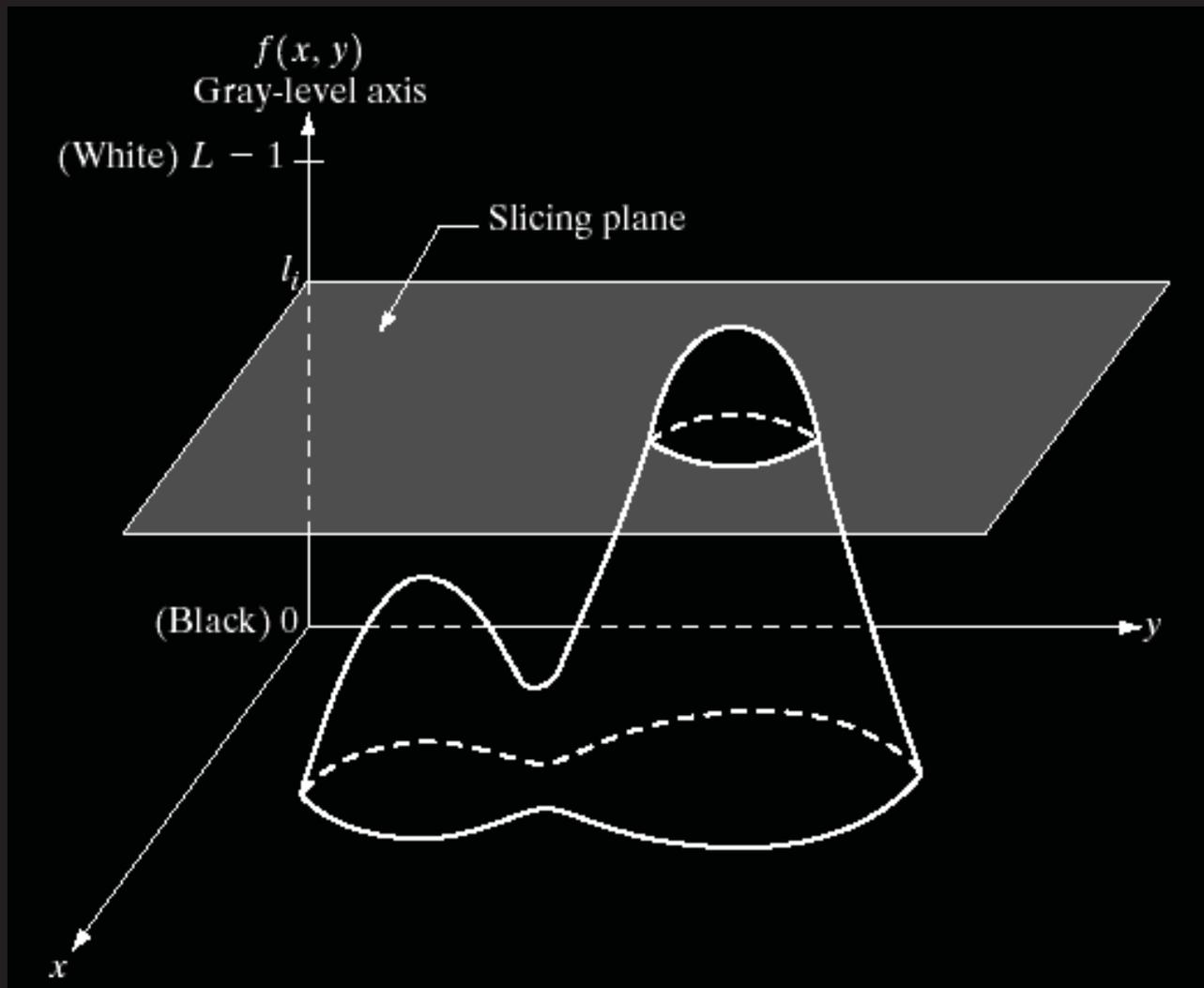
## *Intensity Slicing*

- Intensity slicing and color coding is one of the simplest kinds of pseudocolor image processing
- First we consider an image as a 3D function mapping spatial coordinates to intensities (that we can consider heights)
- Now consider placing planes at certain levels parallel to the coordinate plane
- If a value is one side of such a plane it is rendered in one color, and a different color if on the other side

# Pseudo color Image Processing

## *Intensity Slicing*

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



# Pseudo color Image Processing

## *Intensity Slicing*

In general intensity slicing can be summarized as:

1. Let  $[0, L-1]$  represent the grey scale
2. Let  $I_0$  represent black  $[f(x, y) = 0]$  and let  $I_{L-1}$  represent white  $[f(x, y) = L-1]$  –  $L=256$  gray level
3. Suppose  $P$  planes perpendicular to the intensity axis are defined at levels  $I_1, I_2, \dots, I_p$
4. Assuming that  $0 < P < L-1$  then the  $P$  planes partition the grey scale into  $P + 1$  intervals  $V_1, V_2, \dots, V_{P+1}$

# Pseudo color Image Processing

## *Intensity Slicing*

- Grey level color assignments can then be made according to the relation:

$$f(x,y) = c_k \quad \text{if } f(x,y) \in V_k$$

- where  $C_k$  is the color associated with the  $k^{\text{th}}$  intensity level  $V_k$  defined by the partitioning planes at  $l = k - 1$  and  $l = k$

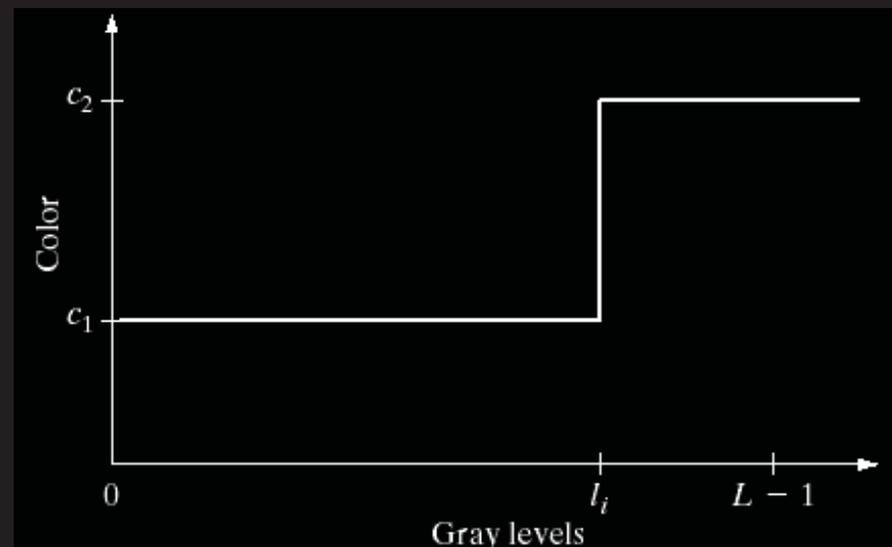
# Pseudo color Image Processing

## *Intensity Slicing*

In general:

Gray scale:  $[0, L - 1]$

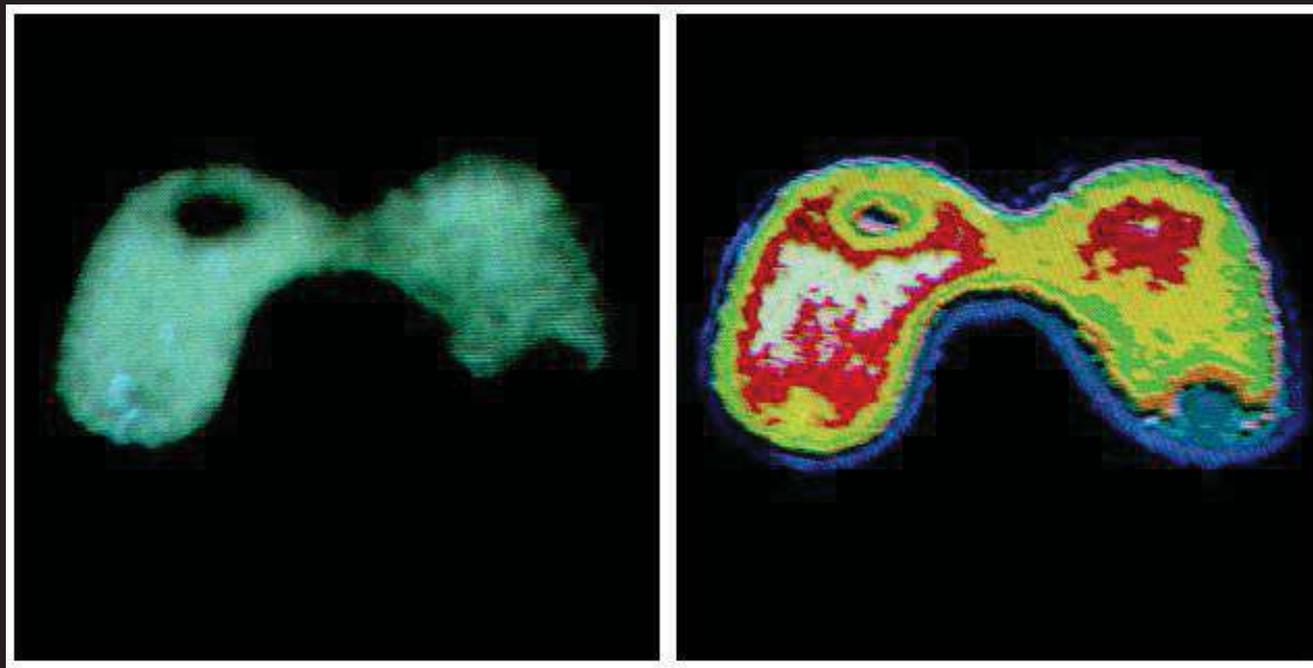
- Black  $[f(x, y) = 0]$ : level  $I_0$ ; White  $[f(x, y) = L - 1]$ :  $I_{L-1}$
- Suppose  $P$  planes perp. to intensity axis: levels  $I_1, I_2, \dots, I_P$
- Planes partition gray scale into  $P+1$  intervals  $V_1, V_2, \dots, V_{P+1}$
- Colour assignment:  $f(x, y) = c_k$  if  $f(x, y) \in V_k$



# Pseudo color Image Processing

## *Intensity Slicing*

- Regions that appear of constant intensity in the monochrome image are actually quite variable!
- Here the gray scale was divided into intervals, without regard of the meaning of the gray levels

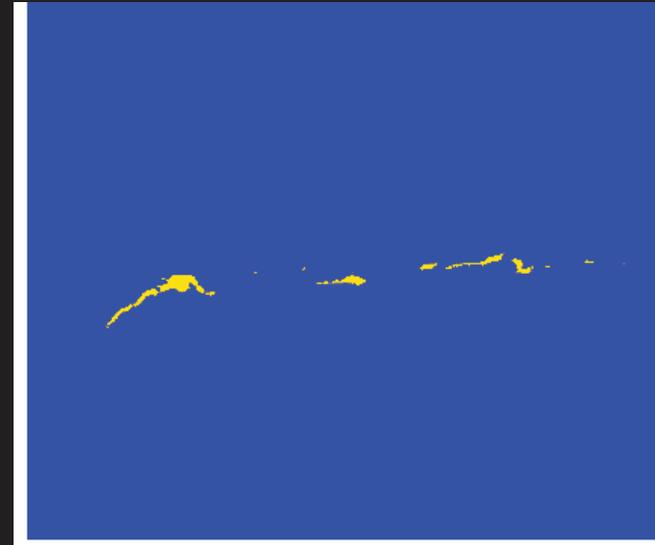
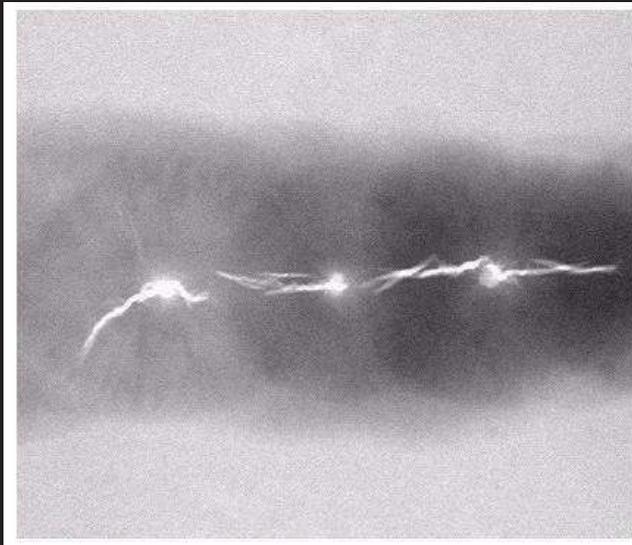


Intensity slicing (human chest)

# Pseudo color Image Processing

## *Intensity Slicing*

Easier to manually detect cracks and porosities

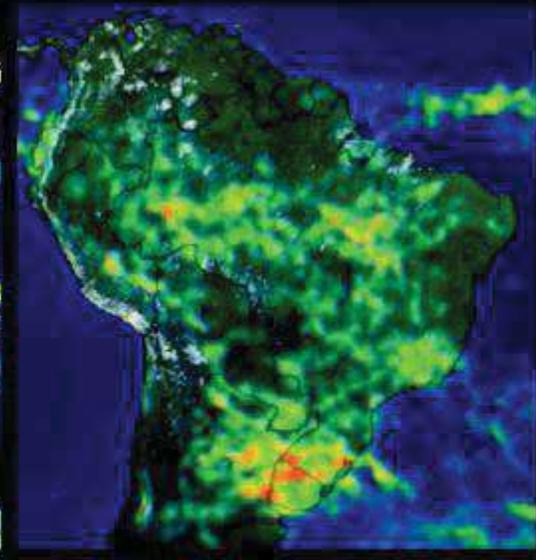
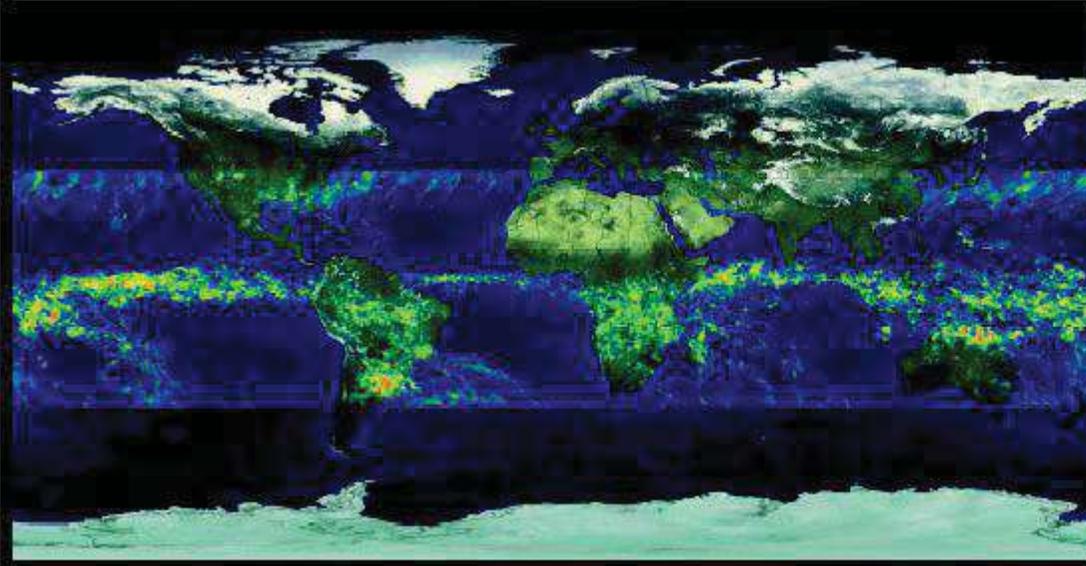
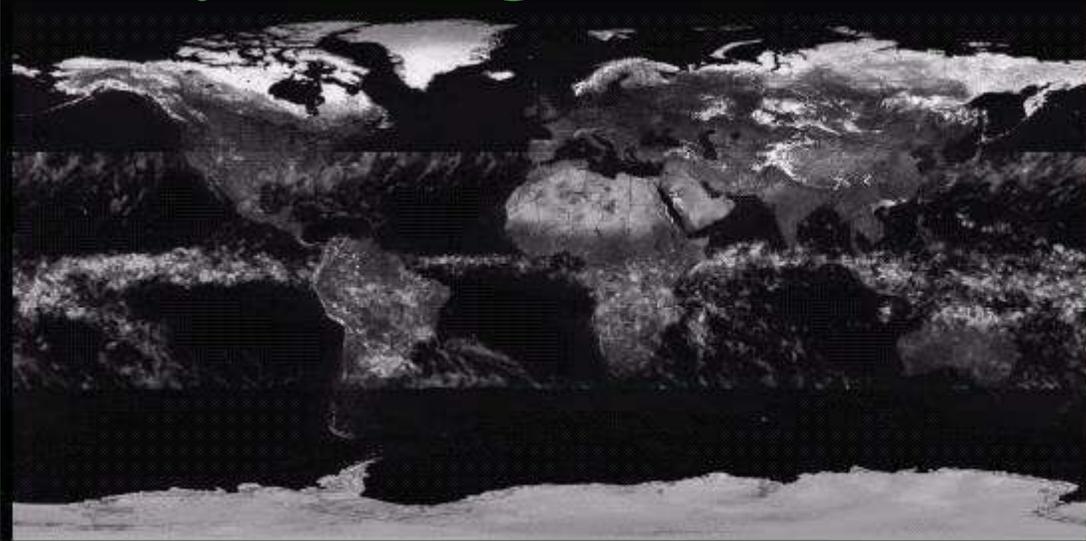


X-ray image of weld

# Pseudo color Image Processing

## *Intensity Slicing*

Tropical rainfall measuring mission (TRMM): a satellite uses a precipitation radar, a microwave imager, and a visible and infrared scanner to detect rain (also over ocean)



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Use of color to highlight rainfall levels

## *Important Notes*

- Color image processing is divided into two major areas: **full-color** and **pseudo-color** processing.
  1. In the **1<sup>st</sup>** category, the images are acquired with a full-color sensor like color TV or color scanner.
  2. In the **2<sup>nd</sup>** category, there is a problem of assigning a color to a particular monochrome intensity or range of intensities.

## *Important Notes*

- Three **quantities** are used to describe the **quality** of a chromatic light source: radiance, luminance and brightness.

**1. Radiance**, watts [W] The total amount of energy that flows from the light source.

**2. Brightness**, is a measure of **intensity** after the **image** has been acquired with a digital camera or digitized by an analog-to-digital converter.

**3. Luminance**, is a measure to describe the perceived brightness of a **color** (or visually perceived brightness).

**Note** we can have high radiance, but low luminance.

## *Important Notes*

- The characteristics generally used to distinguish one color from another are brightness, **hue** and saturation.
1. **Brightness** is a measure of **intensity** after the **image** has been acquired with a digital camera or digitized by an analog-to-digital converter.
  2. **Hue** is an attribute associated with dominant wavelength in a mixture of light waves.
  3. **Saturation** refers to the relative purity or the amount of white light mixed with a hue.

## *Important Notes*

- A color **model** is also called as color **space** or color **system** .
- Its purpose is to facilitate the specification of colors in some standard, generally accepted way.

## *Important Notes*

- Images are represented in the RGB color model consist of three component images one for each primary color.
- When fed into RGB monitor, these three images combine on the phosphor screen to produce a composite color image.
- The **number of bits** used to represent each pixel in RGB space is called the **pixel depth**.

## *Important Notes*

- If an image is given in RGB format then the **saturation** component is obtained by the equation.

$$S = 1 - (3 / (R + G + B)) [\min(R, G, B)].$$

**END**

**Of Lecture**

# Homework

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

# *Aerial Photography & Photogrammetry*

## Lecture 5-1

# Scale of a Vertical Aerial Photograph

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**2017-2022**

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# Reading Chapters

*“Elements of Photogrammetry with Applications in GIS”, by Förstner etl, 2016.*

	<i>Material</i>	<i>Chapter Sections</i>	<i>Page</i>	<i>Exercises page</i>	<b>LABORATORY EXERCISE page</b>
<u>1</u>	Introduction		1	23	
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<u>4</u>	<b>Scale of a Vertical Aerial Photograph</b>		68	84	<b>83</b>
<u>5</u>	Horizontal Measurements - Distance, Bearings and Areas		86		<b>83</b>
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# Lecture Contents

## 4.1 SCALE CLASSIFICATION

### 4.2 The Theory of Scale

#### 4.1 SCALE CLASSIFICATION

##### 4.2.1 Representative Fraction

##### 4.2.2 Photo Scale Reciprocal

##### 4.2.3 Equivalent Scale

## 4.3 TYPES OF SCALE

### 4.3.1 Average Scale

### 4.3.2 Point Scale

## 4.4 VARIATION IN SCALE

## 4.5 BASIC SCALE EQUATIONS

## 4.6 PHOTO SCALE DETERMINATION

Example 1   Example 2   Example 3   Example 4   Example 5

Example 6   Example 7

### 4.6.1 Assumptions

## LABORATORY EXERCISE

# OBJECTIVES

After a thorough understanding of this chapter and completion of the laboratory exercise, you will be able to:

- Define photographic scale and list the three most common methods of expressing it.
- Convert between these three methods.
- Define average scale and point scale.
- List the two primary causes of variation in photo scale within a single photograph.
- List two general equations that can be used to calculate photo scale.
- Compute the average scale of a single photo or photo project, given the focal length and the flying height above the average elevation of the ground.

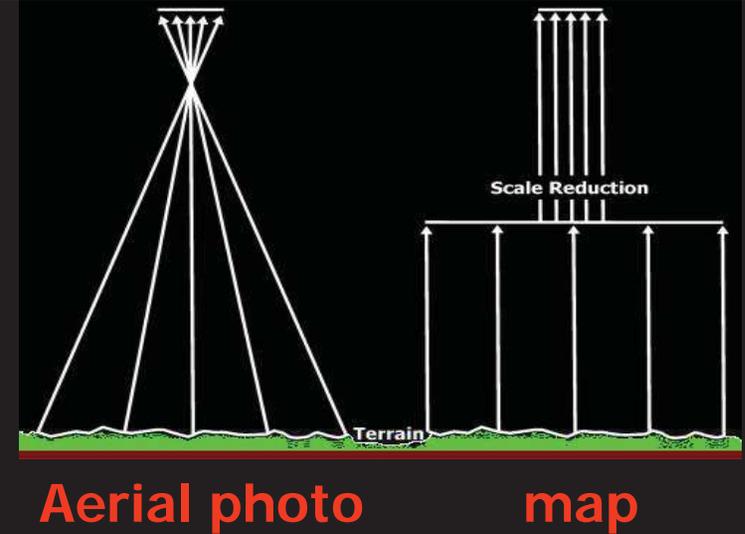
# OBJECTIVES

After a thorough understanding of this chapter and completion of the laboratory exercise, you will be able to:

- **Compute** the **average photo scale** between **two points**, given the photo distance **PD** and the corresponding ground or map distance **MD** (and map scale **MS**) between the same two points.
- **Compute** the **photo scale at a point**, given the **focal** length and the **flying** height above the point.
- **Compute** the **flying height** above mean sea level, given a **point** photo scale, the **focal** length of the camera lens, and the ground elevation above mean sea level at the point of known scale.
- **Compute** the **scale at a point**, given the focal length, the scale at another point, and the elevations of both points.

# Image/photo scale

Scale is the ratio of a distance on an aerial photograph to that same distance on the ground in the real world.



## Scale is expressed in 3 ways:

1. **Representative fraction (RF)**: RF is the **ratio** of distance on the photo to the same distance on the ground, expressed as a simple fraction. It is unitless. e.g. 1:15,000
2. **Photo scale reciprocal (PSR)**: is the **inverse** of RF and is also unitless. e.g. a RF of 1:15,000 is a PSR of 15,000.
3. **Equivalent scale (ES)**: e.g. 1 foot = 12 inch , 1 chain = 66 feet

## 4.1 SCALE CLASSIFICATION

- There are many different classifications of photo scale as to small, medium, and large, but there is **not general agreement**.

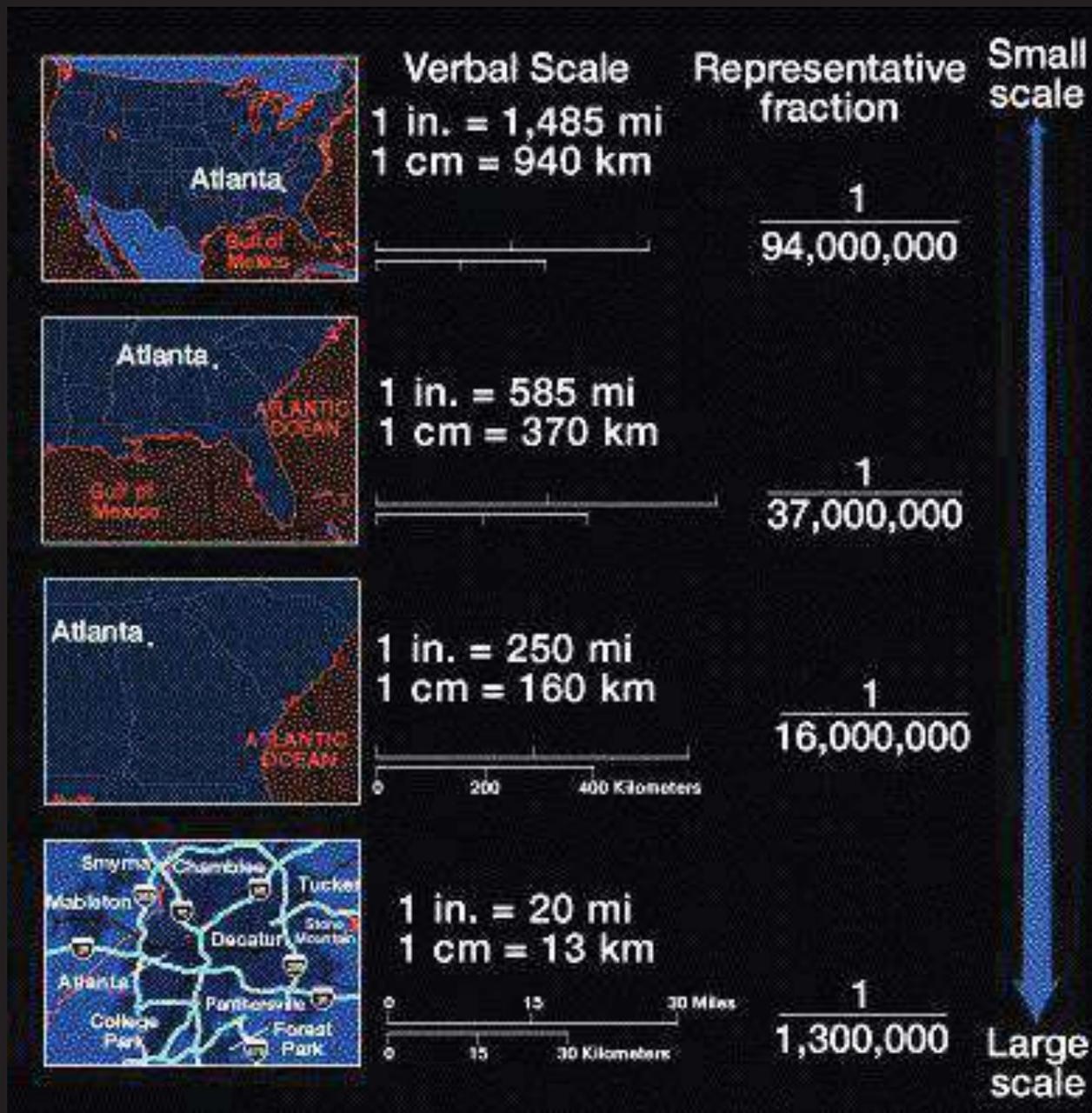
1. **Small** scale = 1:24,000 and smaller

2. **Medium** scale = 1:10,000 to 1:24,000

3. **Large** scale = 1:1,000 to 1:10,000

4. **Very large** scale = 1:1,000 and larger

# 4.2 The Theory of Scale



## 4.2 The Theory of Scale

- Both photographic scale and map scale are defined as a ratio of distances between corresponding points on the photo (or map) and on the ground.
- Scale can be expressed as a RF, PSR, or ES.
- Most maps use ES, whereas photo scale is usually expressed in terms of RF or PSR.

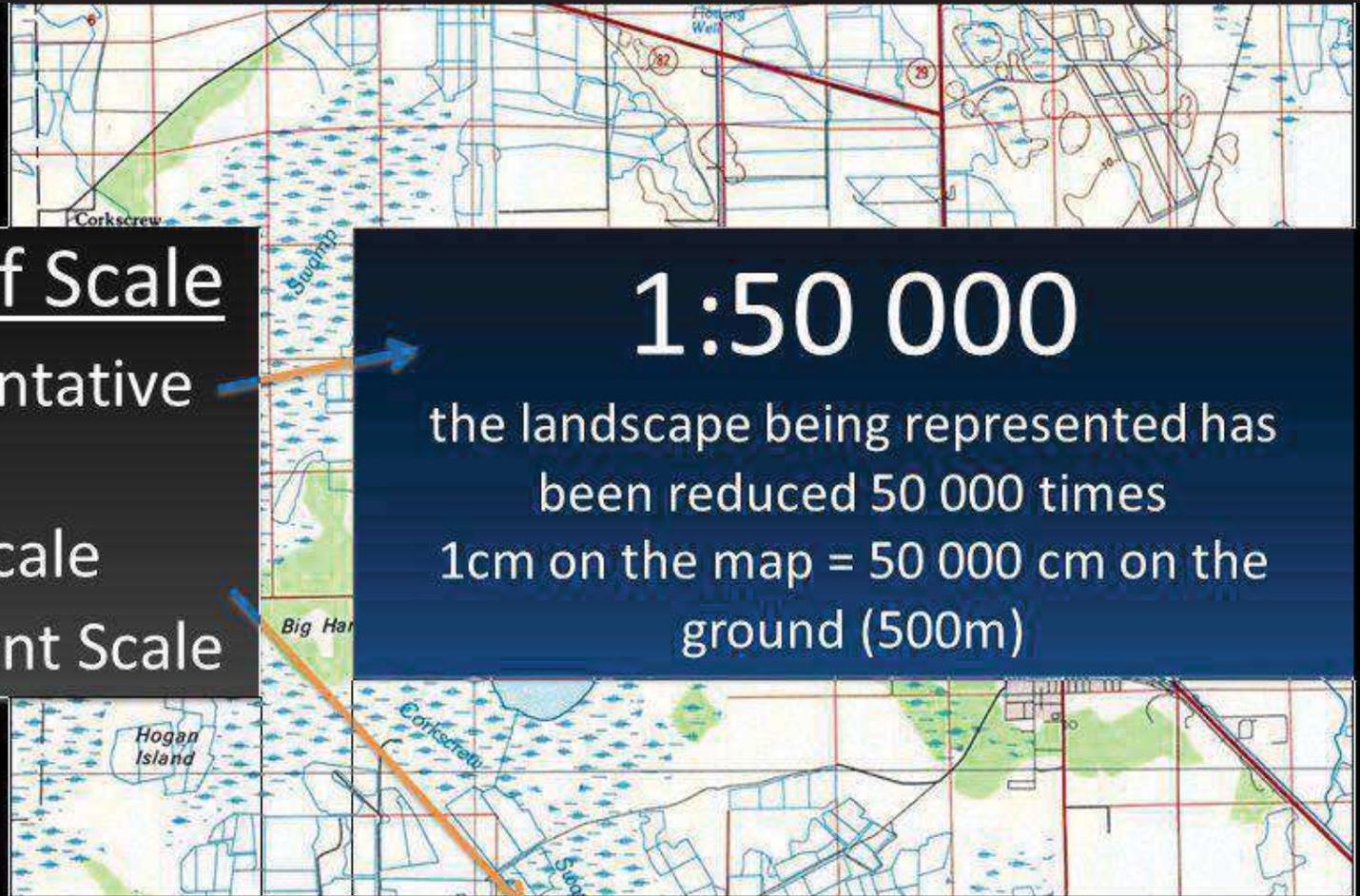
# 4.2 The Theory of Scale

## 3 Types of Scale

1. Representative Fraction
2. Linear Scale
3. Statement Scale

**1:50 000**

the landscape being represented has been reduced 50 000 times  
1cm on the map = 50 000 cm on the ground (500m)



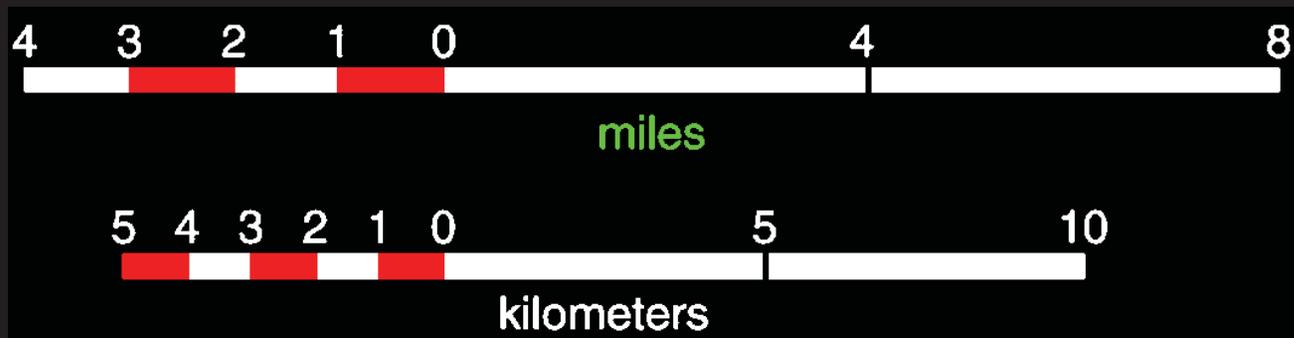
“One kilometre equals two centimetres on the map”

## 4.2 The Theory of Scale

### Representative Fraction

$$1:250,000 \text{ or } \frac{1}{250,000}$$

### Linear Scale



## 4.2 The Theory of Scale

### 4.2.1 Representative Fraction $1:250,000$ or $\frac{1}{250,000}$

- **Representative fraction** is the ratio of a distance on the photo to the same distance on the ground and can be expressed as a simple fraction ( $1/15,840$ ).
- Alternatively, for convenience, it may be printed as  $1:15,840$ . This means that one unit on the photo is equivalent to 15,840 of the same units on the ground.
- Because the units on the photo and on the ground are the same, they cancel out and the **ratio is unitless**.

## 4.2 The Theory of Scale

### 4.2.2 Photo Scale Reciprocal (PSR)

- is the inverse of the **representative fraction**.
- Thus, an RF of 1:15,840 is a PSR of 15,840, meaning that the ground distance is 15,840 times the photo distance.
- An important feature of PSR is that a *smaller numerical value of PSR represents a larger scale* (PSR of 10,000 > PSR 20,000) because PSR is the reciprocal of RF.
- **Think of it this way**: PSR is a number we must multiply the photo distance by to get the actual distance.
- Therefore, **if the PSR is small**, it is closer to the actual object size than if the PSR is large.

## 4.2 The Theory of Scale

### 4.2.3 Equivalent Scale

- A PSR of 15,840, for **example**, is the same as an **equivalent scale** of **4 inches = 1 mile**.
- To convert from an equivalence to an RF or PSR, we have to change the units of measurement so that they are the same.
- For **example**, we change **4 inches = 1 mile** to a representative fraction by setting it up as a ratio and multiplying by unity so that all the units cancel and we get an RF of **1:15,840**, or a PSR of 15,840.

## 4.2 The Theory of Scale

### 4.2.3 Equivalent Scale

$$\text{Equivalence: } 4 \text{ in.} = 1 \text{ mile, or } \left( \frac{4 \text{ in.}}{1 \text{ mile}} \right) = 1$$

$$\text{Multiply by unity: } \left( \frac{4 \text{ in.}}{1 \text{ mile}} \right) \left( \frac{1 \text{ mile}}{5280 \text{ ft}} \right) \left( \frac{1 \text{ foot}}{12 \text{ in}} \right) = \left( \frac{1}{15840} \right) = \text{RF}$$

$$\text{or: } = 15840 = \text{PSR}$$

Similarly, how we can convert RF and PSR back to an equivalent scale by the same process of multiplying by unity so that all units cancel: ??

For this, please return to the text book page 70

## 4.2 The Theory of Scale

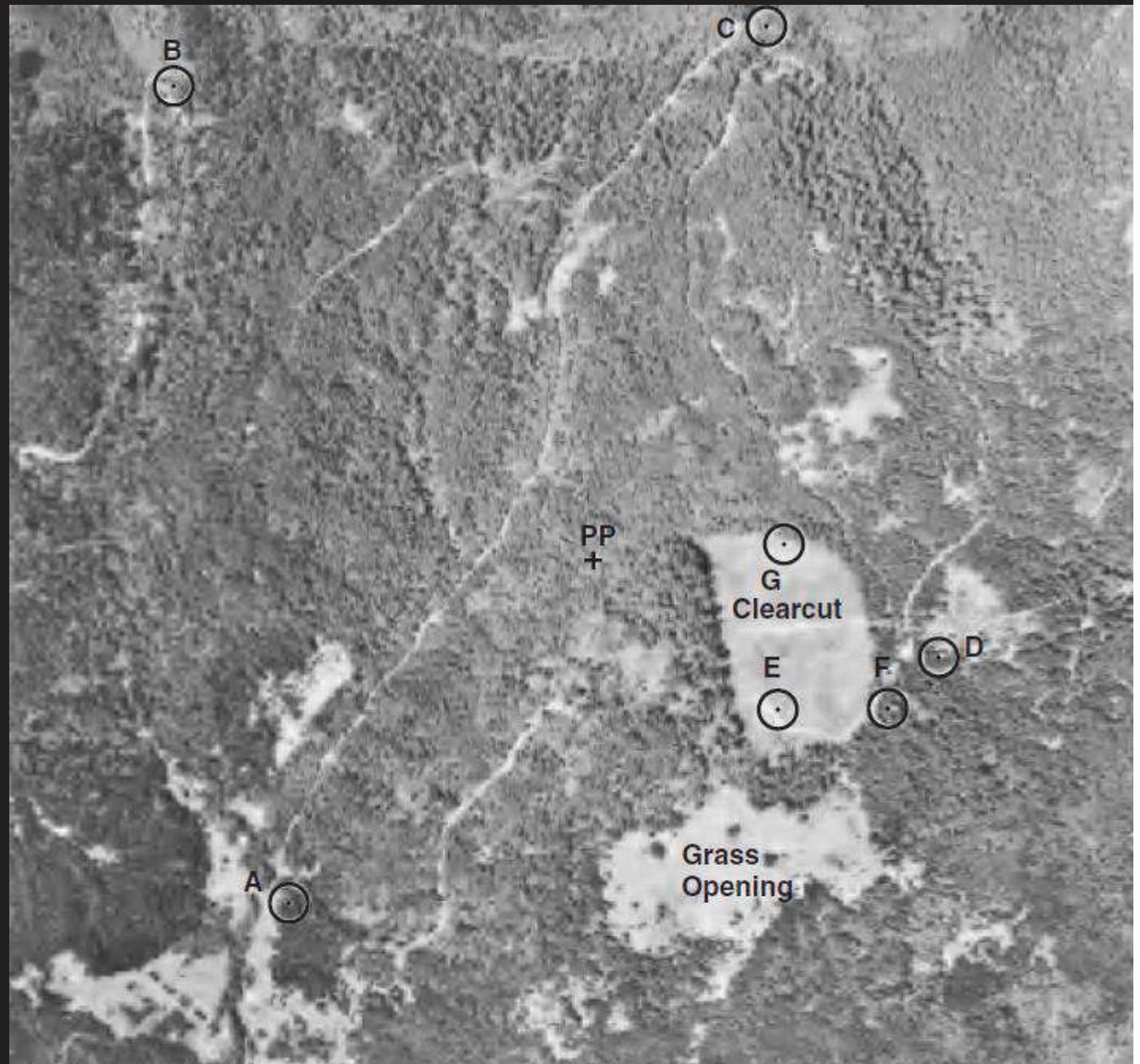
- One unit of measure that is important in natural resource measurements is the **chain**.
- Foresters and surveyors still use the chain measurement, and a number of examples in this text refer to the chain.
  - A chain = 66 feet in length.
  - 1 Foot = 12 in.
  - 1 in. = 2.45 cm

## 4.2 The Theory of Scale

- Photo scale is important because it relates size or distance on the photo to actual size or distance on the ground.
- **For example**, consider the **clearcut** on an aerial photograph shown in **Figure 4.1**.
- Let's assume that we want to **replant it with tree seedlings** spaced **12 feet** apart, with **12 feet** between rows.
- How many seedlings should we order? **To determine this**, we need to **know the size** of the area on the ground.
- The area on the original photo had dimensions of about **(1.0×1.5)** in. (original photo scale), but that doesn't tell us how large the area is on the ground.
- **We need some way** to relate measurements on the photo to actual sizes on the ground.
- **If we know** the scale of the photo, then **we can translate distances and areas** on the photo into corresponding distances and areas on the ground.

**Figure 4.1.**  
There is no way of  
determining the  
area of this clearcut  
without knowing  
the  
photo scale.

This annotated  
photo is to be used  
with the laboratory  
exercises at the  
end  
of Chapters 4 and  
5.



## 4.3 Types of Scale

There are two general types of  
photo scales:

*Average scale*

and

*Point scale.*

## 4.3 Types of Scale 4.3.1 Average Scale

- Average scale can refer to the entire project (**many** photos), a **single** photo, or a **portion** of a photo or may be between **two points**.
- It is the **desired** average scale and is the goal of the aerial photo mission.
- Due to several factors, the **project scale is rarely exactly** the same as the actual scale of the individual aerial photo.

## 4.3 Types of Scale

### 4.3.1 Average Scale

- Primary factors contributing to this difference are tilt, changes in flying height between exposure stations, and differences in elevation of the terrain.
- Except in flat terrain, this average photo scale only approximates the actual scale at all points on the photo.

## 4.3 Types of Scale 4.3.2 Point Scale

- The second and **most exact** type of scale is the point scale.
- It is the photo scale at a point on the ground at a **given elevation**.
- **Every point on the photo at a different elevation has a different point scale.**
- In a vertical photograph, the range in these point scales depends on the **focal length** of the camera lens and the **amount of variation in elevation** of the terrain shown on a given photo.

## 4.4 Variation in Scale

- It is evident from the preceding discussion of types of scales that the *photo scale is not constant over the entire photo project or even within a single photo.*
- The **two primary** causes of these differences in scale over a single photo are **tilt** and **differences in flying** height above the ground caused by differences in **ground elevation**, or topography.
- An additional source of scale variation *among* photos within the same project is **slight altitudinal** changes in flying height of the aircraft between *exposure stations.*

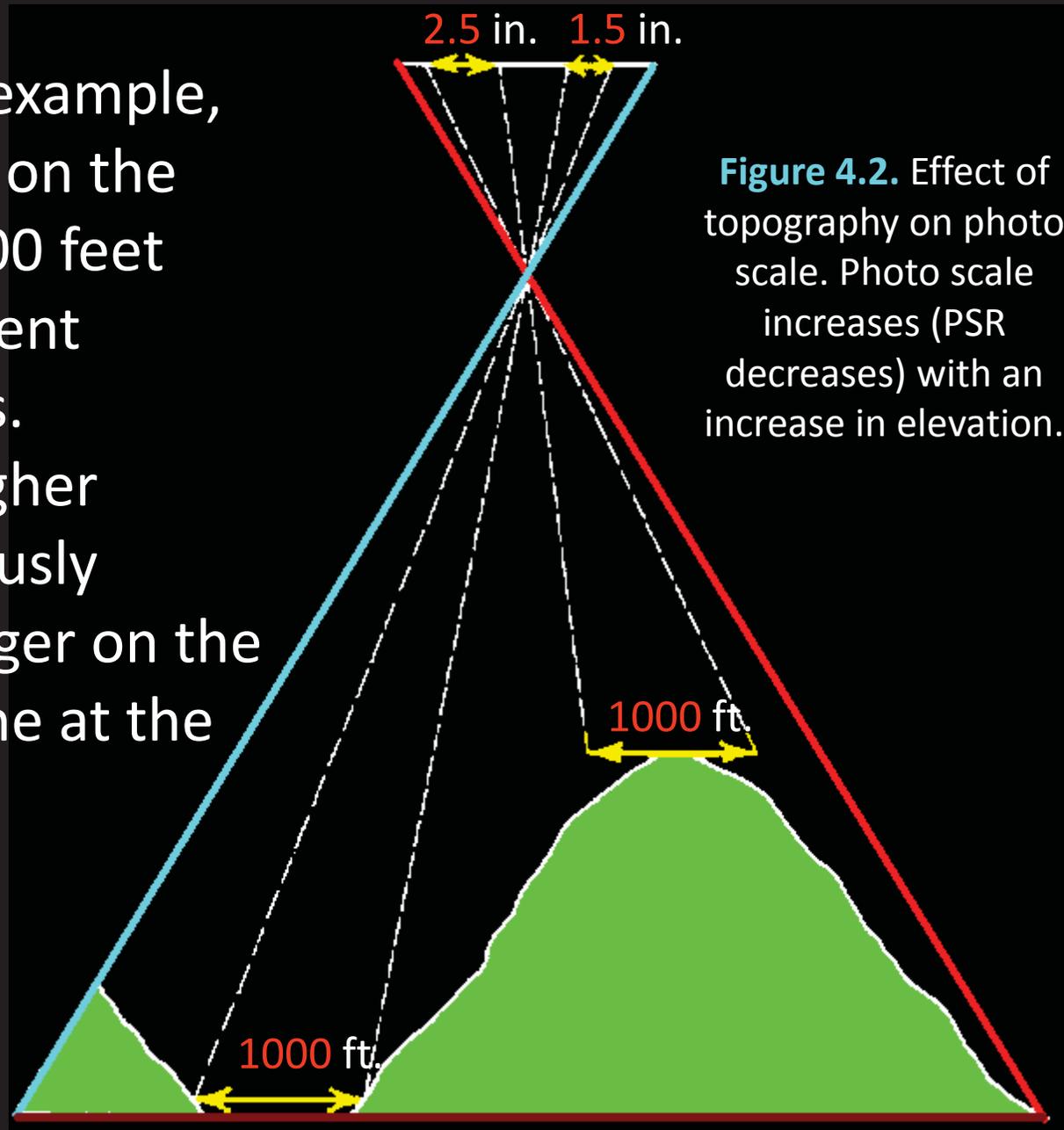
## 4.4 Variation in Scale

- In addition to defining **RF** as **photo distance** divided by **ground distance**, (PD/GD) we can define it as **focal length** divided by **the flying height** above the terrain (f/H).
- **Focal length** remains **constant** on an aerial photo mission, but flying height above the terrain **varies** as the ground elevation changes.
- As a result, **higher** ground elevations have a **larger** scale than lower elevations.
- This means that an object would **appear larger** on an aerial photo if it is at an elevation of 2,000 feet than if the same object were at an elevation of 1,000 feet.

## 4.4 Variation in Scale

## Effect of topography

- In Figure 4.2, for example, you see two lines on the ground, each 1,000 feet long, but at different ground elevations.
- The line at the higher elevation is obviously imaged much longer on the photo than the line at the lower elevation.



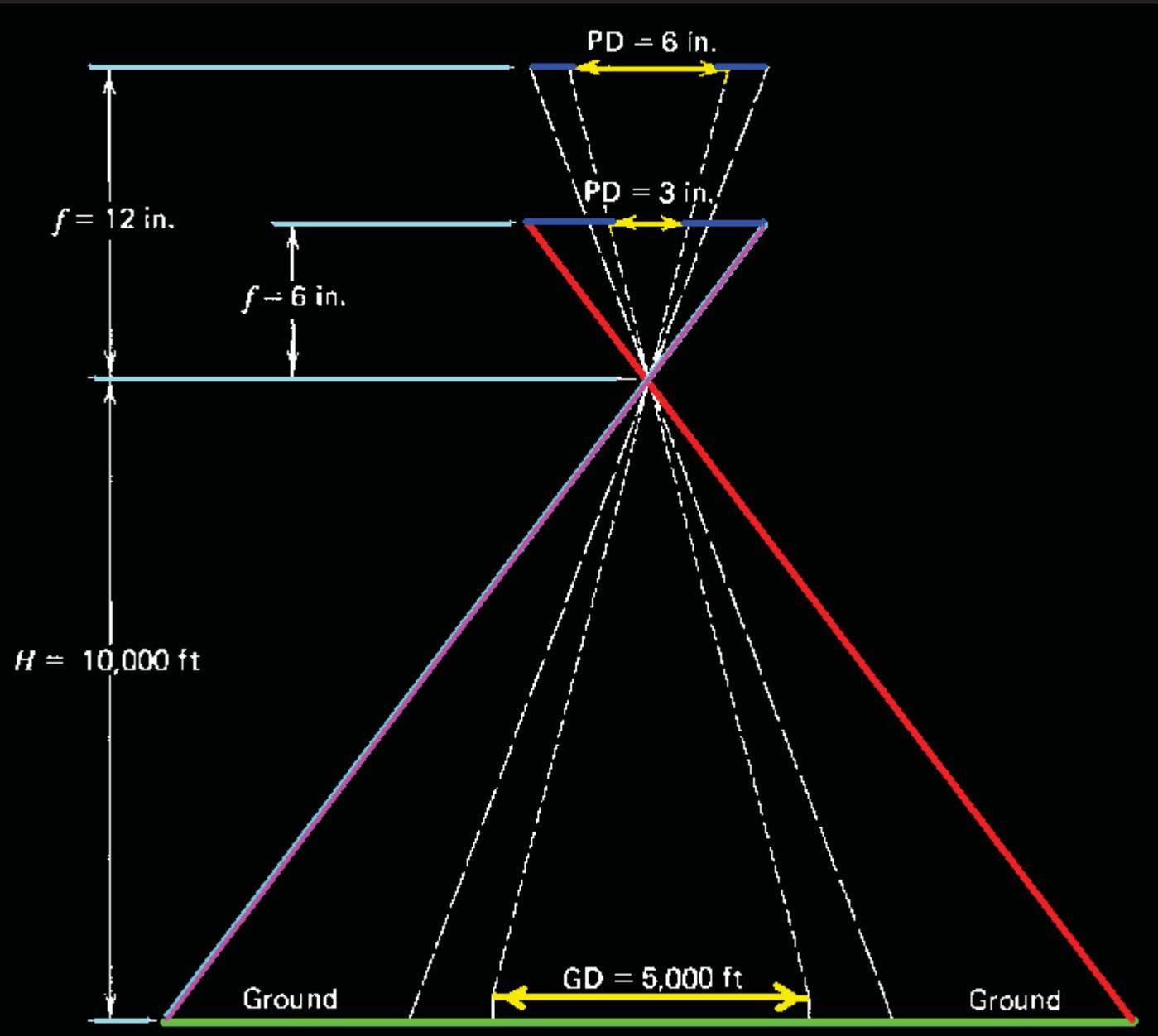
**Figure 4.2.** Effect of topography on photo scale. Photo scale increases (PSR decreases) with an increase in elevation.

## 4.4 Variation in Scale Effect of camera lens focal length

- In addition to the ground elevations and altitude of the plane, the **focal length** of the camera affects the photo scale.
- By using a camera with a longer focal length, the scale of the photos is increased.
- **Figure 4.3** illustrates that by **doubling** the focal length from 6 to 12 in. the RF is doubled (**scale increased**) from 1:20,000 to 1:10,000. When using a focal length of 6 in., the line is 3 in. on the photo, but when we use a focal length of 12 in., the length of the line doubles in length.

## 4.4 Variation in Scale Effect of camera lens focal length

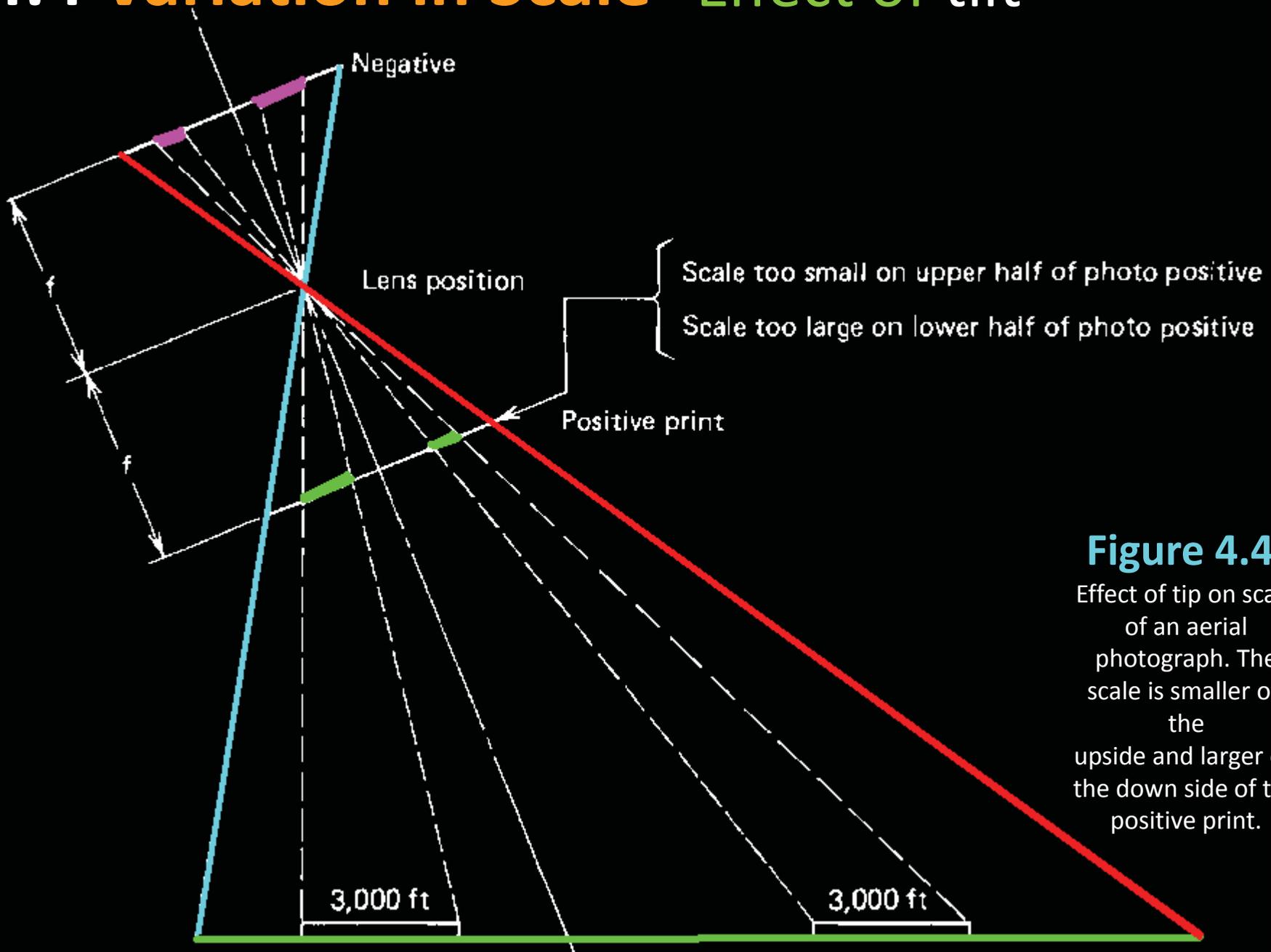
**Figure 4.3.** Effect of camera lens focal length on photo scale. Increasing the focal length from 6 in. to 12 in. doubles the scale, for example, from an RF of 1:20,000 to an RF of 1:10,000.



## 4.4 Variation in Scale Effect of tilt

- Tilt also causes scale variation within a single photograph (Figure 4.4).
- The scale of a tilted photo changes in a regular manner throughout the photo.
- If the scale near the center of the photo is approximately correct, then the scale is smaller on the side of the photo positive that is tilted upward, and larger on the side that is tilted downward.
- The scale changes across the photograph in the direction of the tilt.
- You can see in Figure 4.4 that the length of the 3,000-ft ground line on the photo varies depending on whether it is on the upper or lower half of the photo.

## 4.4 Variation in Scale Effect of tilt



**Figure 4.4.**  
Effect of tip on scale of an aerial photograph. The scale is smaller on the upside and larger on the down side of the positive print.

## 4.5 Basic Scale Equations

- From the geometry of the central projection of an aerial photograph (Figure 4.5)
- we can see that by using similar triangles, photo scale can be **computed** in one of two ways:
  1. **First** by using focal length ( $f$ ) and flying height above the ground ( $H$  or  $A - E$ ), or
  2. **Second** , by using the photo distance ( $PD$ ) and the ground distance ( $GD$ ) between the same two points:

$$RF = \left( \frac{f}{H} \right) = \left( \frac{f}{A - E} \right) \left( \frac{PD}{GD} \right)$$

$$PSR = \left( \frac{H}{f} \right) = \left( \frac{A - E}{f} \right) \left( \frac{GD}{PD} \right)$$

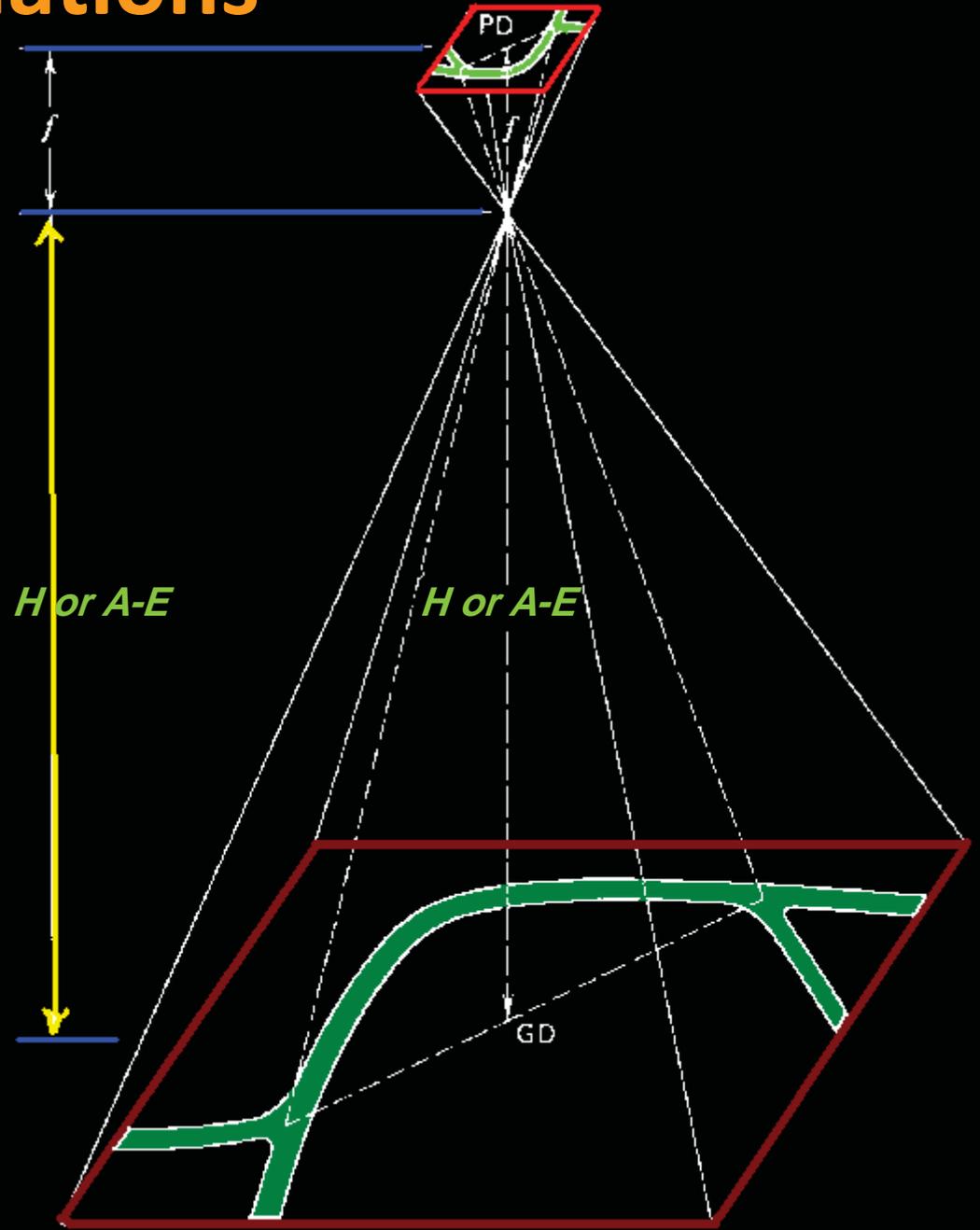
# 4.5 Basic Scale Equations

Where:

- $GD$  = Ground distance
- $PD$  = Photo distance
- $H = A - E$  = Flying height of the aircraft above the ground in feet or meters
- $A$  = Altitude of aircraft above sea level
- $E$  = Ground elevation above sea level
- $f$  = Focal length of the camera lens in the *same* units of measure as for  $H$  (usually ft or m)

**Figure 4.5**

The two basic scale equations can be derived using similar triangles shown in dashed lines.



## 4.6 Photo Scale Determination

- Using the camera **focal length** and **flying height** equation is by far the least expensive.
- All that is required is a knowledge of focal length, flying height above sea level, and ground elevation (no ground visitation is required).
- **Ground elevation** can be **determined** using a topographic map.

# *Aerial Photography & Photogrammetry*

## Lecture 5-2

# Scale of a Vertical Aerial Photograph

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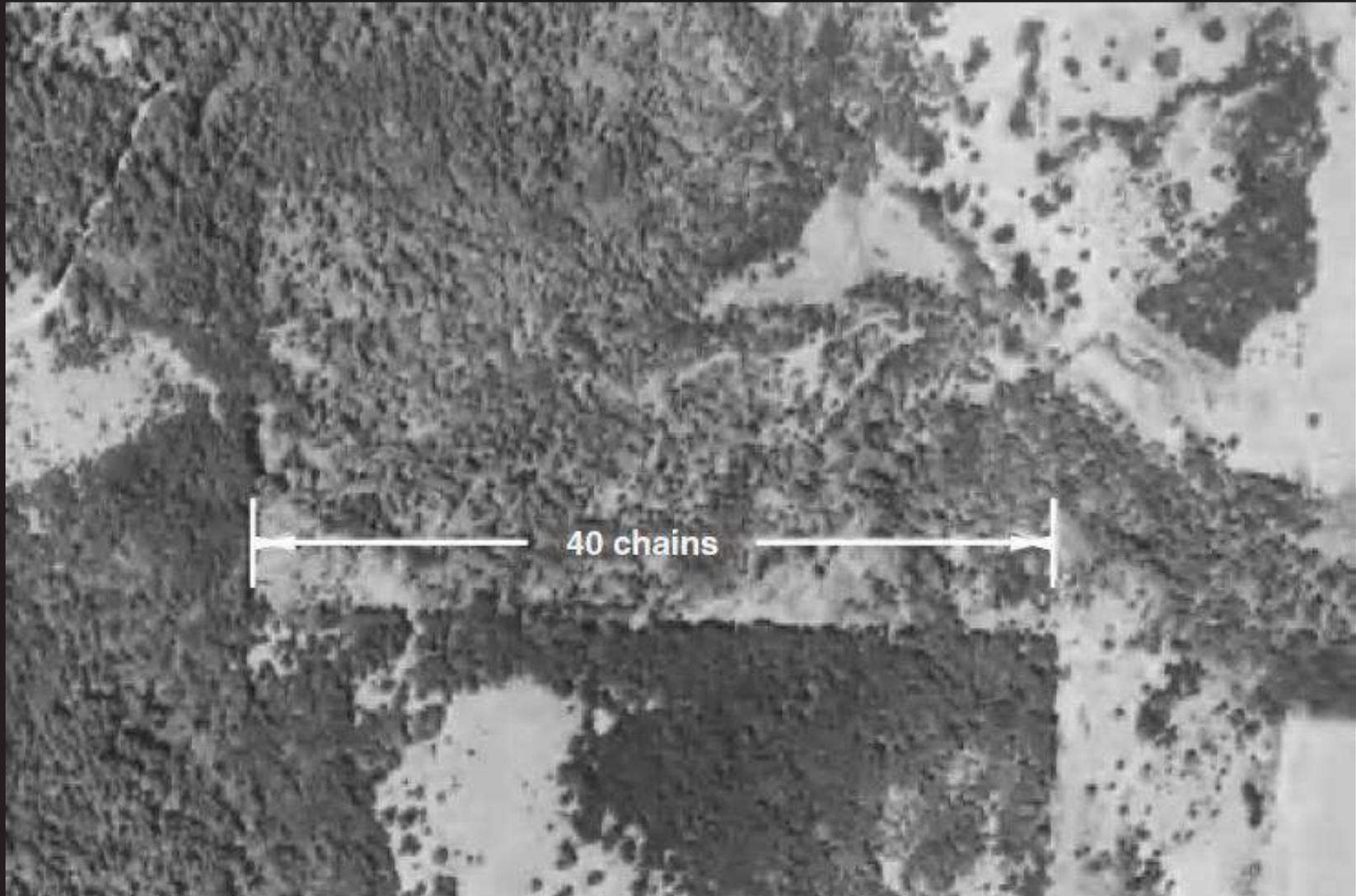
## 4.6 Photo Scale Determination

- Actually, the **photo mission planner** uses an **estimate** of the **average elevation** over the entire project in the original mission planning.
- Using the **other equation** usually (but not always) requires ground **visitations**.
- **See Appendix 4-2**

## 4.6 Photo Scale Determination

- Ground distance, required by this elevation, can be obtained **four different ways**...Each of these ways requires that you positively identify both ends of distances on the photo and map (property boundaries, road junctions, city blocks, etc.):
  1. Measure the **ground distance** directly on the ground.
  2. Measure the **map distance** and compute the ground distance using the map scale.
  3. We **might know** the **ground distance between two points**, in which case **only** the photo measurement would be necessary.
  4. Use the global positioning system (**GPS**) to obtain grid coordinates and then calculate the ground distance.

## 4.6 Photo Scale Determination



**Figure 4.6.** Sometimes photo scale can be calculated if the GD is known, such as the old cutting boundary shown here.

## 4.6 Photo Scale Determination

### Example 1 (Using focal length and flying height)

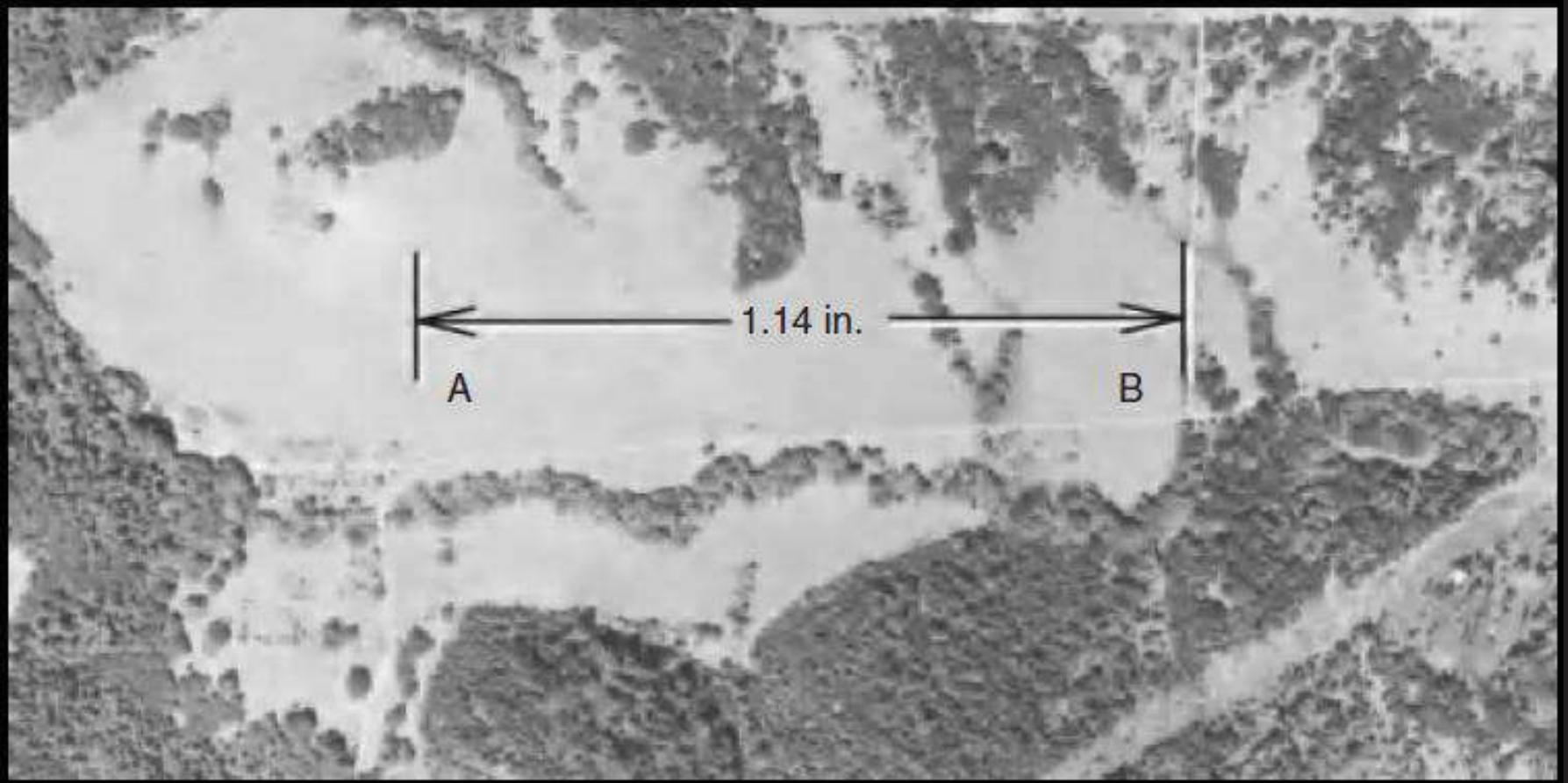
- Suppose the desired **average project** PSR is **14,000** (Figure 4.7).
- This is the nominal scale that was the goal of the photo mission and will vary from photo to photo.
- Let us suppose that the attempted flying height above mean sea level, as estimated by the mission planner, is **8,000** feet and the **focal length** of the camera lens is **6** in.
- From a topographic map of the same area we **estimate the average ground** elevation covered by a specific photo to be **800** feet.
- Therefore, **the average photo PSR** is:

$$PSR_{800} = \frac{A - E}{f} = \frac{8,000 \text{ ft} - 800 \text{ ft}}{0.5 \text{ ft}}$$

or

$$PSR_{800} = \frac{H}{f} = \frac{7,200 \text{ ft}}{0.5 \text{ ft}} = 14,400$$

## 4.6 Photo Scale Determination



**Figure 4.7.** Aerial photo for examples 1, 2, and 3.

## 4.6 Photo Scale Determination

### Example 2

Using *GD* & *PD* for this and all following examples.

- Now let's **determine** the **average scale** between two road junctions (points A and B) on the same photo (Figure 4.7).
- The distance on the original photo, as measured with an engineer's scale, is **1.12** in. and the distance as measured on the ground is **1,393** feet.
- Furthermore, let us assume that the elevation of point **A** was **490** feet and of point **B** was **584** feet. We find **the average PSR** to be:

$$PSR_{537} = \frac{GD}{PD} = \frac{1,393 \text{ ft}}{1.12 \text{ in.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 14,930$$

Notice the use of subscripts, here and elsewhere, for PSR. The 537 in this case indicates that the **PSR is valid for all points** on the photo at an elevation of 537 feet above mean sea level (MSL), which is the average of 490 and 584 feet.

## 4.6 Photo Scale Determination

### Example 2

Using *GD* & *PD* for this and all following examples.

- Also notice that we rounded PSR to the nearest 10.
- This is the usual procedure because measurement errors on the photo, map, or ground usually create even larger errors in PSR determination.
- Because foresters and other natural resources managers frequently measure ground distances in **chains** (66 feet per chain), let's work the same problem using chains.
- Our ground distance would be 21.11 ch. and our PSR calculation would be:

$$PSR_{537} = \frac{GD}{PD} = \frac{21.11 \text{ ch.}}{1.12 \text{ in.}} \times \frac{66 \text{ ft}}{1 \text{ ch.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 14,930$$

## 4.6 Photo Scale Determination

**Example 2**  $PSR_{537} = \frac{GD}{PD} = \frac{21.11 \text{ ch.}}{1.12 \text{ in.}} \times \frac{66 \text{ ft}}{1 \text{ ch.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 14,930$

- This is **not the scale** for **either** point A or B.
- It is the scale for the **average** elevation between the two points (**537** feet).
- This is not the same scale as we obtained for **Example 1**.
- The average scale for **Example 1** was for the entire photo with an estimated average elevation of **800** feet.

## 4.6 Photo Scale Determination

**Example 2**  $PSR_{537} = \frac{GD}{PD} = \frac{21.11 \text{ ch.}}{1.12 \text{ in.}} \times \frac{66 \text{ ft}}{1 \text{ ch.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 14,930$

- Note also that the scale at point A =  $PSR_{490}$  and the scale at point B =  $PSR_{584}$
- If we solve for the two PSR's using the formula in example 1, we get:

$$PSR_{490} = \frac{8,000 \text{ ft} - 490 \text{ ft}}{0.5 \text{ ft}} = 15,020$$

$$PSR_{584} = \frac{8,000 \text{ ft} - 584 \text{ ft}}{0.5 \text{ ft}} = 14,830$$

The average of these two point scales is

**14,930**

, which is what we calculate for  $PSR_{537}$ .

Throughout this book, we stress the use of units. If your **units do not cancel** in RF or PSR calculations, you have set up the solution **wrong**. In all examples, all the units must cancel, leaving the final answer (PSR) unitless.

## 4.6 Photo Scale Determination

### Example 3 average scale

- Instead of measuring the **distance between these same two road junctions** on the ground, suppose that we **have a map** of the same area on which the road junctions can be located.
- Our map has an equivalent scale of **1 in. = 400 ft** and we measure the map distance from point **A** to point **B** as **3.48 in.**
- **What is the average scale** between these two points? **What was the flying** height above sea level?

## 4.6 Photo Scale Determination

### Example 3 Solution

- First, convert map equivalent scale to **map scale reciprocal (MSR)**:

$$\frac{400 \text{ ft}}{1 \text{ in.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 4,800 \text{ MSR}$$

- It should be noted that  $MSR = GD/MD \rightarrow GD = (MD)(MSR)$ .
- Therefore:

$$GD = (3.48 \text{ in.}) \times 4800 \left( \frac{1 \text{ ft}}{12 \text{ in.}} \right) = 1,392 \text{ ft}$$

## 4.6 Photo Scale Determination

### Example 3 Solution

Our  $PSR_{537}$  is then:

$$PSR_{537} = \frac{GD}{PD} = \frac{1,392 \text{ ft}}{1.12 \text{ in.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 14,920$$

This is a slightly different scale than we had in Example 2, but this difference can be attributed to slight errors of measurement or in locating points A and B on the map. For those who would like to work this example in one equation, we have:

$$PSR_{537} = \frac{(MD) \times (MSR)}{PD} = \frac{3.48 \text{ in.} \times 4,800}{1.12 \text{ in.}} = 14,920$$

Now we can calculate the flying height above sea level (A) because we know the scale at an elevation of 537 feet and we know that the focal length is 6 inches.

## 4.6 Photo Scale Determination

### Example 3 Solution

- If we solve one of our scale formulas,

$$PSR = (A - E)/f, \quad \text{for } A, \text{ we get:}$$

$$A = f(PSR) + E = 0.5 \text{ ft}(14,920) + 537 \text{ ft} = 7997 \text{ ft}$$

- Notice that this is slightly different than the planned flying height of 8,000 feet.
- Actually, our answers in Examples 2 and 3 were point scales since they represent the scale on the photograph where the elevation is 537 feet, but it is more convenient to think of them as average scales.

## 4.6 Photo Scale Determination

### Example 3 Solution

- Now that we know the flying height above sea level for this particular photo, we can calculate the photo scale at any point on the photo as long as we know the elevation of that point and the focal length.
- **For example**, let's calculate the **point scale** at **another point** on the same photo.

## 4.6 Photo Scale Determination

### Example 4

From our **contour map** we find the **elevation** of the **new point** to be **475** feet. Thus our PSR at this point becomes:

$$PSR_{475} = \frac{7,997 \text{ ft} - 475 \text{ ft}}{0.5 \text{ ft}} = 15,040$$

For comparison, let's find the point scale at another point on the photo that has an elevation of **1,250** ft.

$$PSR_{1250} = \frac{7,997 \text{ ft} - 1,250 \text{ ft}}{0.5 \text{ ft}} = 13,490$$

- From this example, we can see **how much a change** in ground elevation **GE** affects the photo scale.
- The **difference** in PSR between the last two calculations is **1,550** and the difference in elevation is exactly half that amount, or **775** feet.

## 4.6 Photo Scale Determination

### Example 4

This suggests that we could **derive** a single equation that would allow us to **calculate the PSR at any elevation** on the photo **if we know** the **focal length**, the **PSR** and elevation at **any** one point, and the **elevation** of the point of **unknown PSR**. Here it is:

$$PSR_A = \frac{A - E_A}{f} \quad A = f(PSR_A) + E_A$$
$$PSR_B = \frac{A - E_B}{f} \quad PSR_B = \frac{f(PSR_A) + E_A - E_B}{f}$$
$$PSR_B = PSR_A + \frac{E_A - E_B}{f}$$

Now let's **use this equation** to see if we get the same answer as **before** for the point with an elevation of **475** feet, using the known PSR and elevation from **Examples 3 and 4**.

## 4.6 Photo Scale Determination

### Example 5

$$\begin{aligned}PSR_{475} &= PSR_{537} + \frac{E_{537} - E_{475}}{f} = 14,930 + \frac{537 - 475}{0.5} \\ &= 15,040\end{aligned}$$

or

$$\begin{aligned}PSR_{475} &= PSR_{1250} + \frac{E_{1250} - E_{475}}{f} = 13,490 + \frac{1,250 - 475}{0.5} \\ &= 15,040\end{aligned}$$

- which is **exactly** what we obtained in Example 4.
- **Don't get confused** if the PSR we want to calculate is at a higher elevation than the known PSR.

## 4.6 Photo Scale Determination

**Example 6** Calculate average photo scale at a point

As in our previous example, let's calculate the PSR at 1,250 feet, knowing the PSR at 475 feet:

$$\begin{aligned}PSR_{1250} &= PSR_{475} + \frac{E_{475} - E_{1250}}{f} \\ &= 15,040 + \frac{475 - 1,250}{0.5} = 13,490\end{aligned}$$

which is the same as we obtained before for  $PSR_{1250}$ .

## 4.6 Photo Scale Determination

### 4.6.1 Assumptions

Please for more details refer to the text book,  
page 81

# Laboratory Exercises 2

Please refer to the text book, page 84

For this laboratory exercise you will need to use the annotated aerial photo in Figure 4.1 and the map of the same area in Figure 4.9. Your instructor can alter the exercise by changing the assumptions and/or the data for each problem.

## *Assumptions*

Average project elevation = 900 feet

Focal length of camera lens used = 3 in.

Planned flying height above MSL (A) = 6,900 feet

# *Homework's*

( all )

Questions and Problem

In text book (Page 84)

are required



*Thank you*

Any Questions ?



**END**

**of Lecture**

## 4.6 Photo Scale Determination

Photoscale may also be determined according to:

1. Smallest detail and resolution
2. C-factor and desired minimum contour interval
3. Expected accuracy
4. Enlargement from photo to map in the instrument

**See Appendix 4-1**

## 4.6 Photo Scale Determination

### Example 7 Smallest detail and resolution

- The smallest detail that needs to be seen on the photograph is **1** foot in length.
- If the resolution of the photo is **0.1** mm, **determine** the photoscale.

## 4.6 Photo Scale Determination

### Example 7 Smallest detail and resolution

$$\text{PhotoScale} = \frac{\text{resolution}}{\text{Smallest Details}}$$

$$\text{PhotoScale} = \frac{0.1\text{mm}}{1\text{ft}} = \frac{0.1\text{mm}}{(1\text{ft} \times 12(\text{in} / \text{ft}) \times 254.\text{mm})}$$

$$\text{PhotoScale} = \frac{0.1\text{mm}}{304.8\text{mm}} \cong \frac{1}{3000}$$

# Appendix 4-1

## *PhotoScale* Determining Photoscale

### 2- C-factor and desired minimum contour interval

- Contour interval – difference in elevation between consecutive contour lines

$$C - factor = \frac{\text{flying height}}{\text{contour interval}} = \frac{H}{\Delta h}$$

- C-factor range from 1200 to 1500

## Appendix 4-1

### *PhotoScale* Determining Photoscale

2- C-factor and desired minimum contour interval

### **Example:**

The C-factor of the instrument is given to be 1500. If the desired contour interval is 1 meter, determine the photoscale.

## Appendix 4-1

### *PhotoScale* Determining Photoscale

2- C-factor and desired minimum contour interval

**Solution:**

$$photoscale = \frac{1}{9000}$$

# Appendix 4-1 Accuracy

## PhotoScale

## Determining Photoscale

Mean square error of horizontal position of points:

$$m_h = 0.1\% \circ H = 0.0001H = (10^{-4})H$$

%o – per mil; equivalent to 1/1000

For Cadastral Survey:

$m_h = 10$  cm (urban)

= 30 cm (rural)

# Appendix 4-1

## *PhotoScale* Determining Photoscale

### 3- Expected Accuracy

Example:

Determine the photoscale for an urban area if the camera to be used is a wide-angle camera.

## Appendix 4-1

# *PhotoScale* Determining Photoscale

## 3- Expected Accuracy

Solution:

$$H = \frac{10 \text{ cm}}{0.1\text{‰}} = \frac{10 \text{ cm}}{0.0001} = 100,000 \text{ cm}$$

$$\frac{1}{s_r} = \frac{f}{H} = \frac{6 \text{ in}}{\left(100000 \text{ cm} \times \frac{1 \text{ in}}{2.54 \text{ cm}}\right)} = \frac{1}{6561.67979}$$

$$\text{photoscale} = \frac{1}{6000}$$

# Appendix 4-1

## *PhotoScale* Determining Photoscale

### 4-Enlargement from photo to map

- Using the stereoplotter, there will be an enlargement from the photo to the stereomodel:

$$\text{enlargement} = \frac{Z}{C}$$

where:

Z = projection distance for stereoplotter

C = f = projection distance of camera

# Appendix 4-1

## *PhotoScale* Determining Photoscale 4-Enlargement from photo to map

### Example:

A map with scale 1:5000 was derived from a stereomodel with a scale of 1:8000, using a stereoplotter.

The projection distance of the stereoplotter is twice the focal length of the camera. Determine the scale of the photograph that was used to generate the stereomodel.

## Appendix 4-1

# *PhotoScale* Determining Photoscale

## 4-Enlargement from photo to map

Solution:

$$\text{enlargement} = \frac{Z}{f} = \frac{2f}{f} = 2$$

$$\text{photoscale} = \frac{\text{stereomodel scale}}{2} = \frac{1}{2} \times \frac{1}{8000}$$

$$\text{photoscale} = \frac{1}{16000}$$

# Image/photo scale Appendix 4-2

Importance: Image scale is important because it relates the size or distance on the image to the actual size or distance on the ground.

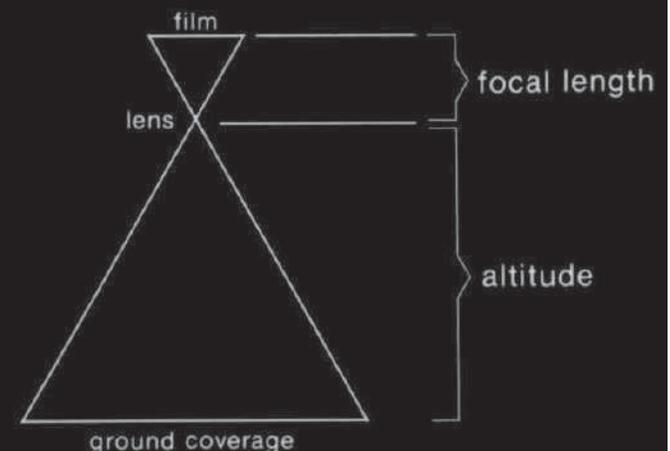
How can u measure scale in an AP?

**1. Arithmetic Proportion:** comparing distance between two points on the airphoto (*Photo Distance*) with the same points on a topographic map.

$$\text{Photo scale} = \frac{\text{Photo Distance}}{\text{Map Distance}} \times \text{Map scale}$$

**2. By camera altitude relationship:**

$$\begin{aligned} \text{Photo scale} &= \text{focal length} / \text{aircraft altitude} \\ &= (f/H) \end{aligned}$$



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

# *Aerial Photography & Photogrammetry*

## Lecture 5-2

# Scale of a Vertical Aerial Photograph

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[faiselgm73@gmail.com](mailto:faiselgm73@gmail.com)

**2017-2022**

*All rights reserved*

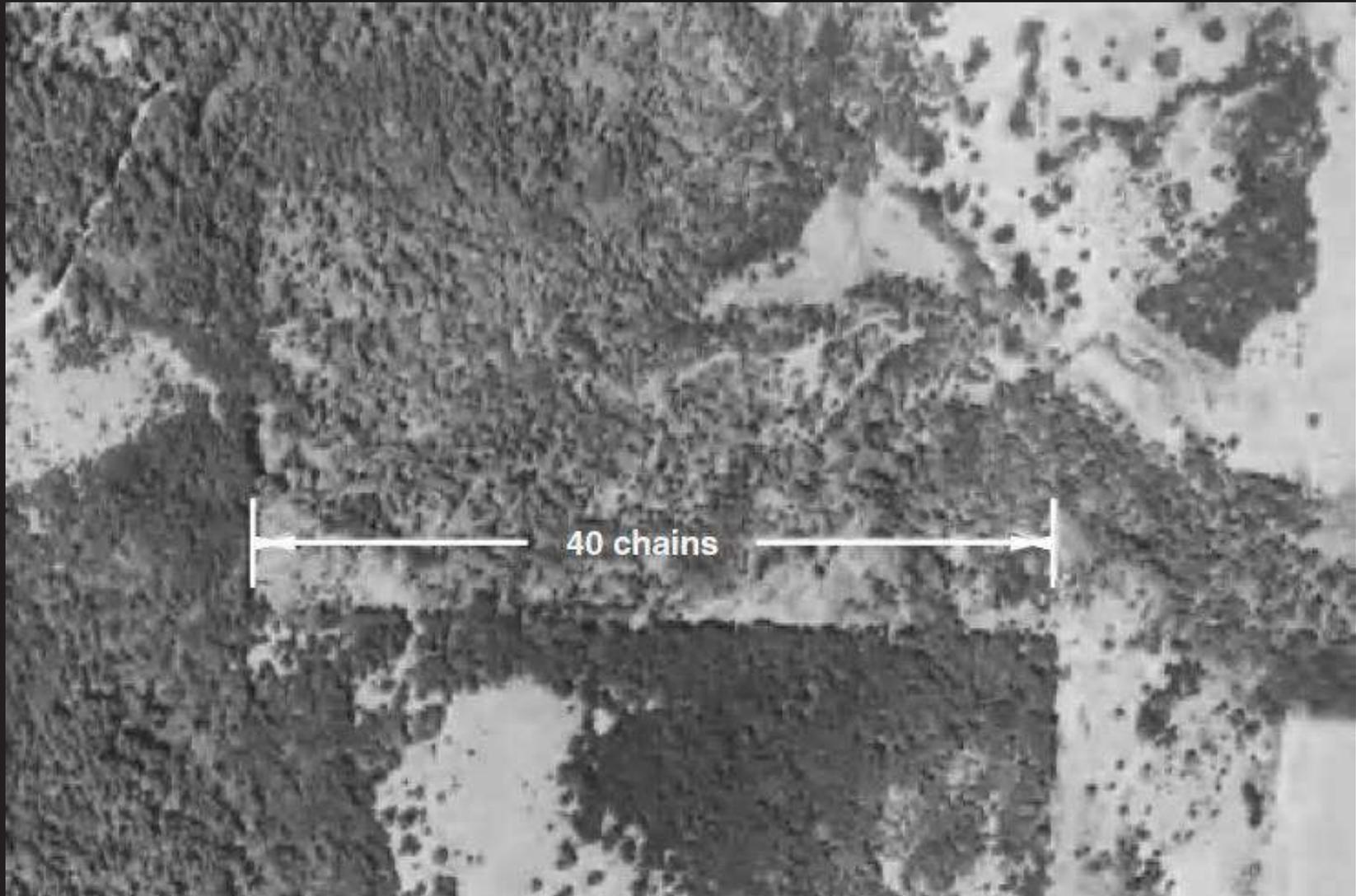
## 4.6 Photo Scale Determination

- Actually, the **photo mission planner** uses an **estimate** of the **average elevation** over the entire project in the original mission planning.
- Using the **other equation** usually (but not always) requires ground **visitations**.
- **See Appendix 4-2**

## 4.6 Photo Scale Determination

- Ground distance, required by this elevation, can be obtained **four different ways**...Each of these ways requires that you positively identify both ends of distances on the photo and map (property boundaries, road junctions, city blocks, etc.):
  1. Measure the **ground distance** directly on the ground.
  2. Measure the **map distance** and compute the ground distance using the map scale.
  3. We **might know** the **ground distance between two points**, in which case **only** the photo measurement would be necessary.
  4. Use the global positioning system (**GPS**) to obtain grid coordinates and then calculate the ground distance.

## 4.6 Photo Scale Determination



**Figure 4.6.** Sometimes photo scale can be calculated if the GD is known, such as the old cutting boundary shown here.

## 4.6 Photo Scale Determination

### Example 1 (Using focal length and flying height)

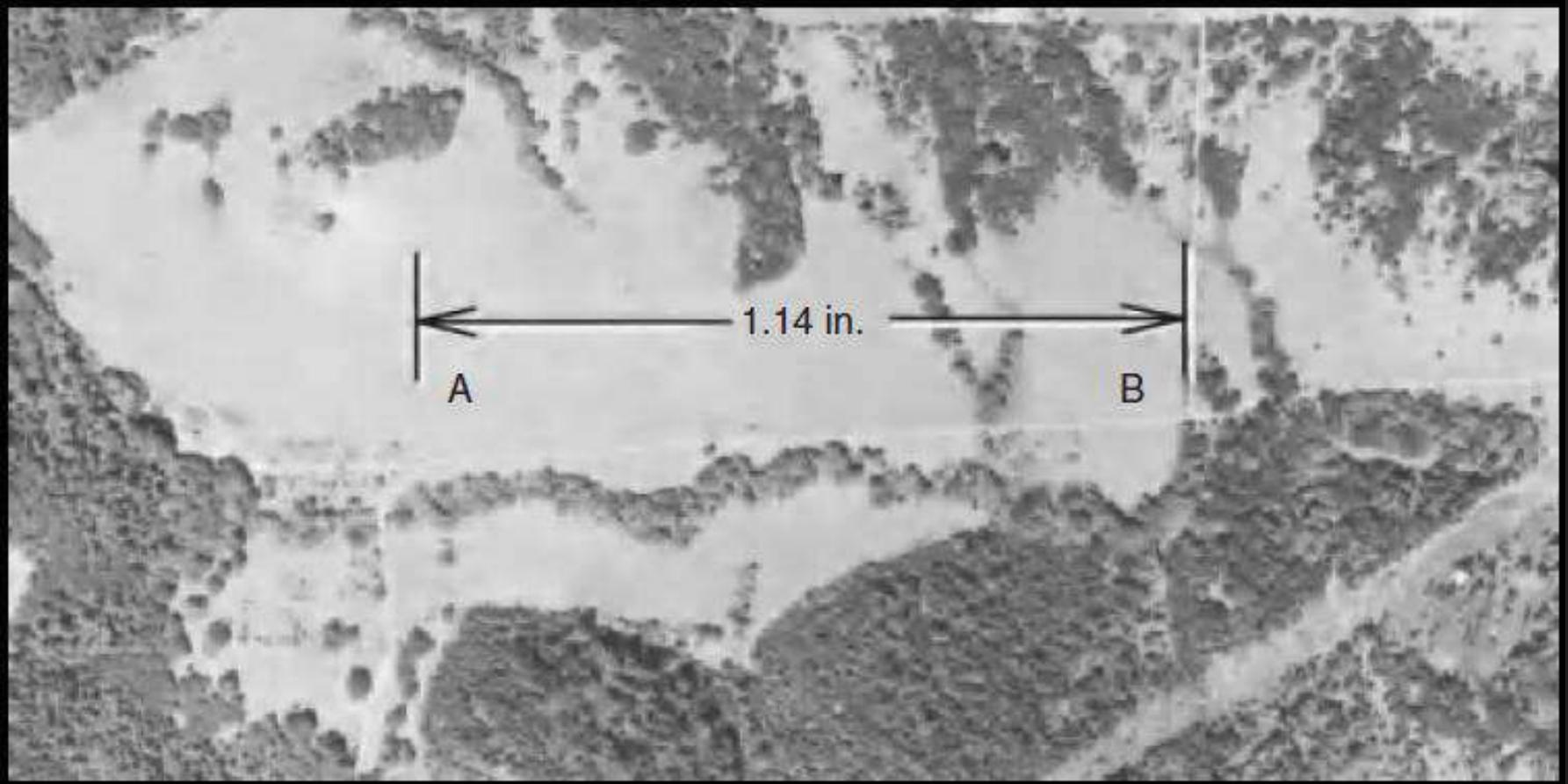
- Suppose the desired **average project** PSR is **14,000** (Figure 4.7).
- This is the nominal scale that was the goal of the photo mission and will vary from photo to photo.
- Let us suppose that the attempted flying height above mean sea level, as estimated by the mission planner, is **8,000** feet and the **focal length** of the camera lens is **6** in.
- From a topographic map of the same area we **estimate the average ground** elevation covered by a specific photo to be **800** feet.
- Therefore, **the average photo PSR** is:

$$PSR_{800} = \frac{A - E}{f} = \frac{8,000 \text{ ft} - 800 \text{ ft}}{0.5 \text{ ft}}$$

or

$$PSR_{800} = \frac{H}{f} = \frac{7,200 \text{ ft}}{0.5 \text{ ft}} = 14,400$$

## 4.6 Photo Scale Determination



**Figure 4.7.** Aerial photo for examples 1, 2, and 3.

## 4.6 Photo Scale Determination

### Example 2

Using *GD* & *PD* for this and all following examples.

- Now let's **determine** the **average scale** between two road junctions (points A and B) on the same photo (Figure 4.7).
- The distance on the original photo, as measured with an engineer's scale, is **1.12** in. and the distance as measured on the ground is **1,393** feet.
- Furthermore, let us assume that the elevation of point **A** was **490** feet and of point **B** was **584** feet. We find **the average PSR** to be:

$$PSR_{537} = \frac{GD}{PD} = \frac{1,393 \text{ ft}}{1.12 \text{ in.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 14,930$$

Notice the use of subscripts, here and elsewhere, for PSR. The 537 in this case indicates that the **PSR is valid for all points** on the photo at an elevation of 537 feet above mean sea level (MSL), which is the average of 490 and 584 feet.

## 4.6 Photo Scale Determination

### Example 2

Using *GD* & *PD* for this and all following examples.

- Also notice that we rounded PSR to the nearest 10.
- This is the usual procedure because measurement errors on the photo, map, or ground usually create even larger errors in PSR determination.
- Because foresters and other natural resources managers frequently measure ground distances in **chains** (66 feet per chain), let's work the same problem using chains.
- Our ground distance would be 21.11 ch. and our PSR calculation would be:

$$PSR_{537} = \frac{GD}{PD} = \frac{21.11 \text{ ch.}}{1.12 \text{ in.}} \times \frac{66 \text{ ft}}{1 \text{ ch.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 14,930$$

## 4.6 Photo Scale Determination

**Example 2**  $PSR_{537} = \frac{GD}{PD} = \frac{21.11 \text{ ch.}}{1.12 \text{ in.}} \times \frac{66 \text{ ft}}{1 \text{ ch.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 14,930$

- This is **not the scale** for **either** point A or B.
- It is the scale for the **average** elevation between the two points (**537** feet).
- This is not the same scale as we obtained for **Example 1**.
- The average scale for **Example 1** was for the entire photo with an estimated average elevation of **800** feet.

## 4.6 Photo Scale Determination

**Example 2**  $PSR_{537} = \frac{GD}{PD} = \frac{21.11 \text{ ch.}}{1.12 \text{ in.}} \times \frac{66 \text{ ft}}{1 \text{ ch.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 14,930$

- Note also that the scale at point A =  $PSR_{490}$  and the scale at point B =  $PSR_{584}$
- If we solve for the two PSR's using the formula in example 1, we get:

$$PSR_{490} = \frac{8,000 \text{ ft} - 490 \text{ ft}}{0.5 \text{ ft}} = 15,020$$

$$PSR_{584} = \frac{8,000 \text{ ft} - 584 \text{ ft}}{0.5 \text{ ft}} = 14,830$$

The average of these two point scales is

**14,930**

, which is what we calculate for  $PSR_{537}$ .

Throughout this book, we stress the use of units. If your **units do not cancel** in RF or PSR calculations, you have set up the solution **wrong**. In all examples, all the units must cancel, leaving the final answer (PSR) unitless.

## 4.6 Photo Scale Determination

### Example 3 average scale

- Instead of measuring the **distance between these same two road** junctions on the ground, suppose that we **have a map** of the same area on which the road junctions can be located.
- Our map has an equivalent scale of **1 in. = 400 ft** and we measure the map distance from point **A** to point **B** as **3.48 in.**
- **What is** the **average scale** between these two points? **What was** the **flying** height above sea level?

## 4.6 Photo Scale Determination

### Example 3 Solution

- First, convert map equivalent scale to **map scale reciprocal (MSR)**:

$$\frac{400 \text{ ft}}{1 \text{ in.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 4,800 \text{ MSR}$$

- It should be noted that  $MSR = GD/MD \rightarrow GD = (MD)(MSR)$ .
- Therefore:

$$GD = (3.48 \text{ in.}) \times 4800 \left( \frac{1 \text{ ft}}{12 \text{ in.}} \right) = 1,392 \text{ ft}$$

## 4.6 Photo Scale Determination

### Example 3 Solution

Our  $PSR_{537}$  is then:

$$PSR_{537} = \frac{GD}{PD} = \frac{1,392 \text{ ft}}{1.12 \text{ in.}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 14,920$$

This is a slightly different scale than we had in Example 2, but this difference can be attributed to slight errors of measurement or in locating points A and B on the map. For those who would like to work this example in one equation, we have:

$$PSR_{537} = \frac{(MD) \times (MSR)}{PD} = \frac{3.48 \text{ in.} \times 4,800}{1.12 \text{ in.}} = 14,920$$

Now we can calculate the flying height above sea level (A) because we know the scale at an elevation of 537 feet and we know that the focal length is 6 inches.

## 4.6 Photo Scale Determination

### Example 3 Solution

- If we solve one of our scale formulas,

$$PSR = (A - E)/f, \quad \text{for } A, \text{ we get:}$$

$$A = f(PSR) + E = 0.5 \text{ ft} (14,920) + 537 \text{ ft} = 7997 \text{ ft}$$

- Notice that this is slightly different than the planned flying height of 8,000 feet.
- Actually, our answers in Examples 2 and 3 were point scales since they represent the scale on the photograph where the elevation is 537 feet, but it is more convenient to think of them as average scales.

## 4.6 Photo Scale Determination

### Example 3 Solution

- Now that we know the flying height above sea level for this particular photo, we can calculate the photo scale at any point on the photo as long as we know the elevation of that point and the focal length.
- **For example**, let's calculate the **point scale** at **another point** on the same photo.

## 4.6 Photo Scale Determination

### Example 4

From our **contour map** we find the **elevation** of the **new point** to be **475** feet. Thus our PSR at this point becomes:

$$PSR_{475} = \frac{7,997 \text{ ft} - 475 \text{ ft}}{0.5 \text{ ft}} = 15,040$$

For comparison, let's find the point scale at another point on the photo that has an elevation of **1,250** ft.

$$PSR_{1250} = \frac{7,997 \text{ ft} - 1,250 \text{ ft}}{0.5 \text{ ft}} = 13,490$$

- From this example, we can see **how much a change** in ground elevation **GE** affects the photo scale.
- The **difference** in PSR between the last two calculations is **1,550** and the difference in elevation is exactly half that amount, or **775** feet.

## 4.6 Photo Scale Determination

### Example 4

This suggests that we could **derive** a single equation that would allow us to **calculate the PSR at any elevation** on the photo **if we know** the **focal length**, the **PSR** and elevation at **any** one point, and the **elevation** of the point of **unknown PSR**. Here it is:

$$PSR_A = \frac{A - E_A}{f} \quad A = f(PSR_A) + E_A$$
$$PSR_B = \frac{A - E_B}{f} \quad PSR_B = \frac{f(PSR_A) + E_A - E_B}{f}$$
$$PSR_B = PSR_A + \frac{E_A - E_B}{f}$$

Now let's **use this equation** to see if we get the same answer as **before** for the point with an elevation of **475** feet, using the known PSR and elevation from **Examples 3 and 4**.

## 4.6 Photo Scale Determination

### Example 5

$$\begin{aligned}PSR_{475} &= PSR_{537} + \frac{E_{537} - E_{475}}{f} = 14,930 + \frac{537 - 475}{0.5} \\ &= 15,040\end{aligned}$$

or

$$\begin{aligned}PSR_{475} &= PSR_{1250} + \frac{E_{1250} - E_{475}}{f} = 13,490 + \frac{1,250 - 475}{0.5} \\ &= 15,040\end{aligned}$$

- which is **exactly** what we obtained in Example 4.
- **Don't get confused** if the PSR we want to calculate is at a higher elevation than the known PSR.

## 4.6 Photo Scale Determination

**Example 6** Calculate average photo scale at a point

As in our previous example, let's calculate the PSR at 1,250 feet, knowing the PSR at 475 feet:

$$\begin{aligned}PSR_{1250} &= PSR_{475} + \frac{E_{475} - E_{1250}}{f} \\ &= 15,040 + \frac{475 - 1,250}{0.5} = 13,490\end{aligned}$$

which is the same as we obtained before for  $PSR_{1250}$ .

## 4.6 Photo Scale Determination

### 4.6.1 Assumptions

Please for more details refer to the text book,  
page 81

# Laboratory Exercises 2

Please refer to the text book, page 84

For this laboratory exercise you will need to use the annotated aerial photo in Figure 4.1 and the map of the same area in Figure 4.9. Your instructor can alter the exercise by changing the assumptions and/or the data for each problem.

## *Assumptions*

Average project elevation = 900 feet

Focal length of camera lens used = 3 in.

Planned flying height above MSL (A) = 6,900 feet

# *Homework's*

( all )

## Questions and Problem

In text book (Page 84)

are required

*Thank you*

Any Questions ?



**END**

**of Lecture**

## 4.6 Photo Scale Determination

Photoscale may also be determined according to:

1. Smallest detail and resolution
2. C-factor and desired minimum contour interval
3. Expected accuracy
4. Enlargement from photo to map in the instrument

**See Appendix 4-1**

## 4.6 Photo Scale Determination

### Example 7 Smallest detail and resolution

- The smallest detail that needs to be seen on the photograph is **1** foot in length.
- If the resolution of the photo is **0.1** mm, **determine** the photoscale.

## 4.6 Photo Scale Determination

### Example 7 Smallest detail and resolution

$$\text{PhotoScale} = \frac{\text{resolution}}{\text{Smallest Details}}$$

$$\text{PhotoScale} = \frac{0.1\text{mm}}{1\text{ft}} = \frac{0.1\text{mm}}{(1\text{ft} \times 12(\text{in} / \text{ft}) \times 254.\text{mm})}$$

$$\text{PhotoScale} = \frac{0.1\text{mm}}{304.8\text{mm}} \cong \frac{1}{3000}$$

# Appendix 4-1

## *PhotoScale* Determining Photoscale

### 2- C-factor and desired minimum contour interval

- Contour interval – difference in elevation between consecutive contour lines

$$C - factor = \frac{\text{flying height}}{\text{contour interval}} = \frac{H}{\Delta h}$$

- C-factor range from 1200 to 1500

## Appendix 4-1

### *PhotoScale* Determining Photoscale

2- C-factor and desired minimum contour interval

### **Example:**

The C-factor of the instrument is given to be 1500. If the desired contour interval is 1 meter, determine the photoscale.

## Appendix 4-1

### *PhotoScale* Determining Photoscale

2- C-factor and desired minimum contour interval

**Solution:**

$$photoscale = \frac{1}{9000}$$

# Appendix 4-1 Accuracy

## PhotoScale

## Determining Photoscale

Mean square error of horizontal position of points:

$$m_h = 0.1\text{‰} H = 0.0001H = (10^{-4})H$$

‰ – per mil; equivalent to 1/1000

For Cadastral Survey:

$m_h = 10 \text{ cm (urban)}$

$= 30 \text{ cm (rural)}$

# Appendix 4-1

## *PhotoScale* Determining Photoscale

### 3- Expected Accuracy

Example:

Determine the photoscale for an urban area if the camera to be used is a wide-angle camera.

## Appendix 4-1

# *PhotoScale* Determining Photoscale

## 3- Expected Accuracy

Solution:

$$H = \frac{10 \text{ cm}}{0.1\text{‰}} = \frac{10 \text{ cm}}{0.0001} = 100,000 \text{ cm}$$

$$\frac{1}{s_r} = \frac{f}{H} = \frac{6 \text{ in}}{\left(100000 \text{ cm} \times \frac{1 \text{ in}}{2.54 \text{ cm}}\right)} = \frac{1}{6561.67979}$$

$$\textit{photoscale} = \frac{1}{6000}$$

# Appendix 4-1

## *PhotoScale* Determining Photoscale

### 4-Enlargement from photo to map

- Using the stereoplotter, there will be an enlargement from the photo to the stereomodel:

$$\text{enlargement} = \frac{Z}{C}$$

where:

Z = projection distance for stereoplotter

C = f = projection distance of camera

# Appendix 4-1

## *PhotoScale* Determining Photoscale 4-Enlargement from photo to map

### Example:

A map with scale 1:5000 was derived from a stereomodel with a scale of 1:8000, using a stereoplotter.

The projection distance of the stereoplotter is twice the focal length of the camera. Determine the scale of the photograph that was used to generate the stereomodel.

## Appendix 4-1

# *PhotoScale* Determining Photoscale

## 4-Enlargement from photo to map

Solution:

$$\text{enlargement} = \frac{Z}{f} = \frac{2f}{f} = 2$$

$$\text{photoscale} = \frac{\text{stereomodel scale}}{2} = \frac{1}{2} \times \frac{1}{8000}$$

$$\text{photoscale} = \frac{1}{16000}$$

# Image/photo scale Appendix 4-2

Importance: Image scale is important because it relates the size or distance on the image to the actual size or distance on the ground.

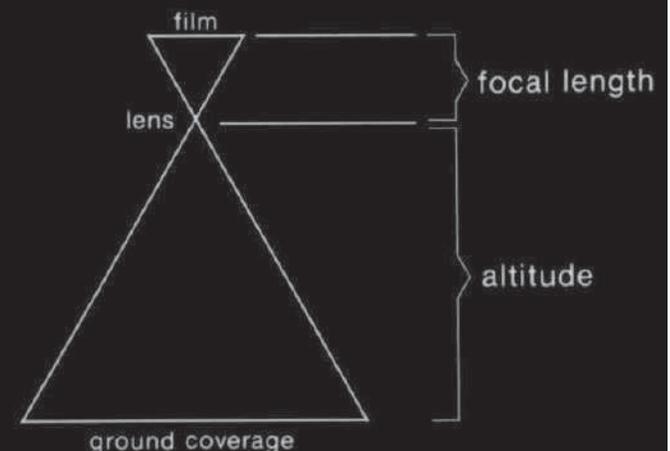
How can u measure scale in an AP?

**1. Arithmetic Proportion:** comparing distance between two points on the airphoto (*Photo Distance*) with the same points on a topographic map.

$$\text{Photo scale} = \frac{\text{Photo Distance}}{\text{Map Distance}} \times \text{Map scale}$$

**2. By camera altitude relationship:**

$$\begin{aligned} \text{Photo scale} &= \text{focal length} / \text{aircraft altitude} \\ &= (f/H) \end{aligned}$$



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السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

# Image Description and Representation

*Lecturer:*

*Dr. Faisal Ghazi Mohammed*

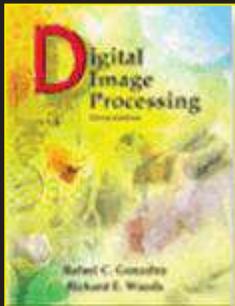
Email: [faisal@scbaghdad.edu.iq](mailto:faisal@scbaghdad.edu.iq)

[faiselgm73@gmail.com](mailto:faiselgm73@gmail.com)

*2023-2020*

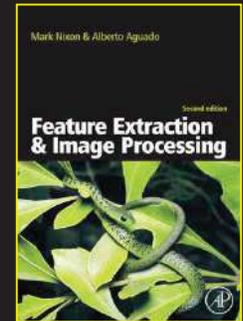
*All rights reserved*

# Reading instructions



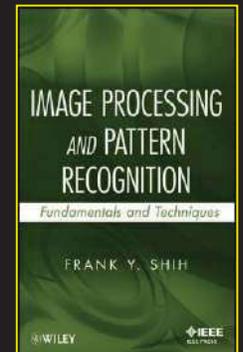
Chapter 11.1 – 11.4  
*Gonzalez & Woods:*  
*Digital Image Processing,*  
*2<sup>nd</sup> ed., 2002.*

Chapter 7 in  
“Feature Extraction and Image  
Processing”, Second edition  
Mark S. Nixon and Alberto S.  
Aguado, 2008



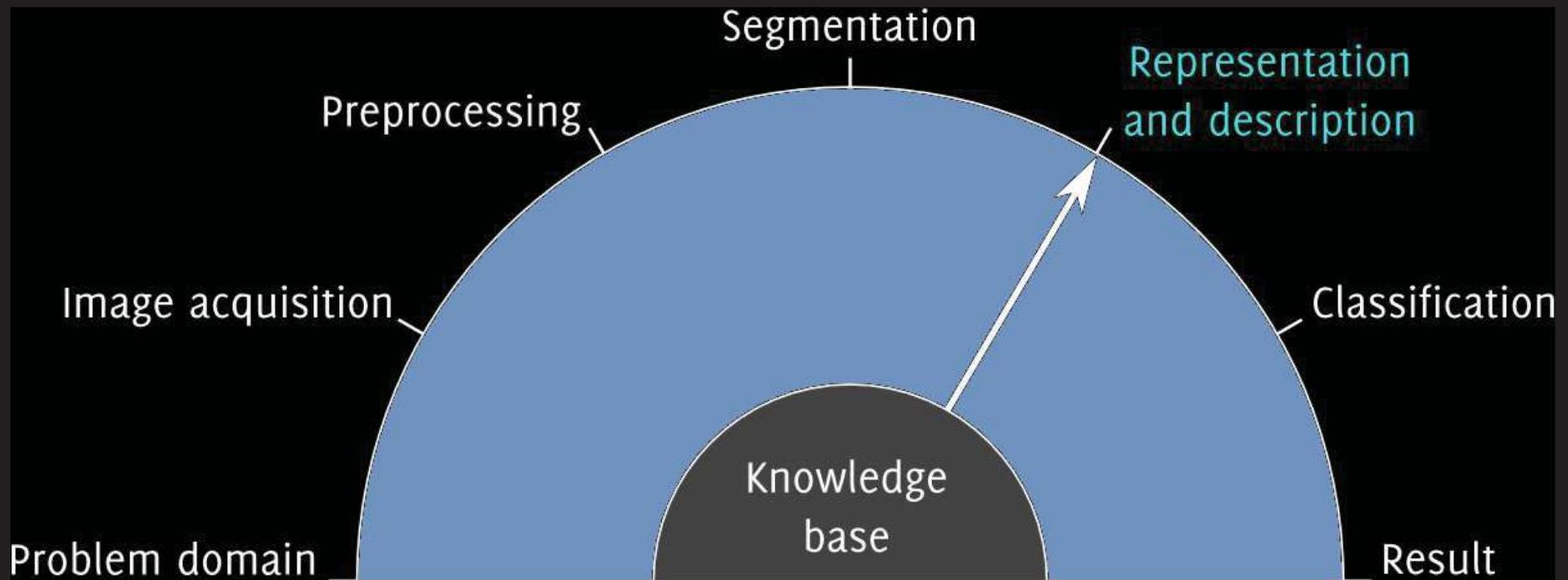
Chapter 8 in  
*Image Processing*  
*Analysis, and Machine*  
*Vision*

Chapter 7 in  
“IMAGE PROCESSING  
AND PATTERN  
RECOGNITION: Fundamentals  
and Techniques:”, FRANK Y.  
SHIH, 2010

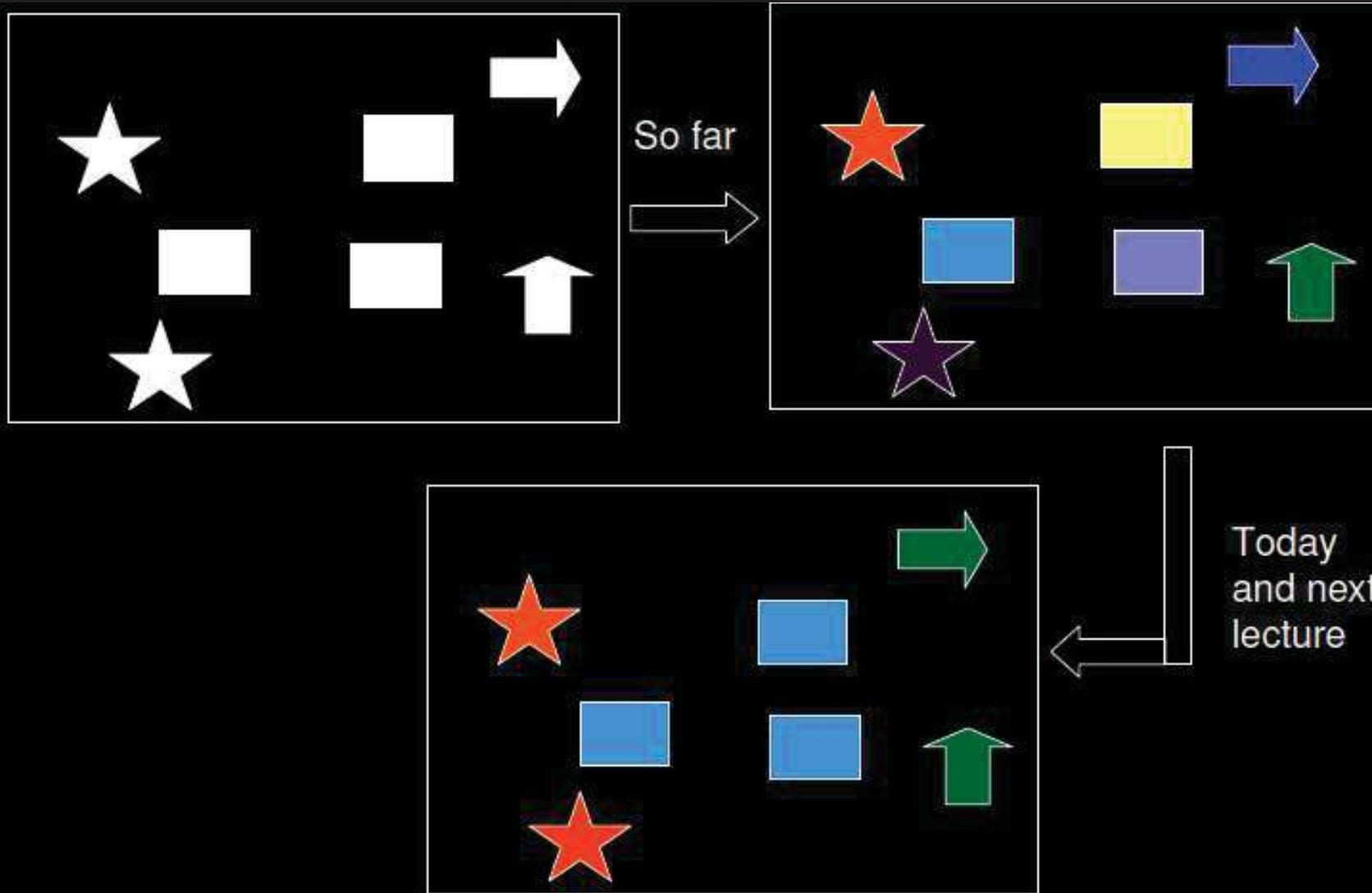




# Our progress in the analysis process



# Our progress in the analysis process



# Contents



*Today we'll look at Object Representation and Description, covering: Today*

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## 2. Shape Representations *(by Boundary)*

- Chain coding
- Signatures
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- Boundary Segments
- Skeletons

## 3. Shape Descriptors *(by Boundary)*

- Simple descriptors
- Shape numbers
- Statistical moments
- Fourier descriptors

# *Contents*



*Today we'll look at Object Representation and Description, covering: **Later***

## **4.Regional Descriptors**

- Simple
- Topological
- Texture( Statistical, Spectral )
- Moments

## **5.Other Descriptors**

- Co-occurrence Matrix
- Principal components

# 1. Introduction



## Definitions

### Object

An object is something which can be seen or touched (tangible), material thing (Oxford dictionary)



### Abstraction

The idea of separating the quality from actual examples, i.e. concentrate on the concept of the object rather than its actual instance.



### Representation

– A way of symbolizing an object

$$r^2 = x^2 + y^2 + z^2$$



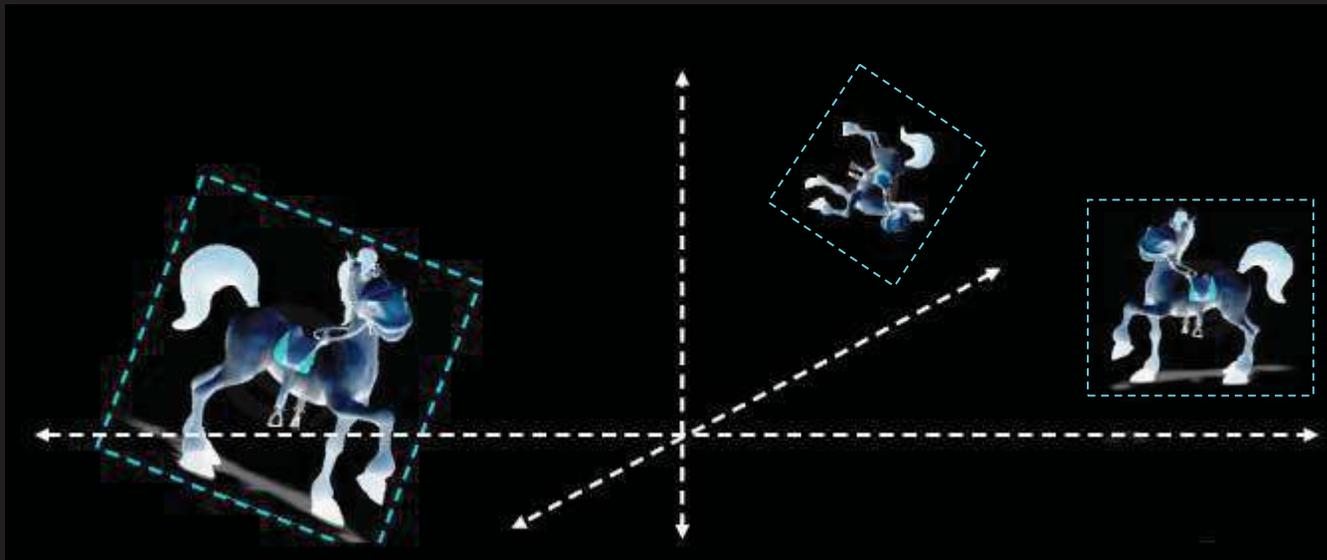
# 1. Introduction

## What is shape?

The most commonly cited definition is given as follows:

Shape is all the geometrical information that remains when location, scale, and rotation effects are filtered out from an object

In other words, a shape is **invariant** to Euclidean similarity transformations of scaling, translation, and rotation. Two objects have the **same** shape if they can be mapped onto each other by a translation, rotation, and scaling.



# 1. Introduction



## Static and Dynamic Shapes

Shapes can be either static or dynamic.

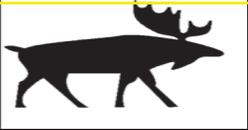
- **Static** shapes are rigid shapes that do not change in time by deformation or articulation. – For example, a model of a car is a static shape,
- While a human face is a **dynamic** shape since it changes while speaking and smiling, for instance.



# 1. Introduction



The shapes are recognized by...

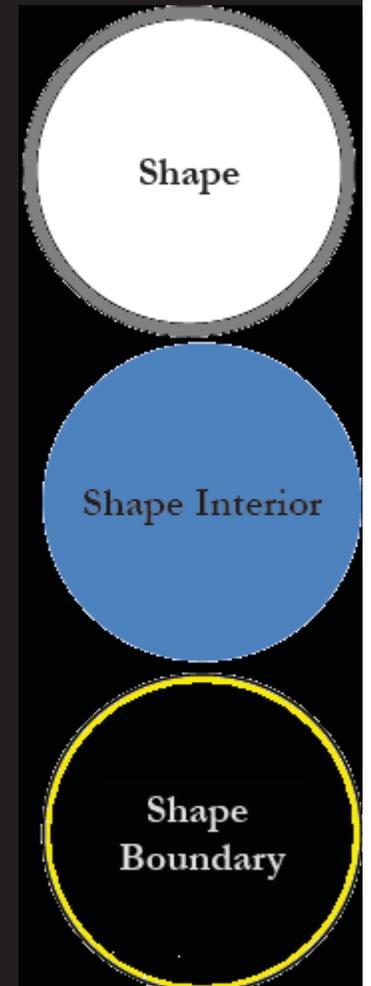
	Texture	Color	Context	Shape
	X	X		
	X			X
				X
				X
				X
			X	X



# *1. Introduction*

## *What is Shape Representation ?*

1. To describe the **boundary** that surrounds an object:
  - **Boundary-based** representation (curves and surfaces)
2. To describe the **region** that is occupied by an
  - **Region-based** representation (images and volumes)
3. To describe an object in terms of **coefficients** in a transform-domain:
  - **Transform-based** shape representation, e.g. Fourier descriptors, spherical harmonics, moments





# *1. Introduction*

## *Binary Image representation strategies*

1. Image **topology**: connectivity and Euler number
2. **Boundary** representation (and description)
  - Chain codes, shape numbers, Fourier descriptors\*
3. **Skeleton** representation
  - String codes, tree grammar\*, automata\*
4. **Fractal** representation
  - Fractal everywhere: coastlines, snow-flakes, leaf veins ...

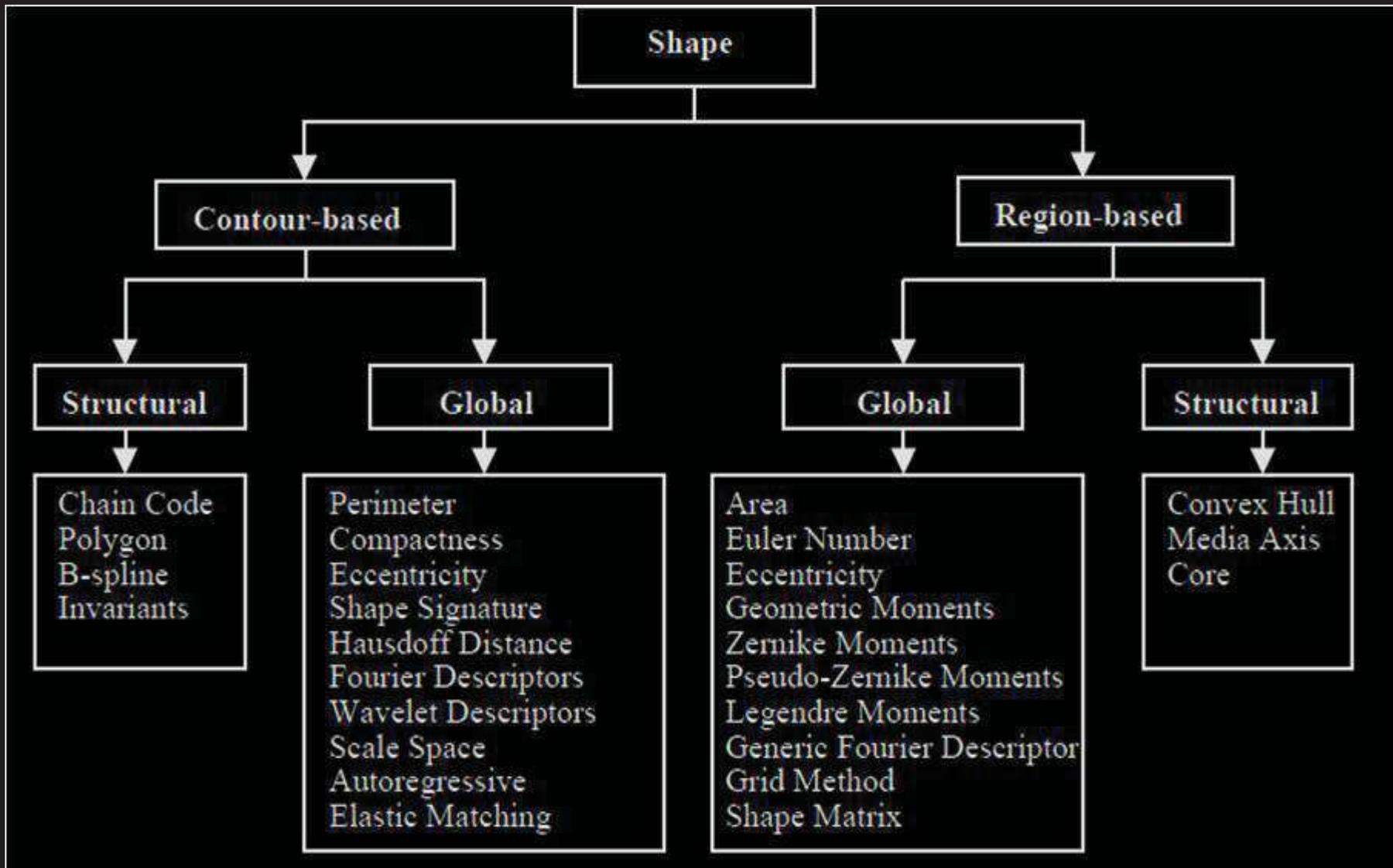
### **Applications:**

- Character/barcode/handwriting recognition

# 1. Introduction



## Classification of shape representation and description techniques



# *1. Introduction*



Shape is not the only,  
but a very  
powerful descriptor of image content

## *2. Shape Representation*



*Two-dimensional shapes can be described in two different ways:*

- 1. Use of the **object boundary** and its **features** (e.g. **boundary length**). This method is directly connected to **edge** and line detection. The resulting description schemes are called **external representations**.*
- 2. Description of the region occupied by the object on the image plane. This method is linked to the **region** segmentation techniques. The resulting representation schemes are called **internal representations**.*



## *2. Shape Representation*

### *What is a Good Representation?*

- There are a variety of ways to represent a shape, however, there are certain attributes/criteria for a representation to be a good one:
  1. Sufficient
  2. Wide domain
  3. Convenient
  4. Sensitive
  5. Unambiguous
  6. Hierarchical
  7. Generative
  8. Stable
  9. Accessible
  10. Efficient

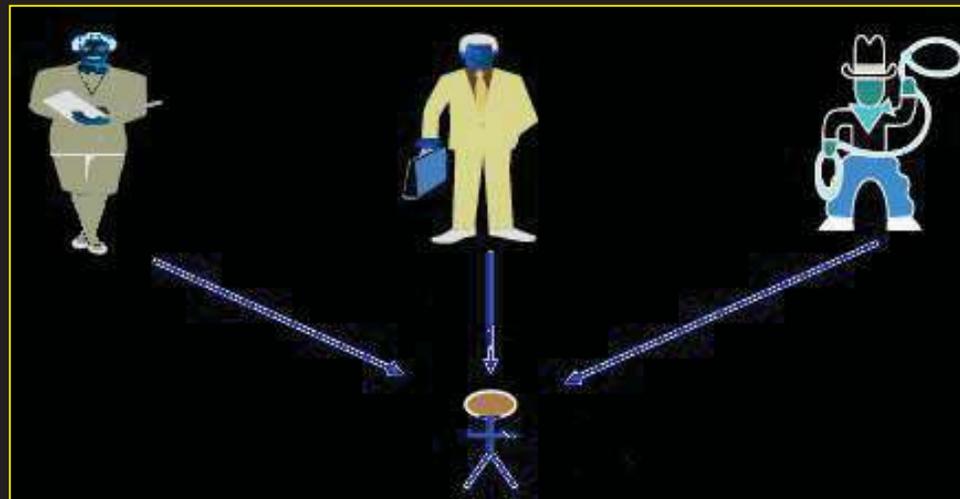
## 2. Shape Representation

### What is a Good Representation?

#### 1. Sufficient

Is this representation sufficient enough? The answer mainly depend on the application.

If we want to detect human versus other classes, this representation might work, but if we want to differentiate *Ahmed* from *Miriam*, this representation is not sufficient.



## *2. Shape Representation*

### *What is a Good Representation?*

#### **2. Uniqueness:-**

This is of crucial importance in object recognition, because each object must have a unique Representation.



Not unique :	dog	dog	dog
unique :	pitbull	collie	cocker-spaniel

Consider the domain to be the animals, those three are dogs however they are different members of the domain, they are not the same dogs, hence the **word dog** as a representation is **not unique** for all member in this domain



## *2. Shape Representation*

### *What is a Good Representation?*

#### *3. Completeness / Unambiguous*

This refers to unambiguous representations *Invariance under geometrical transformations.*

An object/shape may have different representations but no two distinct objects may have a common representation.

3 III

– The number three has different representations (Arabic and roman) however the same representation can not refer to different numbers.

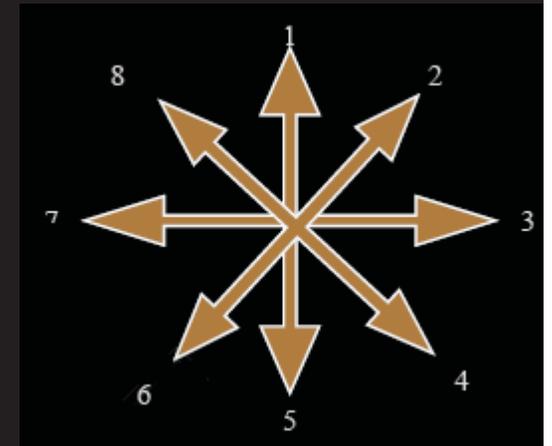
## 2. Shape Representation



### *What is a Good Representation?*

**4. Invariance** under translation, rotation, scaling and reflection is very important for object recognition applications.

Capable of directly generating/recovering the represented shape



Chain-code : 4682 3571

## *2. Shape Representation*



### *What is a Good Representation?*

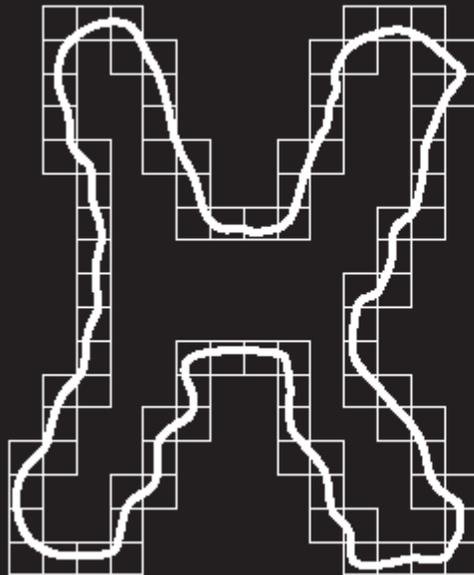
**5. Sensitivity.** This is the ability of a representation scheme to reflect easily the differences between similar objects

**6. Abstraction/rent detail.** This refers to the ability of the representation to represent the basic features of a shape and to abstract from detail. This property is directly related to the noise robustness of the representation.

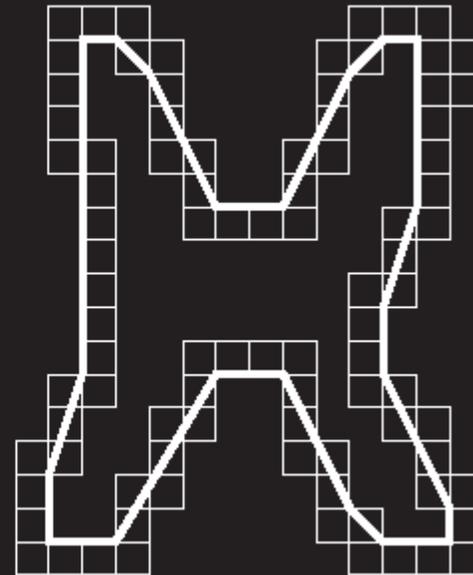
## 2. Shape Representation

### 2.1 Polygon Approximation

Represent an object boundary by a polygon



Object boundary



Minimum perimeter  
polygon

**Minimum perimeter polygon** consists of line segments that minimize distances between boundary pixels.

## *2. Shape Representation*

### 2.1 Polygon Approximation: *Splitting techniques*

- Joint the two furthest points on the boundary  
→ line **ab**
- Obtain a point on the upper segment, that is **c**  
and a point on the lower segment, that is **d**,  
such that the perpendicular distance from  
these points to **ab** is as large as possible
- Now obtain a polygon by joining **c** and **d** with  
a and b
- Repeat until the perpendicular distance is  
less than some predefined fraction of **ab**

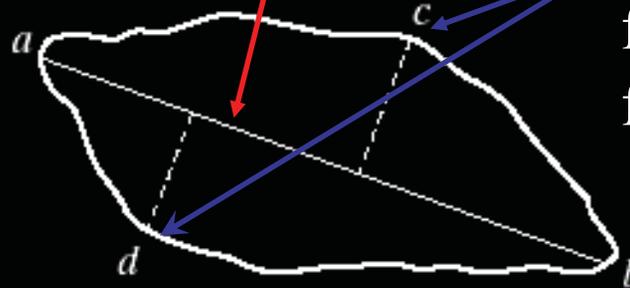
# 2. Shape Representation

## 2.1 Polygon Approximation: *Splitting techniques*

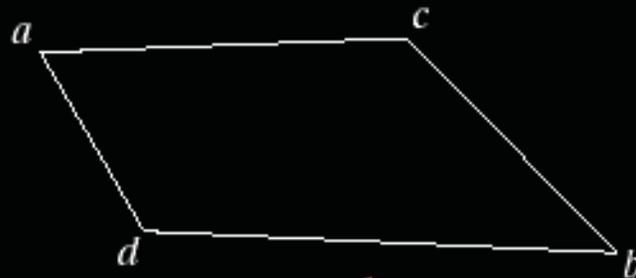
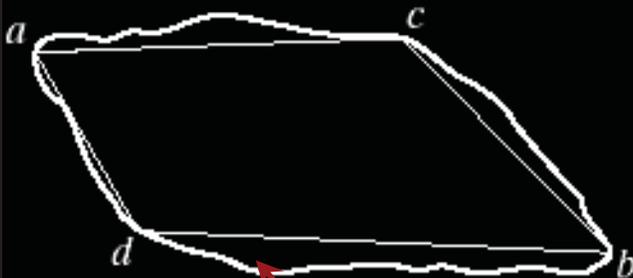
0. Object boundary



1. Find the line joining two extreme points



2. Find the farthest points from the line



3. Draw a polygon

## *2. Shape Representation*

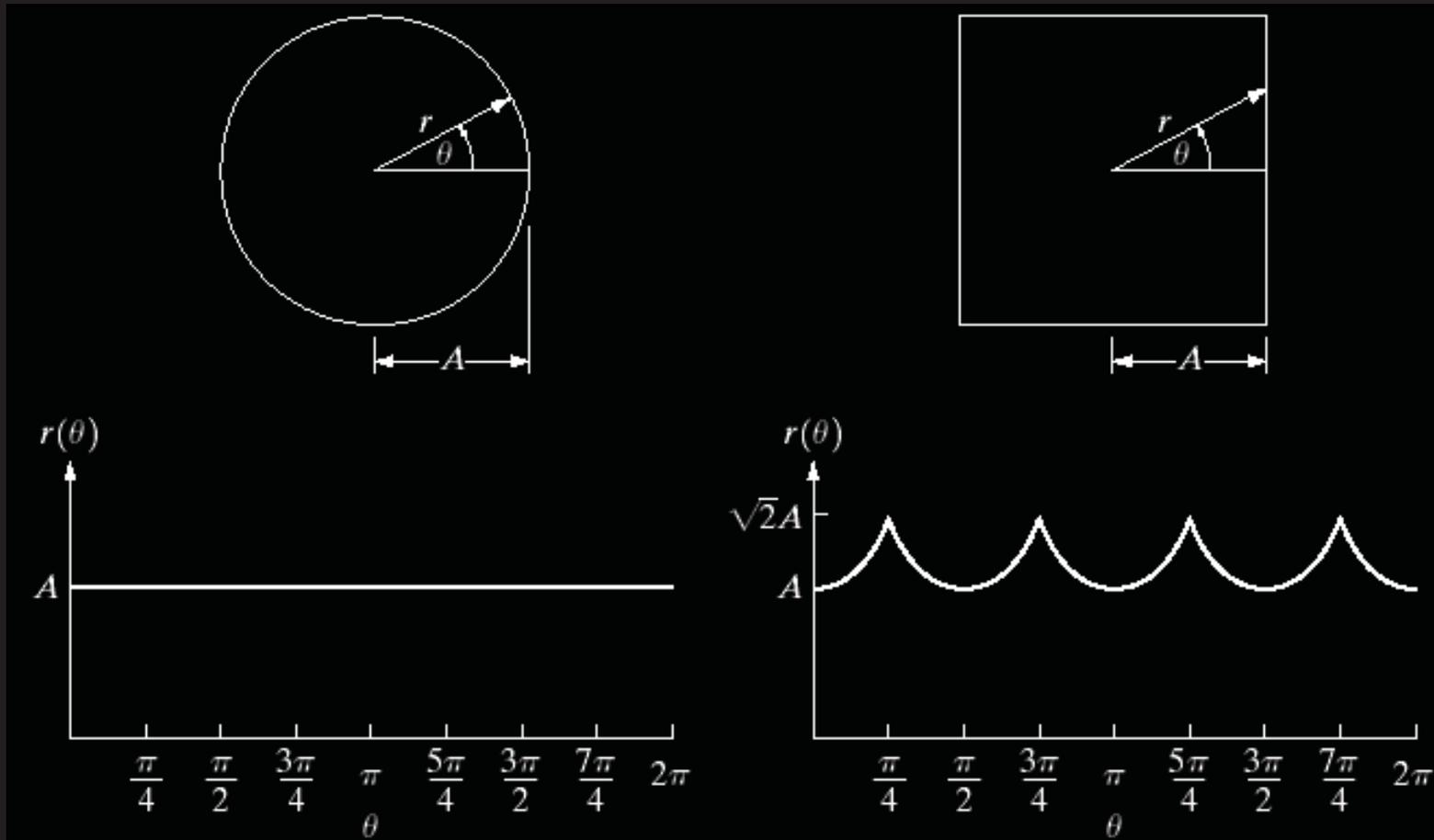
### **2.2 Signatures:** *Distance-Versus-Angle*

- 1-D representation of boundary: generated in various ways Simplest approach: plot  $r(\theta)$
- $r$ : distance from centroid of boundary to boundary point
- $\theta$ : angle with the positive x-axis

# 2. Shape Representation

## 2.2 Signatures: *Distance-Versus-Angle*

Represent an 2-D object boundary in term of a 1-D function of radial distance with respect to  $\theta$ .

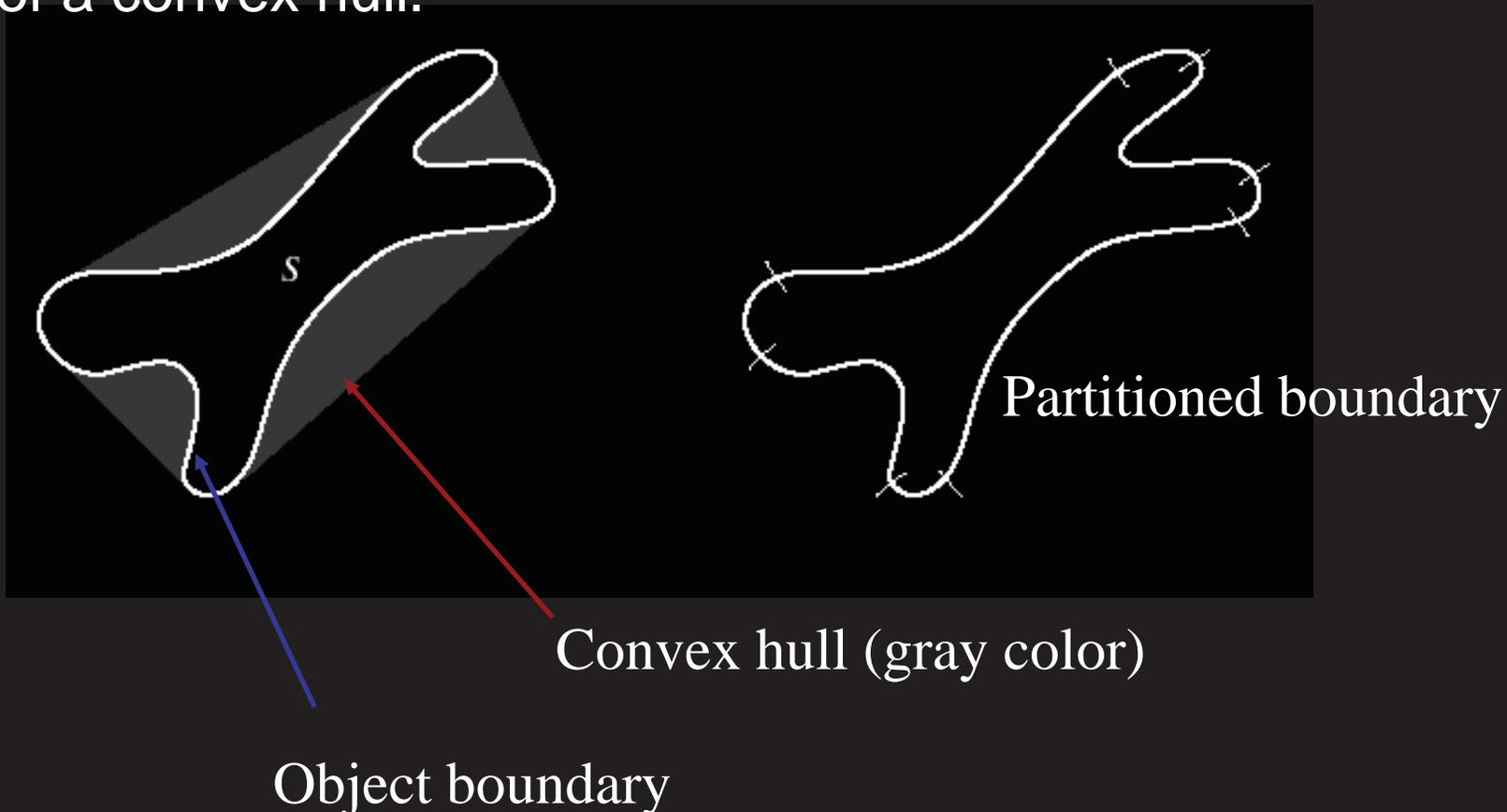


# 2. Shape Representation



## 2.3 Boundary Segments

**Concept:** Partitioning an object boundary by using vertices of a convex hull.



# 2. Shape Representation



## 2.4 Convex Hull Algorithm

**Input** : A set of points on a cornea boundary

**Output**: A set of points on a boundary of a convex hull of a cornea

1. Sort the points by x-coordinate to get a sequence  $p_1, p_2, \dots, p_n$

For the upper side of a convex hull

2. **Put** the points  $p_1$  and  $p_2$  in a list  $L_{\text{upper}}$  with  $p_1$  as the first point

3. **For**  $i = 3$  to  $n$

4. **Do** append  $p_i$  to  $L_{\text{upper}}$

5. **While**  $L_{\text{upper}}$  contains more than 2 points and the last 3 points in  $L_{\text{upper}}$  do not make a right turn

6. **Do** delete the middle point of the last 3 points from  $L_{\text{upper}}$



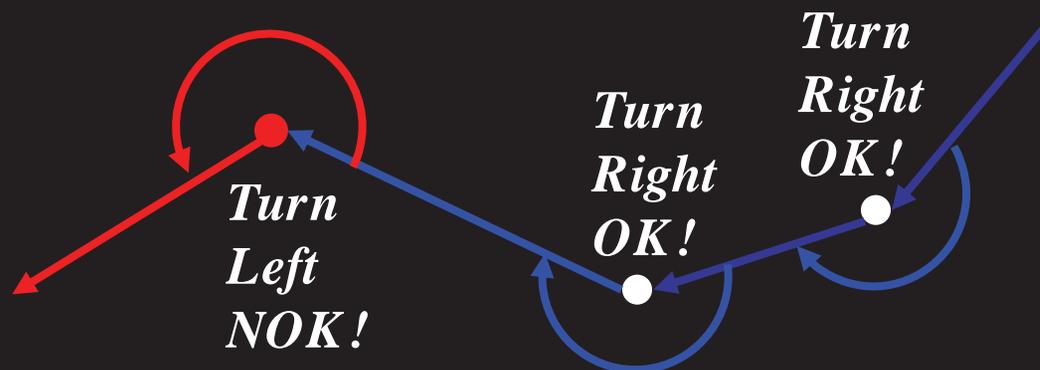
# 2. Shape Representation



## 2.4 Convex Hull Algorithm

### For the lower side of a convex hull

7. **Put** the points  $p_n$  and  $p_{n-1}$  in a list  $L_{lower}$  with  $p_n$  as the first point
8. **For**  $i = n-2$  down to 1
9. **Do** append  $p_i$  to  $L_{lower}$
10. **While**  $L_{lower}$  contains more than 2 points and the last 3 points in  $L_{lower}$  do not make a right turn
11. **Do** delete the middle point of the last 3 points from  $L_{lower}$
12. **Remove** the first and the last points from  $L_{lower}$
13. **Append**  $L_{lower}$  to  $L_{upper}$  resulting in the list  $L$
14. **Return**  $L$

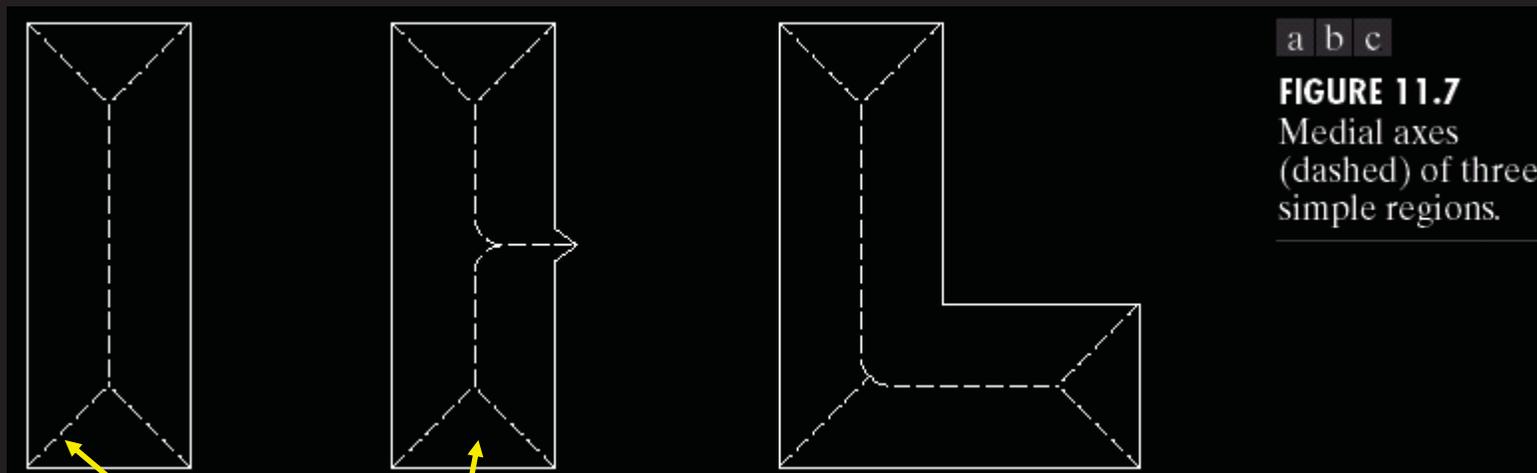


# 2. Shape Representation

## 2.5 Skeletons



Obtained from thinning or skeletonizing processes



a b c

**FIGURE 11.7**  
Medial axes  
(dashed) of three  
simple regions.

Medial axes (dash lines)

# 2. Shape Representation

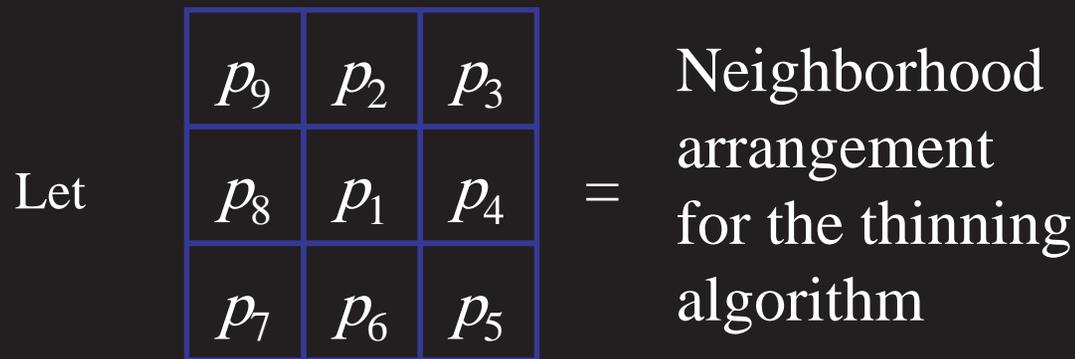


## 2.5 Skeletons: Thinning algorithm

- Concept:**
1. Do not remove end points
  2. Do not break connectivity
  3. Do not cause excessive erosion

Apply only to contour pixels: pixels “1” having at least one of its 8 neighbor pixels valued “0”

**Notation:**



**Example**

0	0	1
1	$p_1$	0
1	0	1

$$N(p_1) = 4$$

$$T(p_1) = 3$$

Let  $N(p_1) = p_2 + p_3 + \dots + p_8 + p_9$

$T(p_1)$  = the number of transition 0-1 in the ordered sequence  $p_2, p_3, \dots, p_8, p_9, p_2$ .



## 2. Shape Representation

### Thinning Algorithm (cont.)

**Step 1. Mark** pixels for **deletion** if the following conditions are true:

a)  $2 \leq N(p_1) \leq 6$

b)  $T(p_1) = 1$  (Apply to all border pixels)

c)  $p_2 \cdot p_4 \cdot p_6 = 0$

d)  $p_4 \cdot p_6 \cdot p_8 = 0$

$p_9$	$p_2$	$p_3$
$p_8$	$p_1$	$p_4$
$p_7$	$p_6$	$p_5$

**Step 2. Delete** marked pixels and go to Step 3.

**Step 3. Mark** pixels for **deletion** if the following conditions are true.

a)  $2 \leq N(p_1) \leq 6$  (Apply to all border pixels)

b)  $T(p_1) = 1$

c)  $p_2 \cdot p_4 \cdot p_8 = 0$

d)  $p_2 \cdot p_6 \cdot p_8 = 0$

**Step 4. Delete** marked pixels and repeat Step 1 until no change occurs.

# 2. Shape Representation



## 2.5 Skeletons: Centers of Maximal Discs

A disc is made of all pixels that are within a given radius  $r$ . The skeleton of a binary object is the union of all disc centers needed to reconstruct the object using the corresponding discs.

### Algorithm

Find the skeleton with Centers of Maximal Discs (CMD)

Completely reversible situation :-)

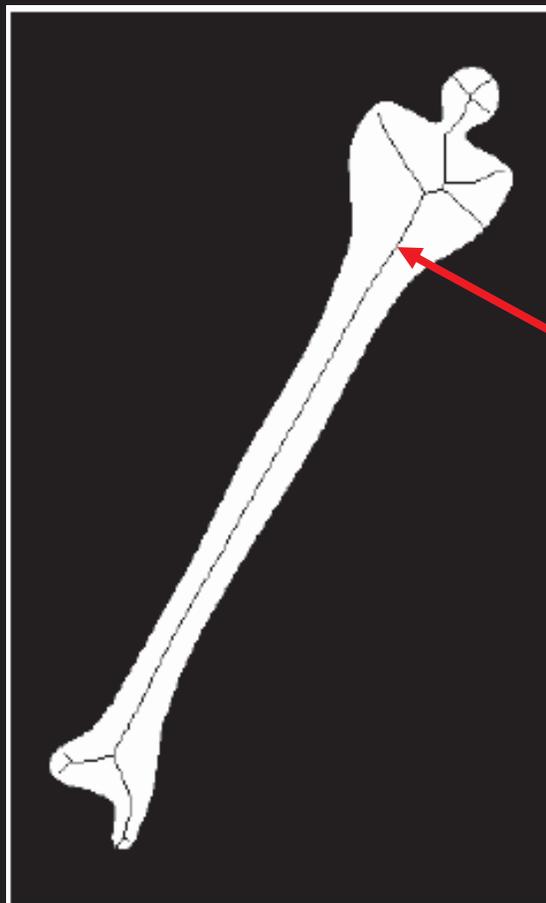
1. Generate DT of object
2. Identify CMDs (smallest set of maxima)
3. Link CMDs

“Pruning” is to remove small branches (no longer fully reversible.)

## 2. Shape Representation



**Example: Skeletons Obtained from the Thinning Alg**



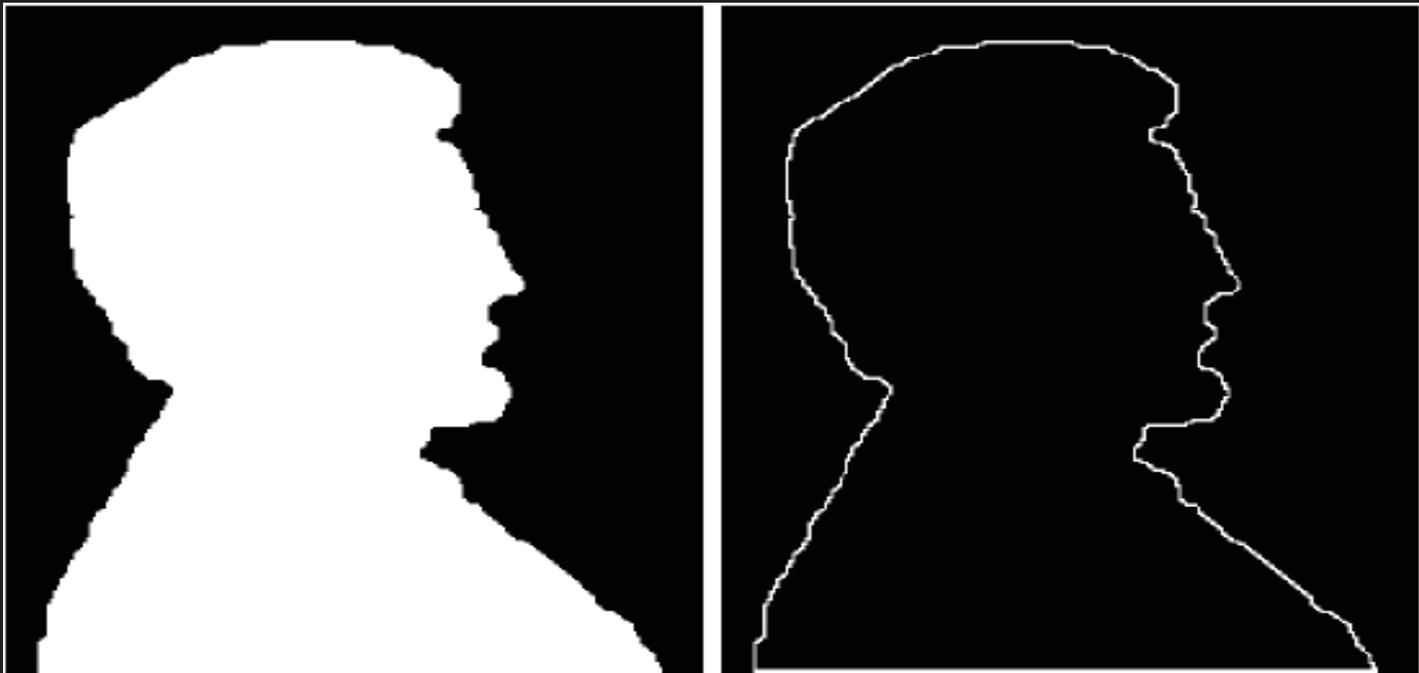
Skeleton

**FIGURE 11.10**  
Human leg bone and skeleton of the region shown superimposed.

## 2. Shape Representation



### Boundary of Binary Shapes



$X$

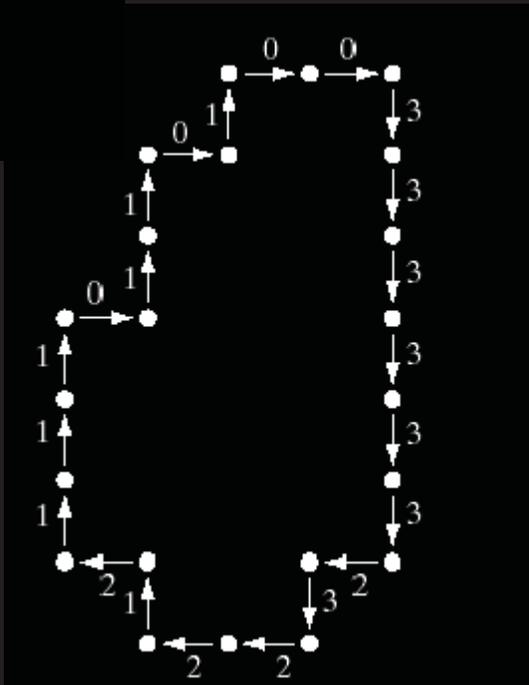
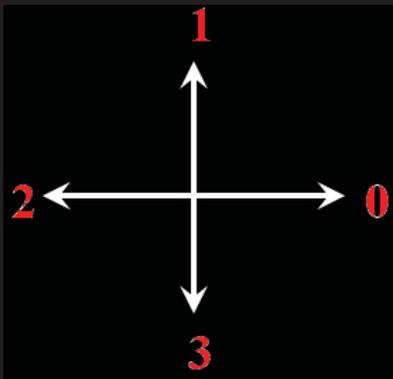
$\partial X$  Boundary

$$\partial X = (X \oplus B) - B$$

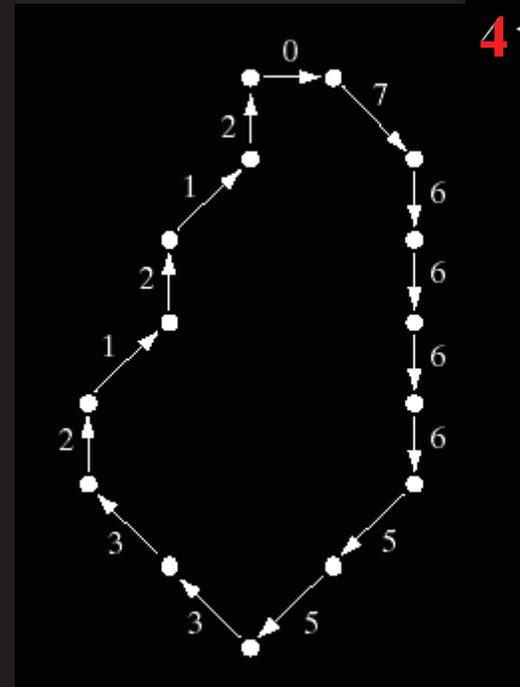
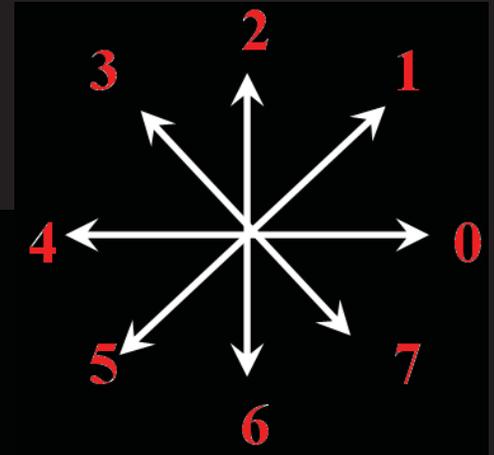
or  $\partial X = X - (X \ominus B)$

# 2. Shape Representation

## Chain Codes: *Boundary Representation*



4-directional chain code:  
00333332322121101101



8-directional chain code:  
076666553321212

# 2. Shape Representation



## Chain Codes: *Boundary Representation*

```
For X = 0 To Wid - 1: For Y = 0 To Hgt - 1
  If ImgE(X, Y) = 1 Then
    ImgE(X, Y) = 0: Ndr = 0: Xr = X: Yr = Y
    Xpnt(L) = Xr: Ypnt(L) = Yr: L = 0
    Form1.Picture3.PSet (X, Y), 0
    NoEdgePoints = NoEdgePoints + 1
    Do
      Flg = True
      For I = 0 To 7: li = (I + Ndr) Mod 8
        Xx = Xr + Xdir(li): Yy = Yr + Ydir(li)
        If ImgE(Xx, Yy) = 1 Then
          ImgE(Xx, Yy) = 0
          Form1.Picture3.PSet (Xx, Yy), 0
          NoEdgePoints = NoEdgePoints + 1
          L = L + 1
          Xpnt(L) = Xx: Ypnt(L) = Yy
          Xr = Xx: Yr = Yy: Ndr = li: Flg = False: I = 7
        End If
      Next I
    Loop Until Flg
    DoEvents
  End If
DoEvents
Next: Next
```

Edge tracking source code written with VB6

```
ReDim Xdir(0 To 7), Ydir(0 To 7)
ReDim Xpnt(10000), Ypnt(10000)
ReDim EdgX(Wid * Hgt), EdgY(Wid * Hgt)
Xdir(0) = 1: Ydir(0) = 0 ' x+1,y
Xdir(1) = 1: Ydir(1) = 1 ' x+1,y+1
Xdir(2) = 0: Ydir(2) = 1 ' x ,y+1
Xdir(3) = -1: Ydir(3) = 1 ' x-1,y+1
Xdir(4) = -1: Ydir(4) = 0 ' x-1,y
Xdir(5) = -1: Ydir(5) = -1 ' x-1,y-1
Xdir(6) = 0: Ydir(6) = -1 ' x ,y-1
Xdir(7) = 1: Ydir(7) = -1 ' x+1,y-1
L = 0 : Ndr = 0
```



## *2. Shape Representation*

### **Chain Codes:** *Boundary Representation*

#### *Problems with the Chain Code*

- Chain code representation is conceptually appealing, yet has the following three **problems**:
  1. Dependent on the **starting** point
  2. Dependent on the **orientation**
  3. Dependent on the **Scaling**
- To use boundary representation in object recognition, we need to achieve invariance to **starting** point and **orientation**
  - **Normalized** codes
  - **Differential** codes



## 2. Shape Representation

Chain Codes: *Boundary Representation*

### *Differentiation Strategy*

change in direction around the border (differences between chain code numbers modulo 4 or 8):

$$1. \quad d_i = \begin{cases} \text{diff}(x_i, x_{i-1}) & \text{if } i \neq 1 \\ \text{diff}(x_i, x_N) & \text{if } i = 1 \end{cases}$$

$$2. \quad \text{if } d_i < 0 \text{ then } d_i = \begin{cases} d_i + 4 & \text{for 4-directional chain codes} \\ d_i + 8 & \text{for 8-directional chain codes} \end{cases}$$

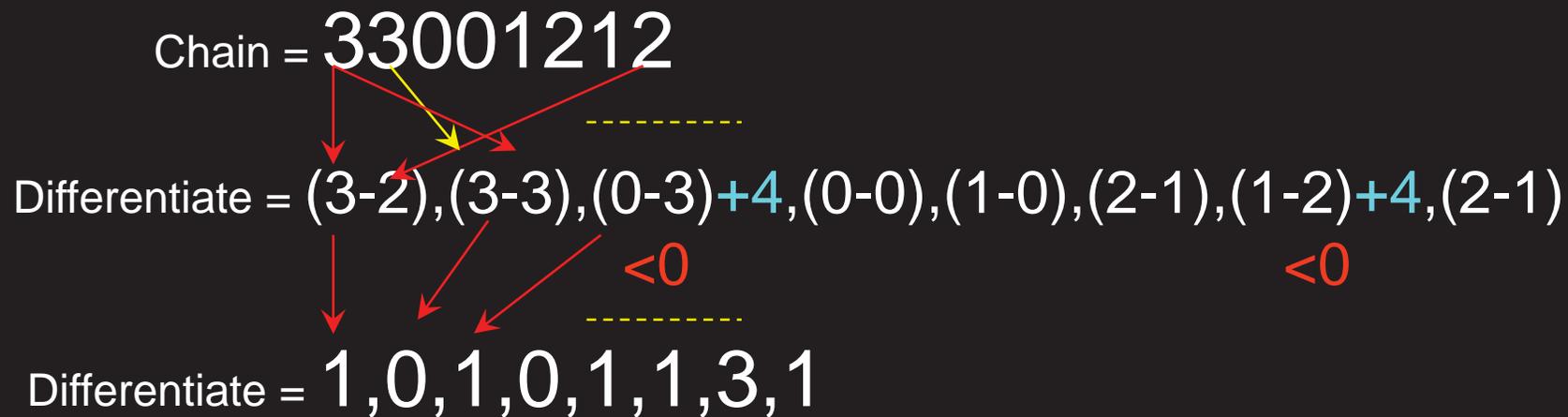


## 2. Shape Representation

Chain Codes: *Boundary Representation*

*Differentiation Strategy*

*Differentiate example:*

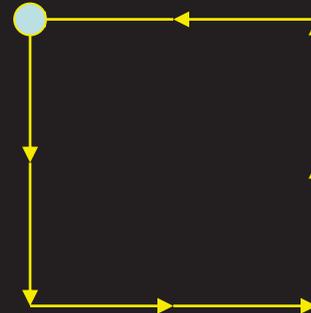
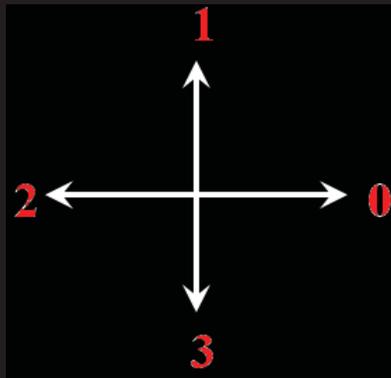




# 2. Shape Representation

Chain Codes: *Boundary Representation*

*Normalization Strategy*



33001122  
Chain code

33001122		00112233
30011223		01122330
00112233		11223300
01122330		12233001
11223300	Sort	22330011
12233001	rows →	23300112
22330011		33001122
23300112		30011223

First row gives the normalized chain code

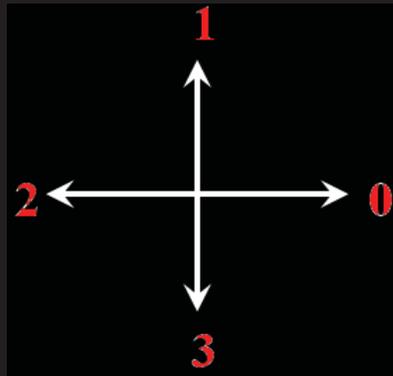
00112233



## 2. Shape Representation

Chain Codes: *Boundary Representation*

*Shape Numbers* = *Normalized Differential Chain Codes*



Differential code:  
 $d_k = c_k - c_{k-1} \pmod{4}$



33001212

33010122

Differentiate

10101131

10113110

Normalize

01011311

01011311

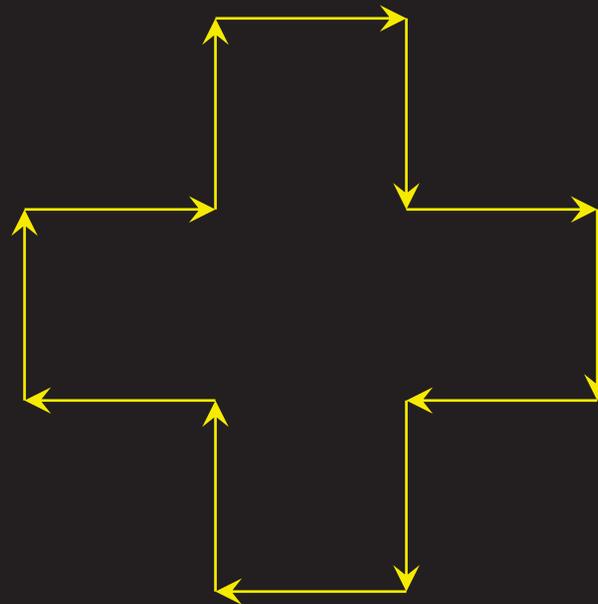
Note that the **shape numbers** of two objects related by 90° rotation are indeed identical



## *2. Shape Representation*

**Chain Codes:** *Boundary Representation*

**Shape Numbers=** *Normalized Differential Chain Codes*



**Homework:** What are the shape numbers of this shape?



## *2. Shape Representation*

### *Representations vs. Descriptors*

After the segmentation of an image, its regions or edges are represented and described in a manner appropriate for further processing.

#### *Shape representation:*

the **ways** we store and represent the objects

- Perimeter (and all the methods based on perimeter)
- Interior (and all the methods ...)

#### *Shape descriptors:*

**features** characterizing object shapes.

- The resulting feature values should be useful for discrimination between different object types.

# 3. Shape Descriptors



## What Are Descriptors?

In general, descriptors are some set of numbers that are produced to describe a given shape. The shape may not be entirely **reconstructable** from the descriptors, but the descriptors for different shapes should be different **enough** that the shapes can be **discriminated**.

## Shape Description methods:

### Shape Represented by its Boundary

1. Statistical Moments
2. Shape Numbers,
3. Fourier Descriptors,

### Shape Represented by its Interior

1. Topological Descriptors
2. Moment Invariants



## *3. Shape Descriptors*

### *3.1 A few simple descriptors are:-*

- **A(X)** is the area of the figure X

The number of pixels in the shape. Your text describes algorithms for calculating the area from quadtree or chain-coding representations.

- **P(X)** is the perimeter of the figure X

**Perimeter(P):** the number of pixels in the boundary of the shape.

- **D<sub>A</sub>(X)** and **D<sub>P</sub>(X)** are the diameters of circles with area **A(X)** and with perimeter **P(X)** , respectively

$$D_A(X) = 2\sqrt{\frac{A(x)}{\pi}} \quad D_P(X) = \frac{P(x)}{\pi}$$

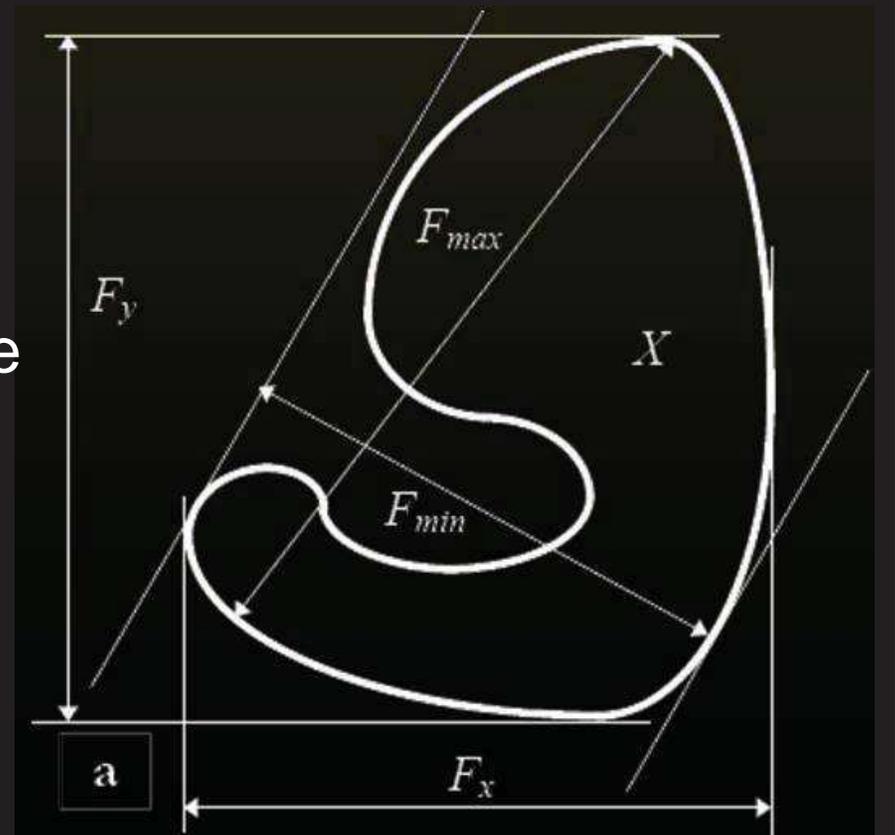
## 3. Shape Descriptors

### 3.1 A few simple descriptors are:-

- **Diameter** =  $\max_{i,j} [D(p_i, p_j)]$  = major axis

$F_x(X)$ ,  $F_y(X)$  are the orthogonal projections of the figure  $X$  on the  $x$  and  $y$  axes.

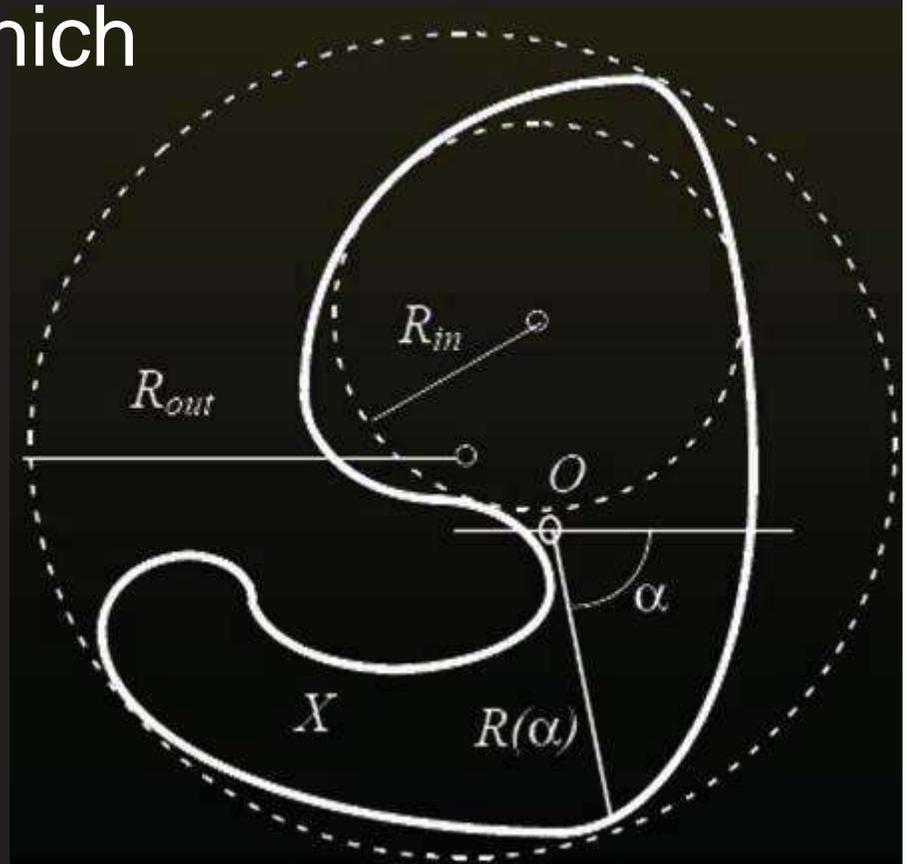
$F_{min}(X)$  and  $F_{max}(X)$  are the minimal and the maximal orthogonal projections of the figure on a line.



## 3. Shape Descriptors

### 3.1 A few simple descriptors are:-

- $R_{in}(X)$  and  $R_{out}(X)$  are the radii of the largest disc in  $X$  and the smallest disc which completely includes  $X$
- Basic **rectangle**  
= major  $\times$  minor





## 3. Shape Descriptors

### 3.1 A few simple descriptors are:-

- **(Non-)Compactness or (Non-)Circularity.** How closely-packed the shape is (not):  $\text{perimeter}^2/\text{area}$ . The most compact shape is a circle ( $4\pi$ ). All other shapes have a compactness larger than  $4\pi$ .
- **Eccentricity.** The ratio of the length of the longest chord of the shape to the longest chord perpendicular to it. (This is one way to define it—there are others.)  $\text{Ecc} = \text{major} / \text{minor}$



## *3. Shape Descriptors*

### *3.1 A few simple descriptors are:-*

- **Elongation.** The ratio of the height and width of a rotated minimal bounding box. In other words, rotate a rectangle so that it is the smallest rectangle in which the shape fits. Then compare its height to its width.
- **Rectangularity.** How rectangular a shape is (how much it fills its minimal bounding box): *area of object/area of bounding box*. This value has a value of 1 for a rectangle and can, in the limit, approach 0 (picture a thin X).
- **Orientation.** The overall direction of the shape.



## *3. Shape Descriptors*

### *3.1 A few simple descriptors are:-*

- **Circularity ratio:** the ratio of the area of the shape to the area of a circle (the most compact shape) having the same perimeter. (*area-perimeter ratio*)

$$M = \frac{4\pi \times \text{area}}{(\text{perimeter})^2}$$

- *For a circle, the ratio is one; for a square, it is  $\pi / 4$ ; for an infinitely long and narrow shape, it is zero.*



## 3. Shape Descriptors

### 3.1 A few simple descriptors are:-

- $\bar{x}(X)$  and  $\bar{y}(X)$  are the coordinates of the center of gravity (also called **center of mass** or **centroids**)
  - they are determined just by averaging the coordinates of each point of the figure  $X$  as:

$$\bar{x} = \frac{u_{10}}{u_{00}} \quad \bar{y} = \frac{u_{01}}{u_{00}}$$

Where  $u_{pq}$  is the regular **moment** of a shape in an  $M$  by  $N$  binary image is defined as:

$$u_{pq} = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} i^p j^q f(i, j)$$

$f(x,y)$  is the intensity of the pixel (either 1 or 0) at the coordinates  $(x,y)$ ,  $p+q$  is said to be the order of the moment.

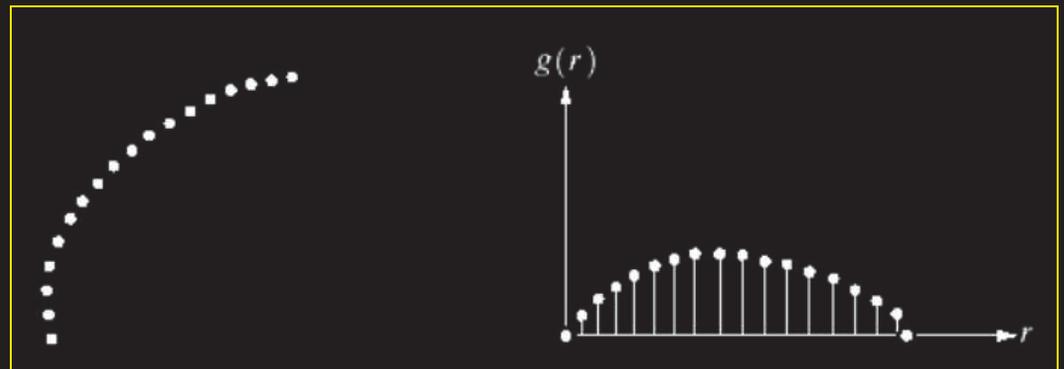


## *3. Shape Descriptors*

### *3.2 Statistical moments:*

1. Useful for describing the shape of boundary segments (or other curves)
2. Suitable for describing the shape of convex deficiencies
3. The histogram of the function (segment curve) can also be used for calculating moments
  - 2<sup>nd</sup> moment gives spread around mean (variance)
  - 3<sup>rd</sup> moment gives symmetry around mean (skewness)

Boundary segment and representation as 1D function



## *3. Shape Descriptors*

### *3.2 Statistical moments:*

Object uniquely defined by infinite sequence of moments  $M_{p,q}$  :

$$m_{p,q} = \int_{Obj} x^p y^q dx dy$$

In terms of pixels  $[1,n] \times [1,m]$  image  $f(x,y)$ :

$$m_{p,q} = \sum_{x=1}^n \sum_{y=1}^m f(x,y) x^p y^q$$

### *3. Shape Descriptors*

#### *3.2 Statistical moments:*

$$m_{p,q} = \sum_{x=1}^n \sum_{y=1}^m f(x,y) x^p y^q$$

# 3. Shape Descriptors

## 3.2 Statistical moments:

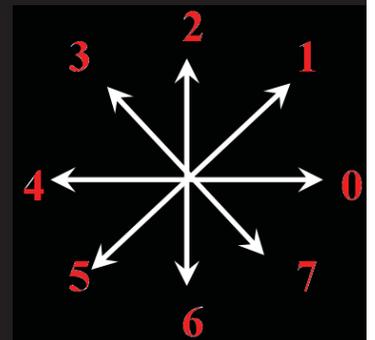
*Example:* A binary image, object is indicated by one's

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	0	0	0	0
2	0	1	1	1	1	1	0	0	0
3	0	1	1	1	1	1	0	0	0
4	0	0	1	1	1	1	1	0	0
5	0	0	1	1	1	1	1	0	0
6	0	0	1	1	1	1	0	0	0
7	0	0	0	1	1	0	0	0	0
8	0	0	0	0	0	0	0	0	0

**Run Length** encoding of the image:  
(0,11,3,5,5,4,5,5,5,4,4,6.2,13)

**Question:** How to describe the object?

**Boundary** Chain code of the object:  
starting pixel: (2,1)  
(0,0,7,6,7,6,5,5,4,3,2,2,3,2,1)



## *3. Shape Descriptors*

### *3.2 Statistical moments:*

The moments of an object ( $\mathcal{O}$ ) are defined as:

$$m_{p,q} = \sum_{(x,y) \in \mathcal{O}} x^p y^q ; p \geq 0; q \geq 0$$

where  $(x,y)$  is the coordinates of a pixel in  $\mathcal{O}$ .

Consider,

1.  $p = 0; q = 0 \rightarrow m_{00} = \sum_{(x,y) \in \mathcal{O}} x^0 y^0$

$\Rightarrow$  Size of  $\mathcal{O}$ , i.e. no. of pixels with value of 1

$$m_{00} = 29 \text{ (example)}$$

## 3. Shape Descriptors

### 3.2 Statistical moments:

2.  $p=1; q=0; \rightarrow$   $m_{10} = \sum_{(x,y) \in O} x^1 y^0$   
 $m_{10} = 100$  (example)

3.  $p=0; q=1; \rightarrow$   $m_{01} = \sum_{(x,y) \in O} x^0 y^1$   
 $m_{01} = 111$  (example)

#### Centroids (Center of Mass)

$$(x_c, y_c) = \left( \frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}} \right)$$

$$(x_c, y_c) = (100/29, 111/29) = (3.45, 3.83) \text{ (example)}$$

# 3. Shape Descriptors

## 3.2 Statistical moments:

### Central Moments

The central moments of an object ( $O$ ) are defined as :

$$\mu_{kj} = \sum_{(x,y) \in O} (x - x_c)^k (y - y_c)^j; \quad k \geq 0; j \geq 0$$

where  $(x,y)$  is the coordinates of a pixel in  $O$  and  $(x_c, y_c)$  is the centroid of the object.

### Note:

- Invariant to translation of object. **Q: what about Rotation?**
- If the origin is translated to the centroid, the central moments become the standard moments
- $\mu_{11}$  is called a product moment
- $\mu_{20}, \mu_{02}$  are moments of inertia of the object w.r.t. the  $x$  and  $y$  axes through the centroid.

# 3. Shape Descriptors

## 3.2 Statistical moments:

	0	1	2	3	4	5	6	7	8	9	(x)
0	0	0	0	0	0	0	0	0	0	0	
1	0	0	0	0	0	0	0	0	0	0	
2	0	1	1	1	0	0	0	0	0	0	
3	0	0	0	1	1	1	0	0	0	0	
4	0	0	0	0	0	1	1	1	0	0	
5	0	0	0	0	0	0	0	0	0	0	
(y)											

$$m_{00} = 9; m_{10} = 36; m_{01} = 27;$$
$$(x_c, y_c) = (36/9, 27/9) = (4, 3);$$

$$\mu_{10} = 0; \mu_{01} = 0;$$
$$\mu_{11} = 12; \mu_{20} = 30; m_{02} = 6;$$

### Image Translated:

0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0	0
2	0	0	0	1	1	1	0	0	0	0
3	0	0	0	0	0	1	1	1	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0

$$m_{00} = 9; m_{10} = 36; m_{01} = 18;$$
$$(x_c, y_c) = (36/9, 18/9) = (4, 2);$$

$$\mu_{10} = 0; \mu_{01} = 0;$$
$$\mu_{11} = 12; \mu_{20} = 30; m_{02} = 6;$$

## *3. Shape Descriptors*

### *3.2 Statistical moments:*

#### Moment invariants

- Translation invariant: central moments

$$\mu_{p,q} = \int_{Obj} \left( x - \frac{m_{1,0}}{m_{0,0}} \right)^p \left( y - \frac{m_{0,1}}{m_{0,0}} \right)^q dx dy$$

- Invariant under uniform scaling

$$\eta_{p,q} = \frac{\mu_{p,q}}{\mu_{0,0}^{(p+q+2)/2}}$$



## *3. Shape Descriptors*

### *3.2 Statistical moments:*

#### Moment invariants

Rotation invariant:

$$\varphi_1 = \mu_{20} + \mu_{02}$$

$$\varphi_2 = (\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2$$

$$\varphi_3 = (\mu_{30} - 3\mu_{12})^2 + (3\mu_{21} - \mu_{03})^2$$

$$\varphi_4 = (\mu_{30} + \mu_{12})^2 + (\mu_{21} + \mu_{03})^2$$

$$\varphi_5 = (\mu_{30} - 3\mu_{12}) + (\mu_{30} + \mu_{12}) [(\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2] + \\ (3\mu_{21} - \mu_{03}) + (\mu_{21} + \mu_{03}) [3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2]$$

$$\varphi_6 = (\mu_{20} - \mu_{02}) [(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2] + 4\mu_{11} (\mu_{30} + \mu_{12}) (\mu_{21} + \mu_{03})$$

Reflection and rotation invariant:

$$\varphi_7 = (3\mu_{21} - \mu_{03})(\mu_{30} + \mu_{12}) [(\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2] + \\ (\mu_{30} - 3\mu_{12})(\mu_{21} + \mu_{03}) [3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2]$$

# *Similarity Measures*

	<b>Direct</b>	<b>Feature based</b>
<b>Boundary</b>	length of a boundary , Chain code, Arc Decomposition (ASR-Algorithm)	Central Dist. Fourier Distance histogram ...
<b>Area (point set)</b>	Hausdorff ...	Statistical Moments Zernike Moments ...
<b>Structure</b>	Skeleton ...	---

# *Important Notes*

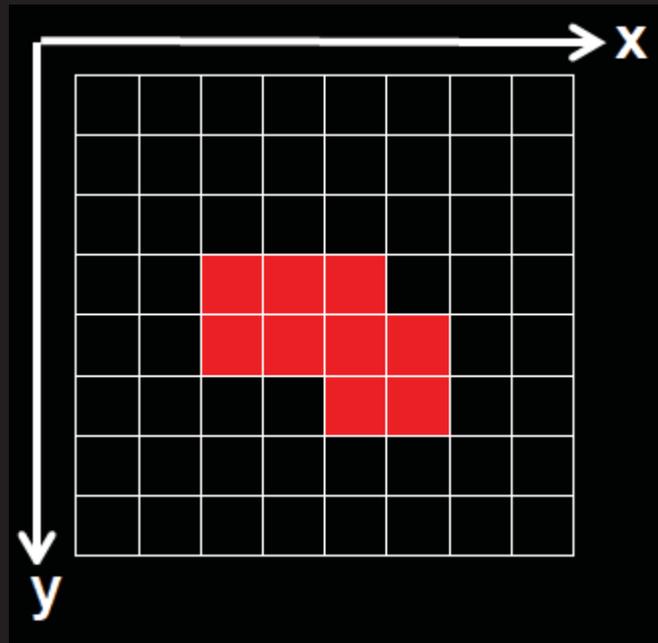
- The **area** of a region is defined by the **total number of pixels** in the region.
- The **perimeter** is given the number of **pixels along** the length of the boundary of the region.
- The **length of a boundary** is one of the simple boundary descriptor.
- The length of the boundary is approximately given by the number of pixels along that boundary.

# *Important Notes*

- The **minor axis** of a boundary is defined as the line perpendicular to the **major axis** and of such length that a box passing through the outer four points of intersection of the boundary with the two axes completely encloses the boundary.
- **Eccentricity**, which is the ratio of major axis to the minor axis which is one of the important parameter that is used to describe a boundary.

## Quiz

Compute the center for the following shapes using moments



Discrete Point Sets

# *Important Notes*

- SS

**END**

**Of Lecture**



## *3. Shape Descriptors*

### *3.1 Fourier Descriptor*

1. Useful for comparing objects. Very difficult to interpret geometrically. 2D problem reduced to 1D problem.
2. Write **x** and **y** coordinates as two vectors, and use each coordinate pair as a complex number. ) **x-axis** treated as real axis and **y** as imaginary axis.
3. Fourier transforming the new coordinates generates the Fourier descriptors.
4. Inverse transforming all these descriptors regenerates the original coordinates. If only some of the descriptors are used in the inverse transform an approximation of the original object is the result.

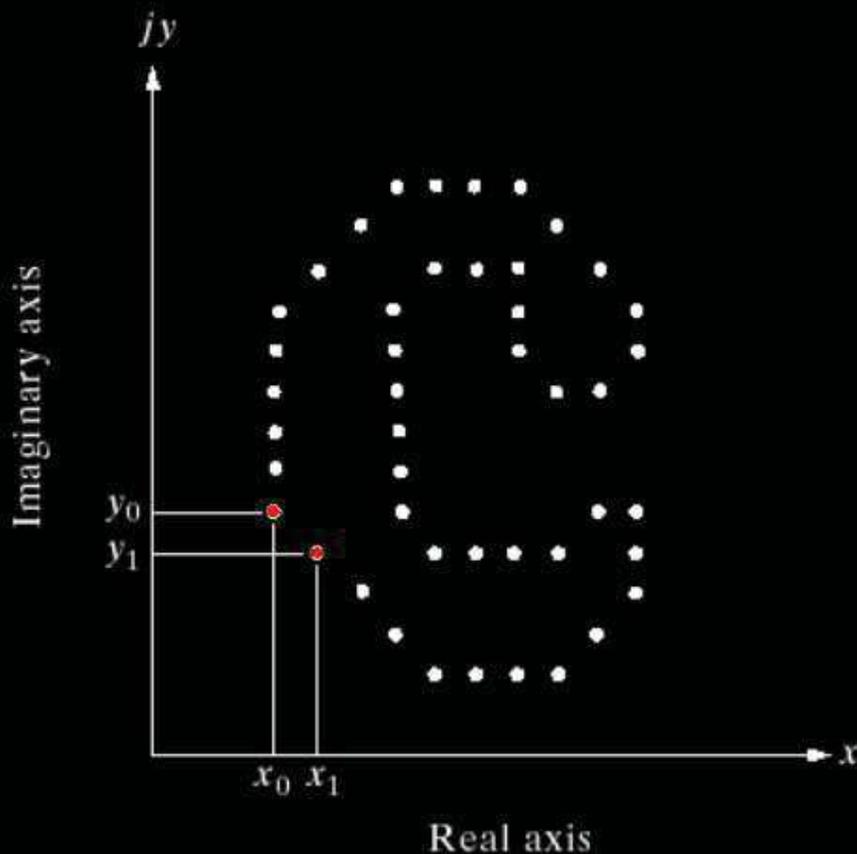


# 3. Shape Descriptors

## 3.1 Fourier Descriptor

Represent boundary as sequence of coordinates.

$$s(k) = [x(k), y(k)], k = 0, 1, 2, \dots, K - 1$$



Treat each coordinate pair as a complex number (reduces problem from 2D to 1D).  
 $s(k) = x(k) + jy(k)$



## 3. Shape Descriptors

### 3.1 Fourier Descriptor

From the DFT of the complex number we get the Fourier descriptors (the complex coefficients,  $a(u)$ ).

$$a(u) = \frac{1}{K} \sum_{k=0}^{K-1} s(k) e^{-j2\pi uk/K}, \quad u = 0, 1, 2, \dots, K-1$$

The inverse Fourier transform of these coefficients restores  $s(k)$ .

$$s(k) = \sum_{u=0}^{K-1} a(u) e^{j2\pi uk/K}, \quad k = 0, 1, 2, \dots, K-1$$

We can create an approximate reconstruction of  $s(k)$  using only the first  $P$  Fourier coefficients.

$$\hat{s}(k) = \sum_{u=0}^{P-1} a(u) e^{j2\pi uk/K}, \quad k = 0, 1, 2, \dots, K-1$$



**M = NoPoints - 1**

ReDim Dx(1 To M), Dy(1 To M), dT(1 To M), St(M)

ReDim A(1 To Order), B(1 To Order), C(1 To Order), D(1 To Order)

MaxX = X(0): MinX = X(0): MaxY = Y(0): MinY = Y(0)

**For I = 1 To M**

If MaxX < X(I) Then MaxX = X(I) Else If MinX > X(I) Then MinX = X(I)

If MaxY < Y(I) Then MaxY = Y(I) Else If MinY > Y(I) Then MinY = Y(I)

**Next I**

Lx = MaxX - MinX: Ly = MaxY - MinY

**For I = 1 To M: J = I - 1**

Dx(I) = (X(I) - X(J)) / Lx: Dy(I) = (Y(I) - Y(J)) / Ly

dT(I) = Sqr(Dx(I) \* Dx(I) + Dy(I) \* Dy(I))

If dT(I) > 0 Then Dx(I) = Dx(I) / dT(I): Dy(I) = Dy(I) / dT(I) Else Dx(I) = 0: Dy(I) = 0

**Next I**

St(0) = 0: St(1) = dT(1)

**For I = 2 To M: St(I) = St(I - 1) + dT(I): Next I**

T = St(M)

Pi = 4 \* Atn(1): P = 2 \* Pi / T: R = T / (2 \* Pi \* Pi)

**For N = 1 To Order: Q = P \* N: S = R / (N \* N)**

SmA = 0: SmB = 0: SmC = 0: SmD = 0

**For I = 1 To M: A1 = Q \* St(I): A2 = Q \* St(I - 1)**

C1 = Cos(A1): C2 = Cos(A2): Cc = C1 - C2

S1 = Sin(A1): S2 = Sin(A2): Ss = S1 - S2

SmA = SmA + Dx(I) \* Cc: SmB = SmB + Dx(I) \* Ss

SmC = SmC + Dy(I) \* Cc: SmD = SmD + Dy(I) \* Ss

**Next I**

A(N) = SmA \* S: B(N) = SmB \* S

C(N) = SmC \* S: D(N) = SmD \* S

Print #1, Format(A(N), "###0.000 "); Format(B(N), "###0.000 "); Format(C(N), "###0.000

"); Format(D(N), "###0.000 ")

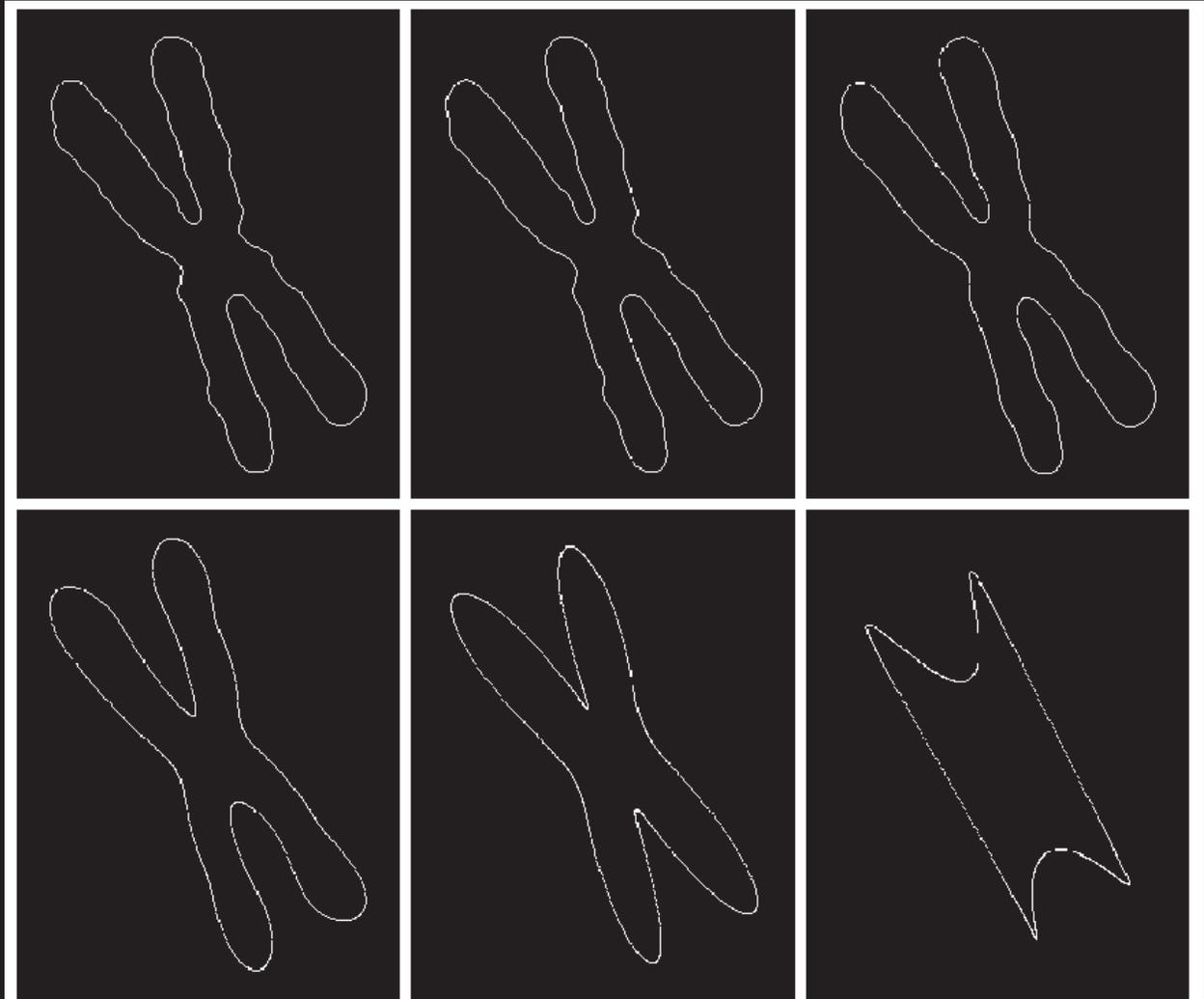
**Next N**

Fourier descriptor source  
code, written with VB6

# *3. Shape Descriptors*

## *3.1 Fourier Descriptor*

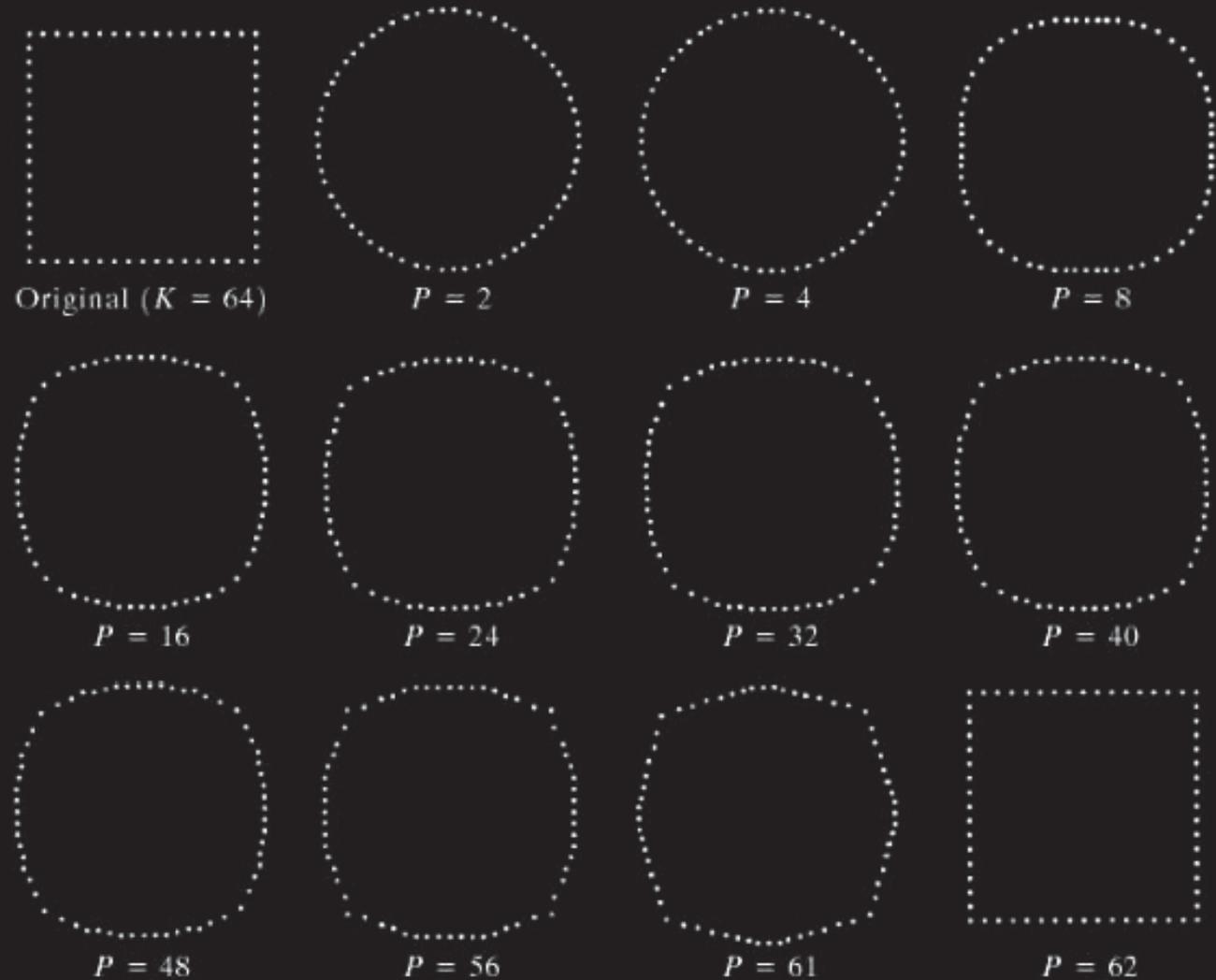
Boundary reconstructed using 546, 110, 56, 28, 14 and 8 Fourier descriptors out of a possible 1090 descriptors.



# 3. Shape Descriptors

## 3.1 Fourier Descriptor

This boundary consist of 64 points,  $P$  is the number of descriptors used in the reconstruction of the boundary



# *Applications of Binary Image Processing and Analysis*

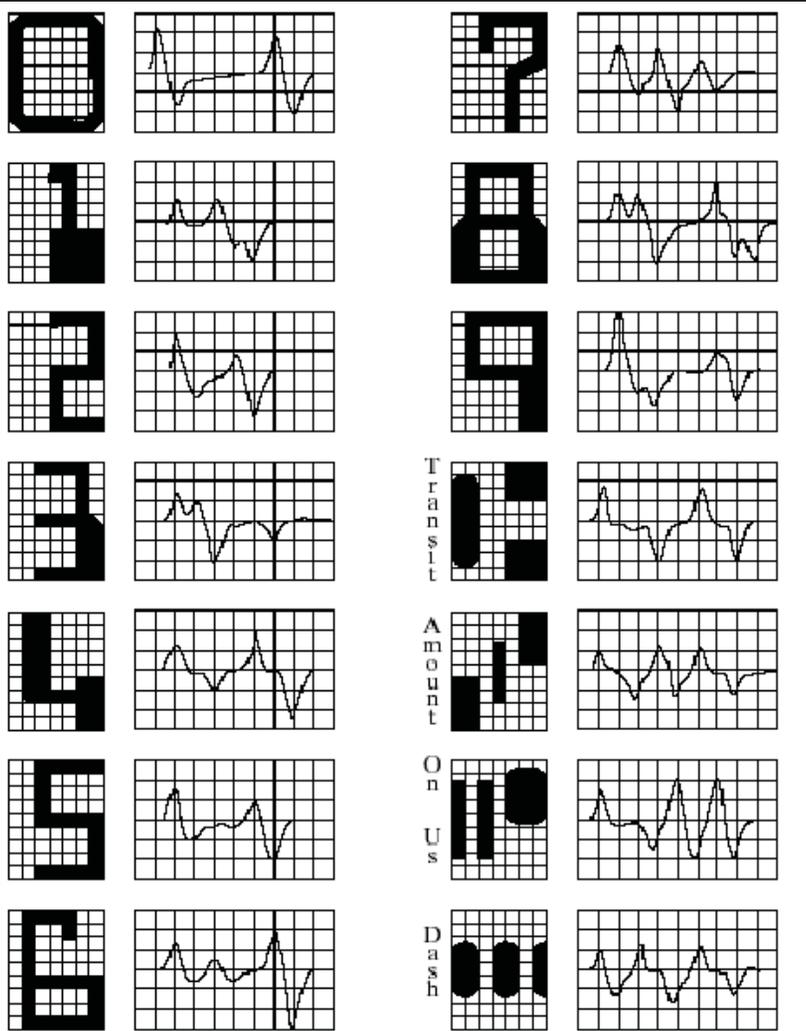


- Optical Character Recognition (OCR)
  - Tax form processing, Google Books, ...
- Barcode recognition
  - Grocery shopping
- Handwriting recognition
  - Biometrics, forensics
- Fingerprint recognition
  - Biometrics, forensics

# Applications of Binary Image Processing and Analysis



## Bank Note Character Recognition



American Banker's  
Association E-13B  
Font character set:

14 characters  
9-by-7 grid

Distinct 1D signature  
is generated as the  
reading head moves  
from left to right and  
detects the change  
of ink area under the  
head

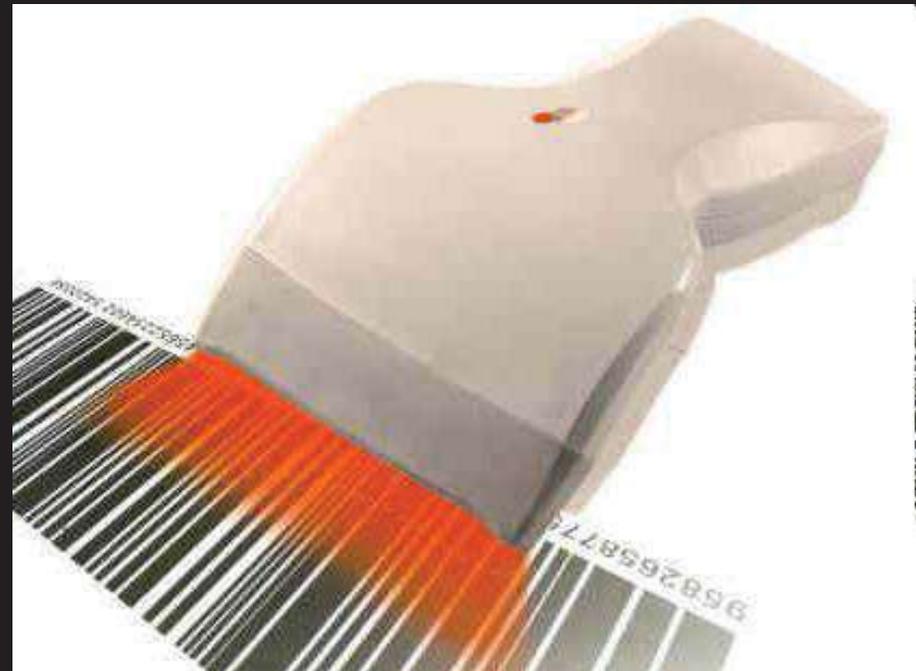
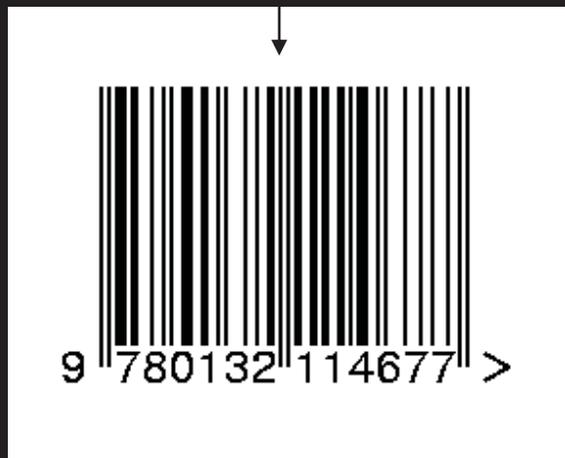
# *Applications of Binary Image Processing and Analysis*



## Barcode recognition



optical scanner

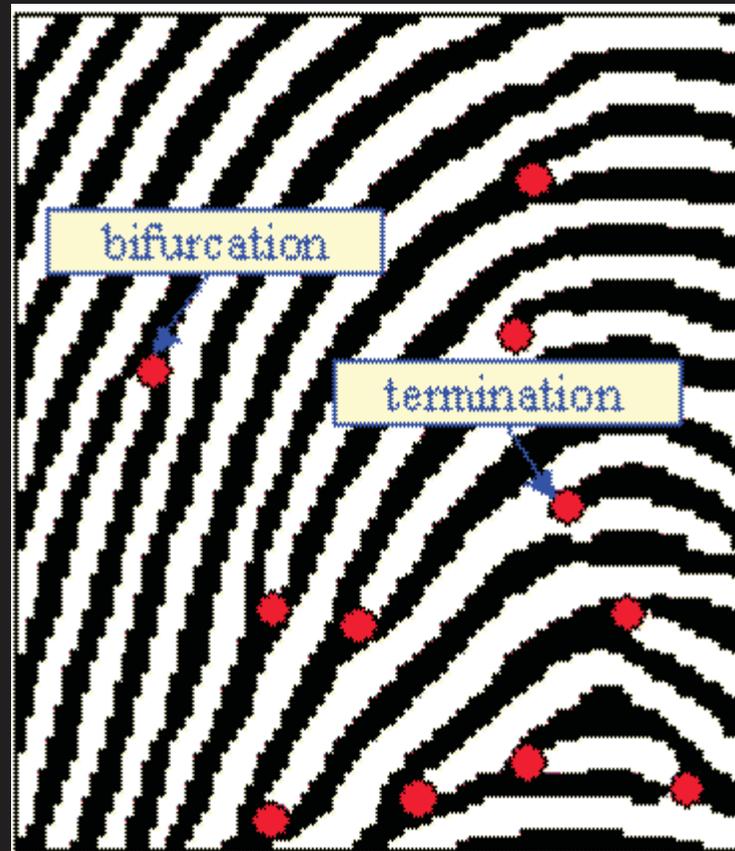


Laser scanner

# *Applications of Binary Image Processing and Analysis*



## Minutiae-based Fingerprint Recognition



# *Applications of Binary Image Processing and Analysis*



## Handwriting Recognition





# *Similarity Measures*

## **Classes of Similarity Measures:**

Similarity Measure depends on

- **Shape Representation**
  - **Boundary**
  - **Area (discrete: = point set)**
  - **Structural (e.g. Skeleton)**
- **Comparison Model**
  - **feature vector**
  - **direct**



# *Similarity Measures*

## Feature Based Coding

This category defines all approaches that determine a feature-vector for a given shape.

Two operations need to be defined:

1. a **mapping** of shape into the feature space and
2. a **similarity** of feature vectors.

Representation



Feature Extraction



Vector Comparison



# *Similarity Measures*

## **Vector Comparison methods:**

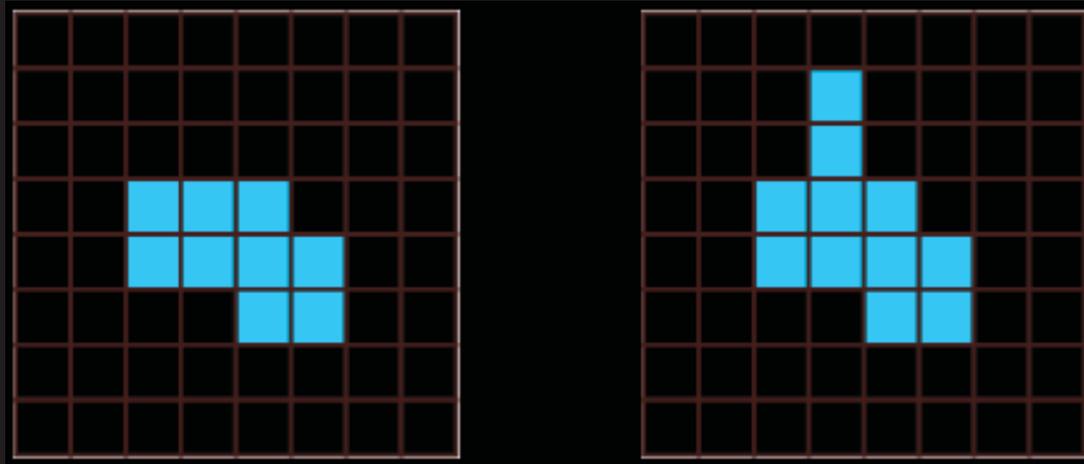
1. (Discrete) Moments
2. Shape A,B given as:
  - Area (continuous) or
  - Point Sets (discrete)



# *Similarity Measures*

## Exercise:

Please compute all 7 moments for the following shapes, compare the vectors using different comparison techniques

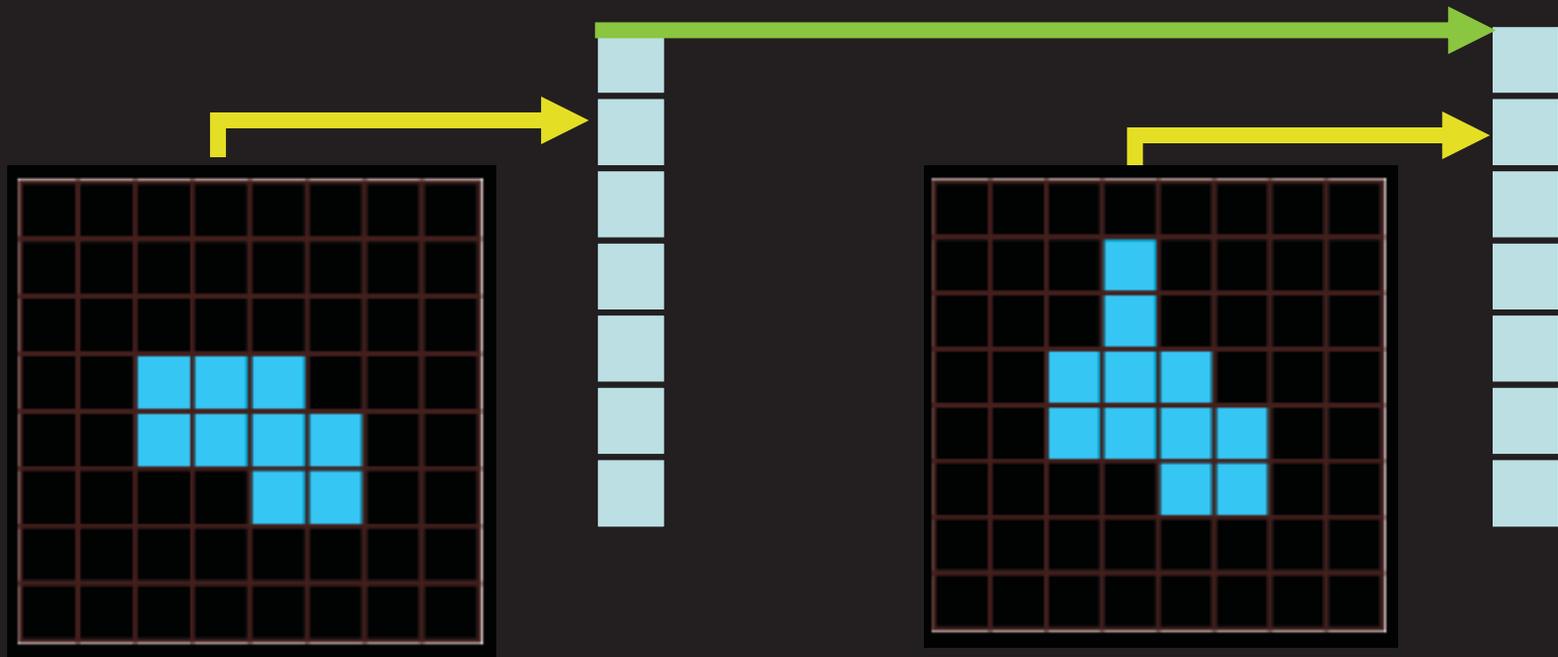


Discrete Point Sets

# *Similarity Measures*

## Exercise:

**Result:** each shape is transformed to a 7-dimensional vector. To compare the shapes, compare the vectors (how?).





# *Summary*

- There are lots of ways to try to describe/quantify the shape of an object from points on the boundary
- Remember the objectives:
  - Compact representation
  - Invariant to as many transformations as possible (translation, rotation, scale, projection, etc.)
  - Relatively insensitive to small variations
- Useful structural representations
  - Boundary (chain codes and shape numbers)
  - Skeleton (string code and tree grammar)
- Various important applications
  - Some are more successful than others
  - Always the tradeoff between cost and performance

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

السَّلَامُ عَلَيْكُمْ وَرَحْمَةُ اللَّهِ وَبَرَكَاتُهُ

**Lecture**

# **Digital Image Processing**

## *Image Classification and Pattern Recognition*

*(Pattern Recognition => Classification)*

*Lecturer:*

*Dr. Loay E. George and Dr. Faisal G. Mohammed*

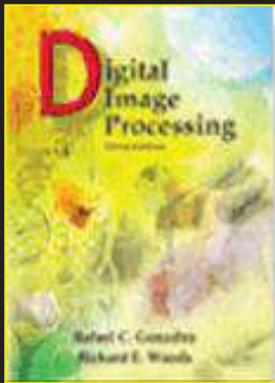
Email: [faisal@scbaghdad.edu.iq](mailto:faisal@scbaghdad.edu.iq)

[faiselgm73@gmail.com](mailto:faiselgm73@gmail.com)

*2020-2021*

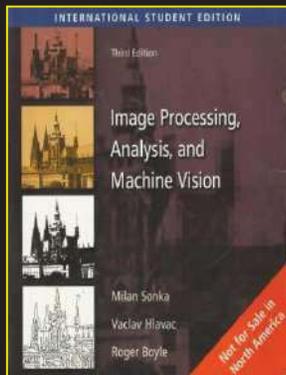
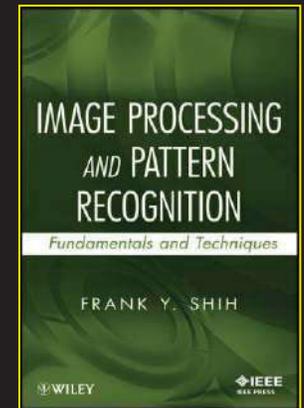
*All rights reserved*

# Reading instructions



Chapter 12  
**“Digital Image Processing”**,  
Gonzalez & Woods: 2<sup>nd</sup> ed.,  
2002.

Chapter 9  
**“IMAGE PROCESSING  
AND PATTERN  
RECOGNITION:  
Fundamentals and  
Techniques”**,  
FRANK Y. SHIH, 2010



Chapter 9  
**“Image Processing  
Analysis, and  
Machine Vision”**,  
Milan Sonka, Vaclav Hlavac  
and Roger Boyle, 2008

# *Contents*

*Today we'll look at color image, covering:*

1. Basics of Classification
2. Supervised Classification
  - Process (e.g. collecting training data)
  - Algorithms
3. **Accuracy Assessments**
4. Unsupervised Classification
  - Algorithms

# Learning objectives

In this lecture we will use the image dataset that we created in the last lecture to build an image classifier.

# Learning outcomes

- Today's lecture focuses on the procedures used in the digital (automated) classification of remotely sensed imagery
- At the end of today's lecture (and its associated practical) you should be able to:
  1. Describe the basic approaches to image classification and understand the differences between unsupervised and supervised methods
  2. Select (and justify your choice of) an appropriate classification method for the analysis and extraction of land cover data from remotely sensed imagery

# *1. An overview on classification*

## Classification

- Classification is a procedure for **sorting pixels** and assigning them to specific categories.
- Characterize pixels using features
  - original band gray values
  - algebraic combinations of the original bands
  - texture measures
  - ....
- The set of characterizations is called a **feature vector**  
e.g.,  $\mathbf{x} = (k1, k2, k3)$  where, for example:  
k1 = gray value in band 1  
k2 = gray value in band 2  
k3 = ratio of gray values in bands 2 and 3

# *1. An overview on classification*

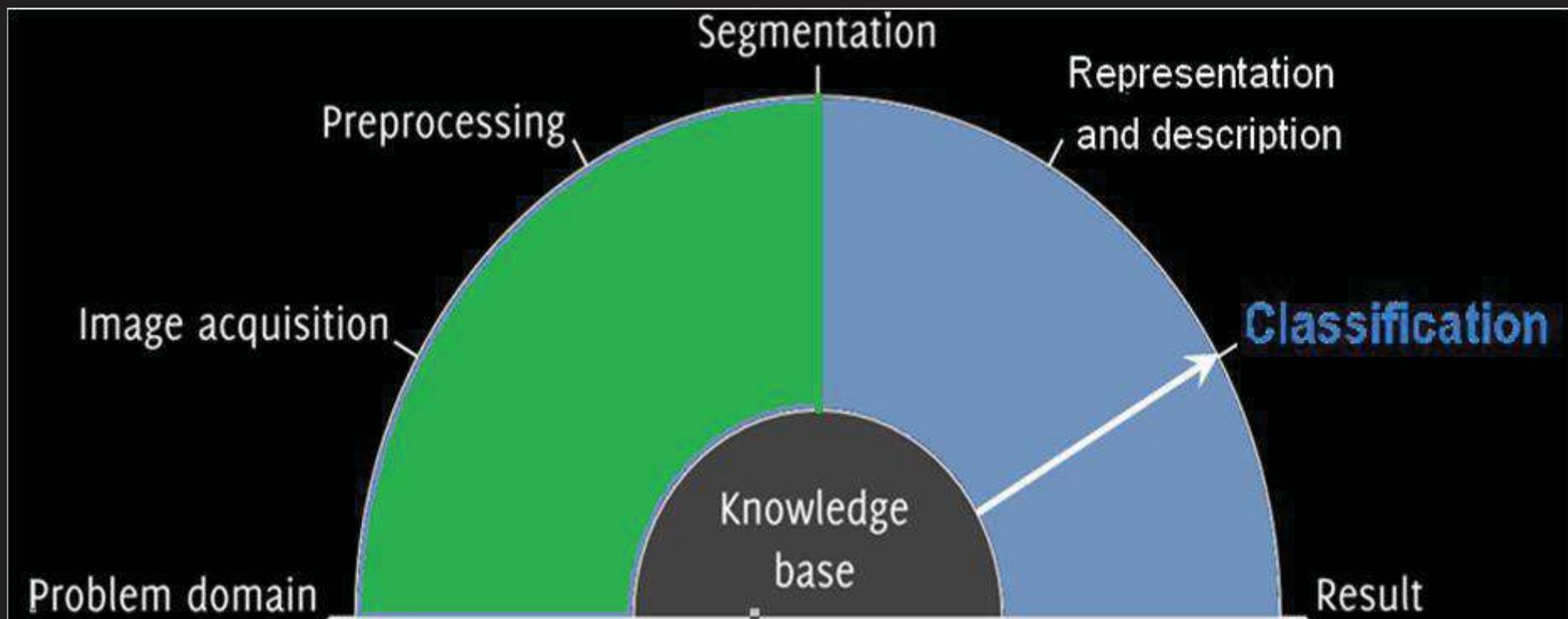
## Pattern Recognition

- Pattern recognition in remote sensing has been based on the intuitive notion that ***pixels belonging to the same class should have similar gray values in a given band.***
- Given two spectral bands, pixels from the same class plotted in a two-dimensional histogram should appear as a localized cluster.
- If  $n$  images, each in a different spectral band, are available, pixels from the same class should form a localized cluster in  $n$ -space.

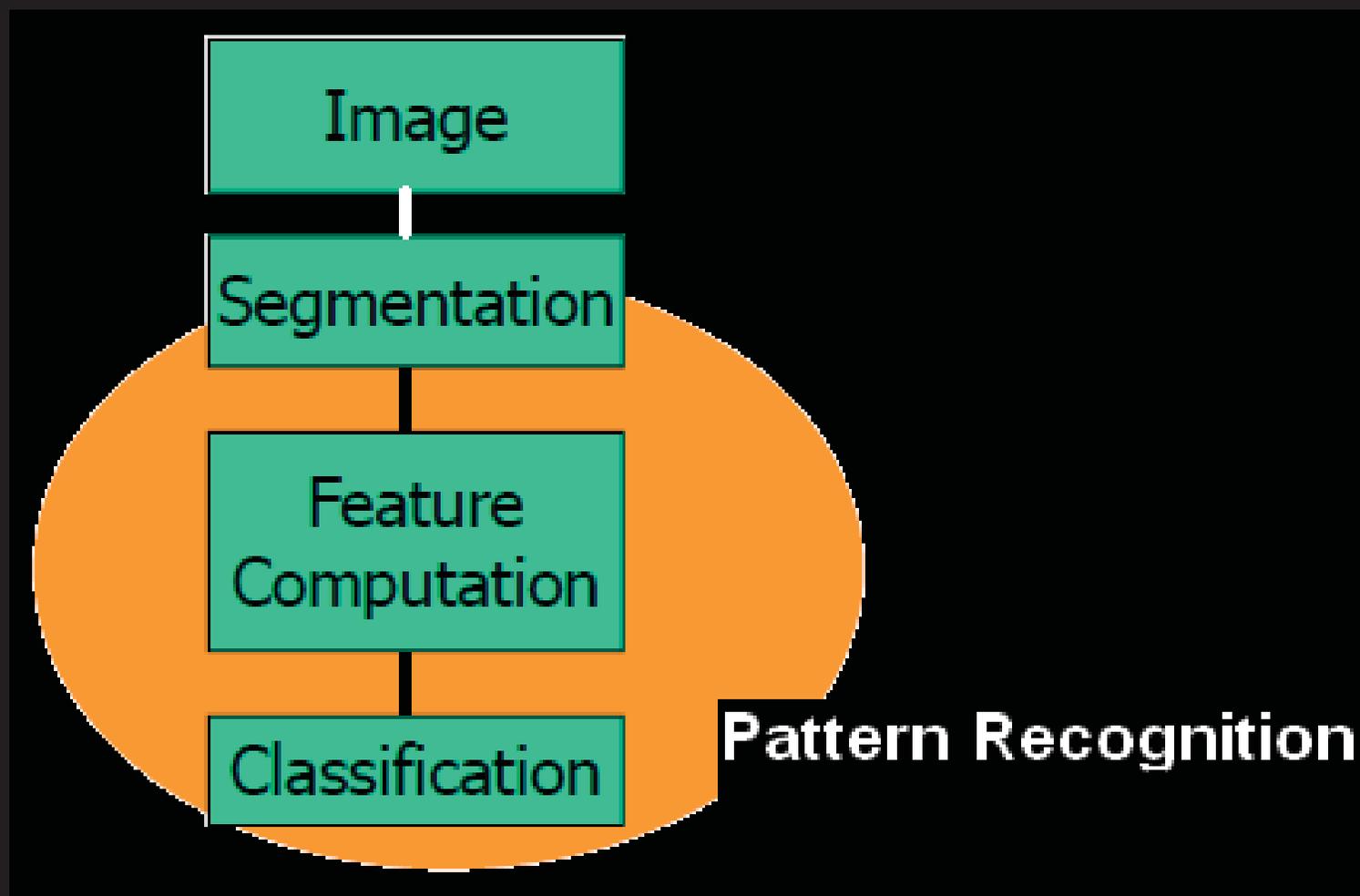
# *1. An overview on classification*



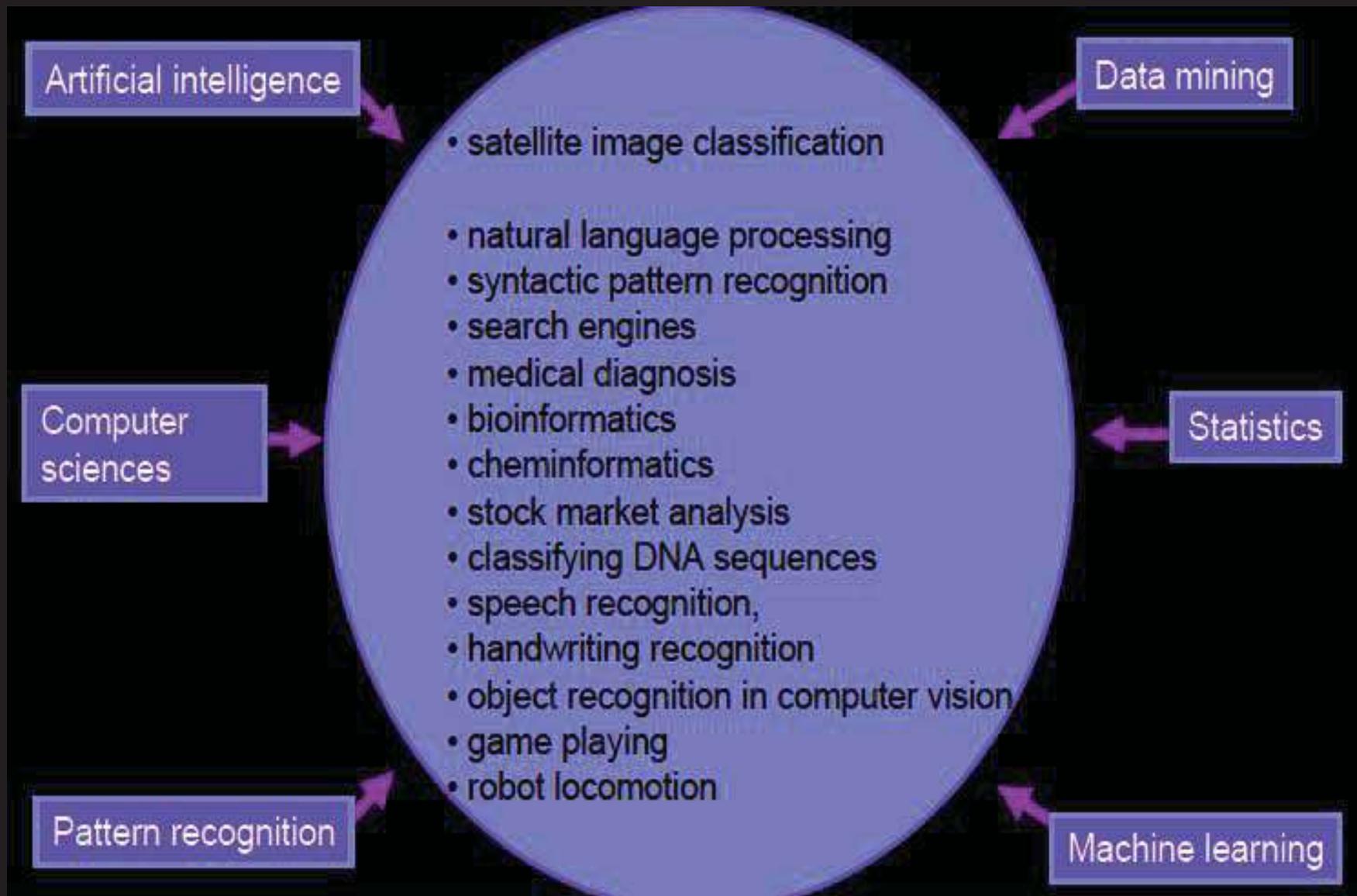
## *Digital Image Analysis Process*



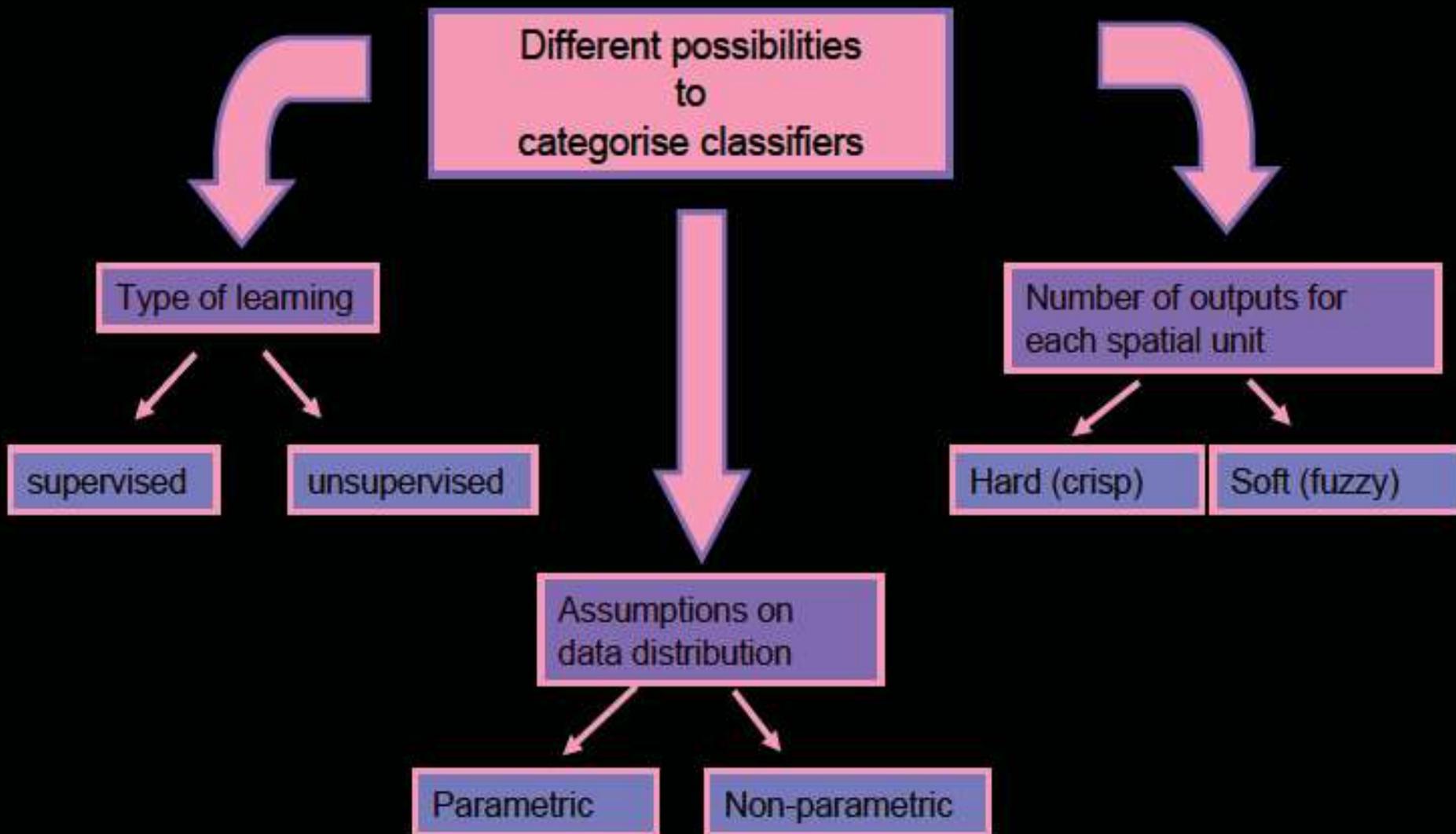
# *1. An overview on classification*



# *1. An overview on classification*



# *1. An overview on classification*



# 1. An overview on classification



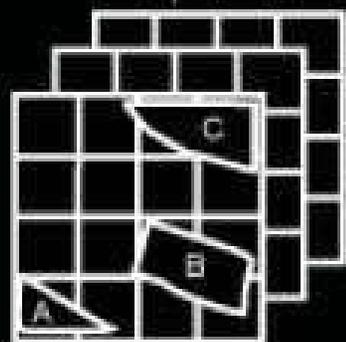
Type of learning

Supervised  
classification

Unsupervised  
classification



A = water  
B = agriculture  
C = rock



DN's

98	178	183	180
96	87	177	181
12	96	98	87
14	11	89	98

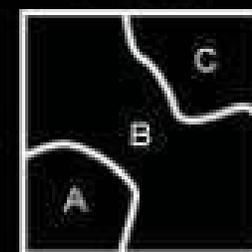
Algorithm

B	C	C	C
B	B	C	C
A	B	B	B
A	A	B	B

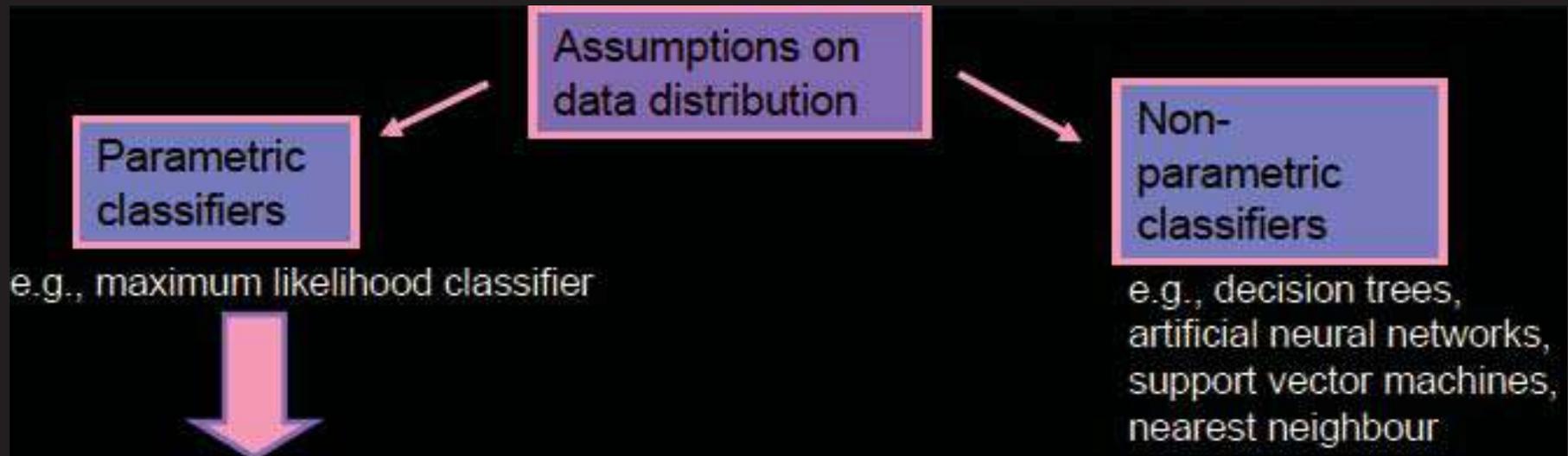
Spectral Classes

Class Identification

A = water  
B = agriculture  
C = rock



# 1. An overview on classification



- They suffer from the **Hughes** phenomenon (i.e. curse of dimensionality), and consequently it might be difficult to have a significant number of training pixels.
- These classifiers rely on assumptions of data distribution.
- The performance of a parametric classifier depends largely on how well the data match the pre-defined models and on the accuracy of the estimation of the model parameters.
- Traditionally most classifiers have been grounded to a significant degree in statistical decision theory.
- They are not adequate to integrate ancillary data (due to difficulties on classifying data at different measurement scales and units).

# 1. An overview on classification



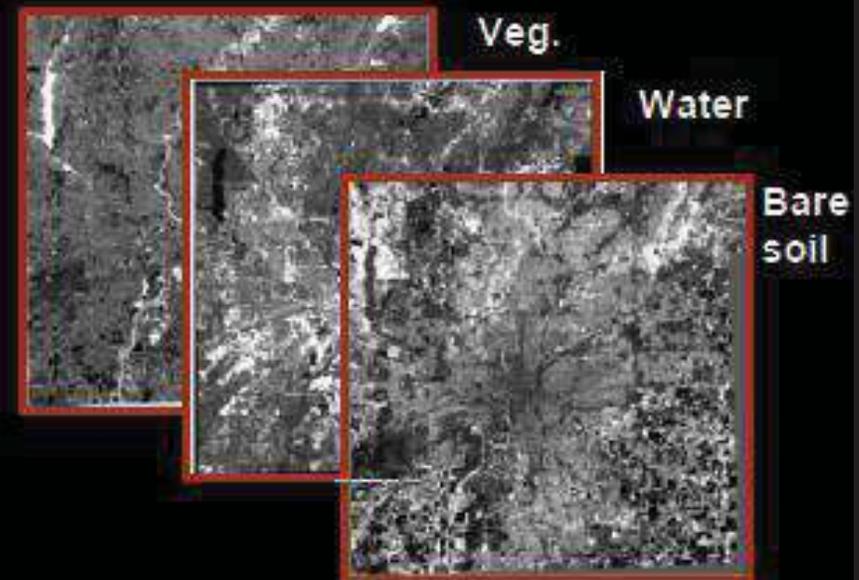
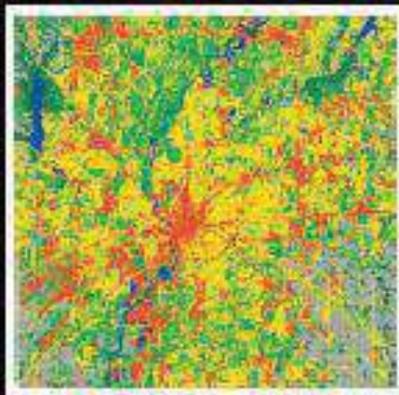
Number of outputs for each spatial unit

Hard (crisp) classification

Soft (fuzzy) classification

each pixel is forced or constrained to show membership to a single class.

each pixel may display multiple and partial class membership.





# *1. An overview on classification*

## Overall objective of classification

- Automatically categorize all pixels in an image into land cover classes or themes

## Three pattern recognitions approaches are

1. Spectral pattern recognition
2. Spatial pattern recognition
3. Temporal pattern recognition

## Selection of classification

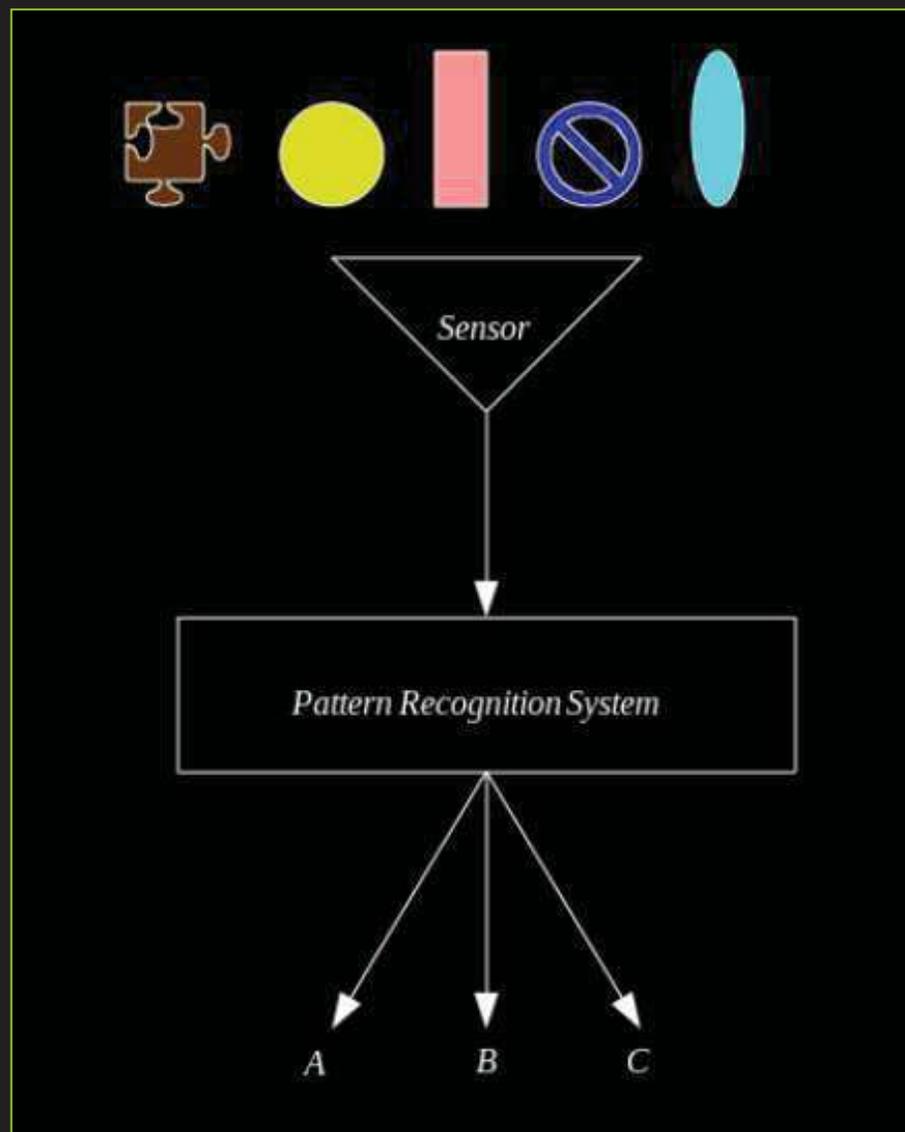
- No single “right” approach
- Depend on
  - The nature of the data being analyzed
  - The computational resources available
  - The intended application of the classified data

# *1. An overview on classification*



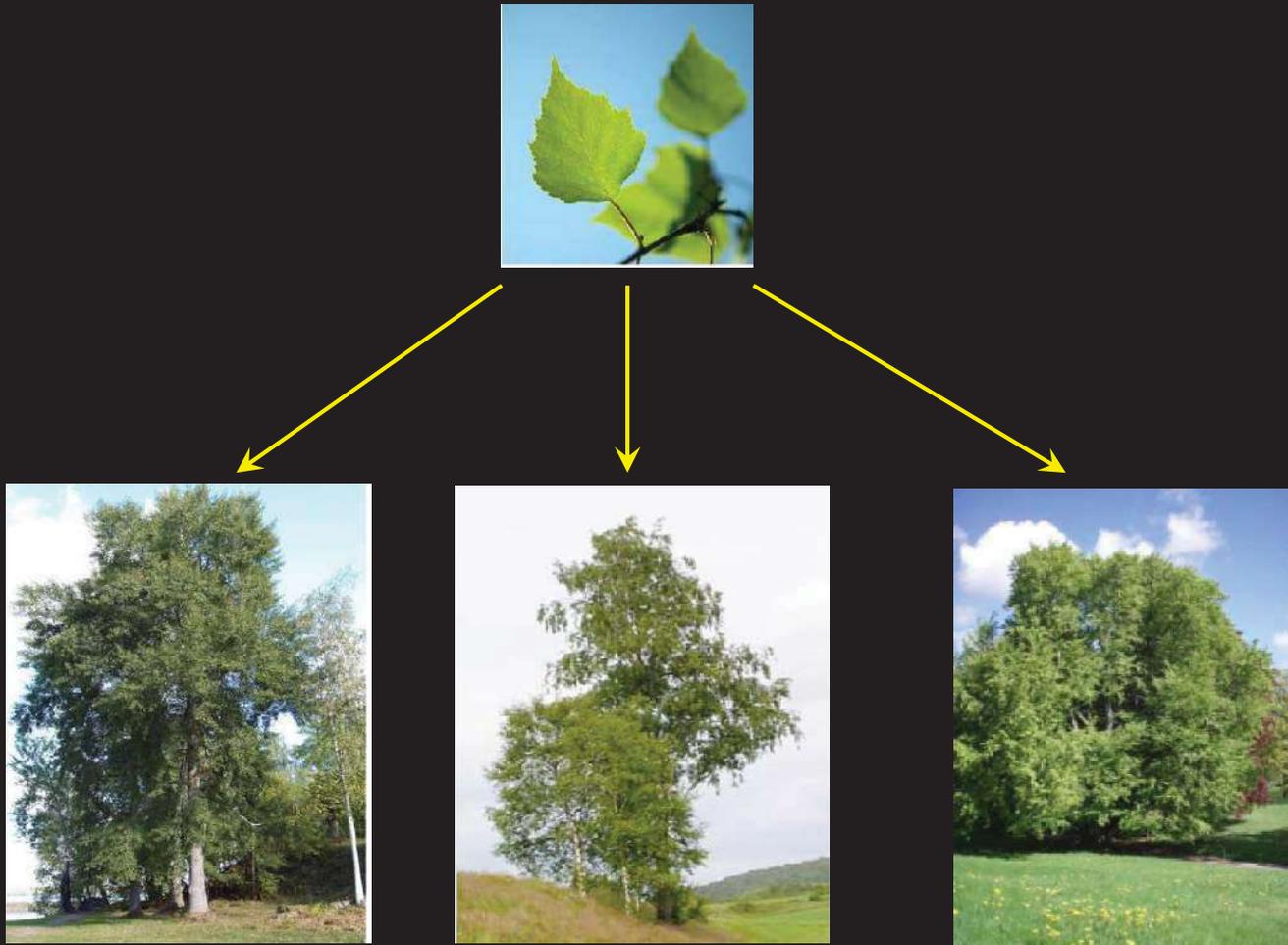
## What is classification ?

- Classification is a process in which individual items (objects/patterns/image regions/pixels) are grouped based on the similarity between the item and the description of the group.



# *1. An overview on classification*

## Classification - Example



Classification of leaves



# *1. An overview on classification*

## *Terminology*

- Object = pattern = point = sample = vector
- Feature = descriptor = attribute = measurement
- Classifier = decision function (boundary)
- Class
- Cluster



# *1. An overview on classification*

## *Terminology*

### *Classification Decision Rules*

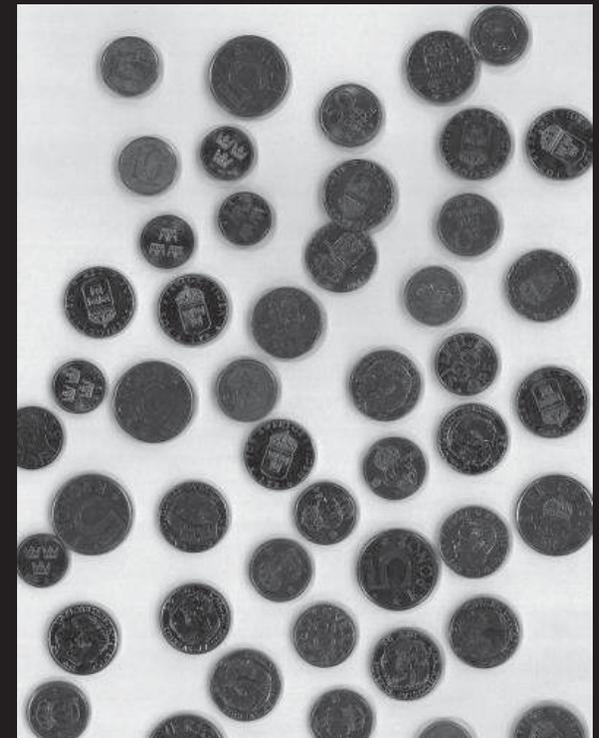
- Decision Rules = for sorting pixels into classes
- **Parametric** = based upon statistical parameters (mean & standard deviation)
- **Non-Parametric** = based upon objects (polygons) in feature space

# *1. An overview on classification*



## Features (Descriptors)

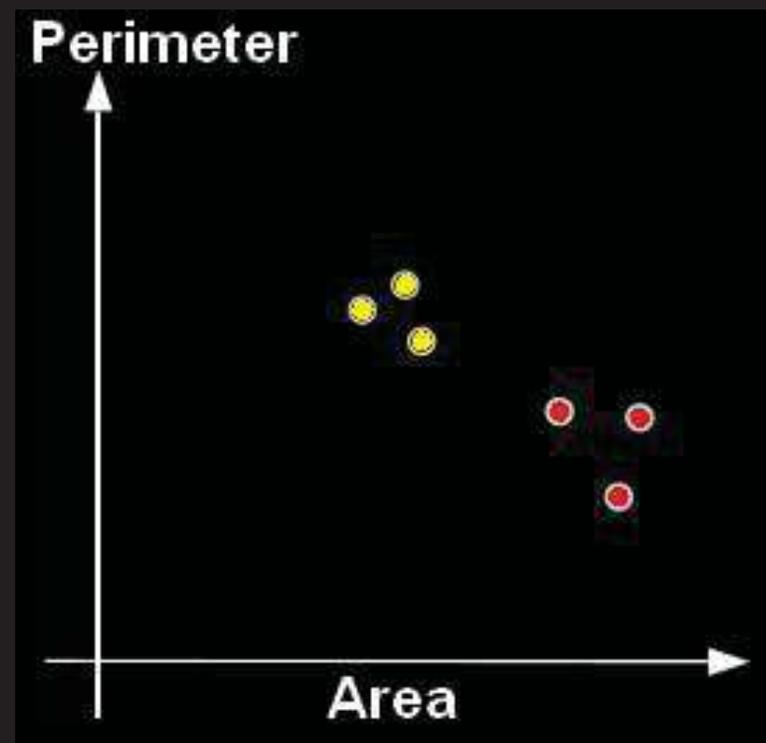
- Features are the individual measurable heuristic properties of the phenomena
- Discriminating (effective) features
- Independent features
- Features:
  - area, perimeter
  - texture
  - color
  - ...



# *1. An overview on classification*



Data set and a representation in the feature space



# *1. An overview on classification*



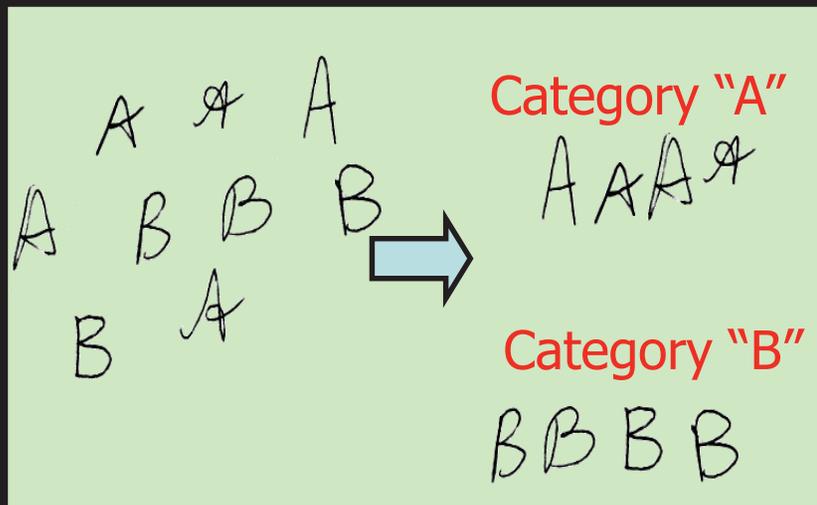
## What are good features ?

- Each pattern is represented in terms of  $n$  features  $x = (x_1; \dots; x_n)$
- The goal is to choose those features that allow pattern vectors belonging to different classes to occupy compact and disjoint regions
- It is application dependent
- You might try many, many features, until you find the right ones
- Often, people compute 100s of features, and put them all in a Classifier
  - The classifier will figure out which ones are good
  - This is wrong!!!

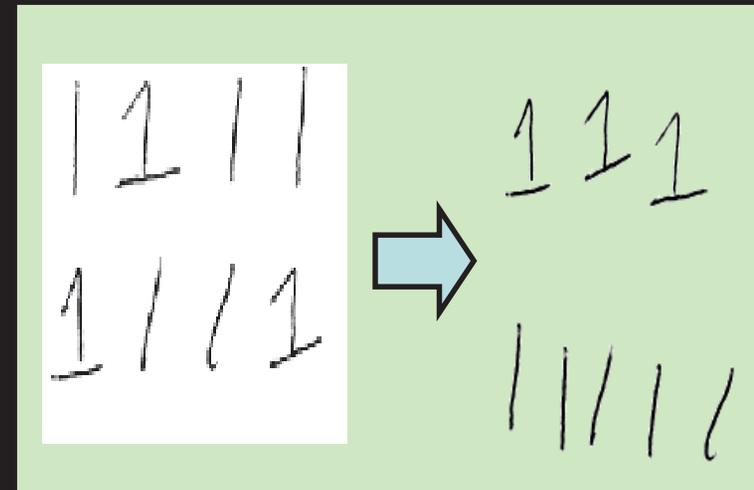
# 1. An overview on classification

## Classification vs. Clustering

- Classification (known categories)
- Clustering (creation of new categories)



Classification (Recognition)  
(Supervised Classification)



Clustering  
(Unsupervised Classification)



# *1. An overview on classification*

## Classification Approaches

1. Unsupervised: self organizing
2. Supervised: training
3. Hybrid: self organization by categories
4. Spectral Mixture Analysis: sub-pixel variations.



# *1. An overview on classification*

## Examples of classification algorithms include:

- Linear classifiers
  1. Fisher's linear discriminant
  2. Logistic regression
  3. Naive Bayes classifier
  4. Perceptron
- Support vector machines
  1. Least squares support vector machines
- Quadratic classifiers
- Kernel estimation
  1. k-nearest neighbor
- Boosting (meta-algorithm)
- Decision trees
  1. Random forests
- Neural networks
- Gene Expression Programming
- Bayesian networks
- Hidden Markov models
- Learning vector quantization

### **The most widely used classifiers are the:**

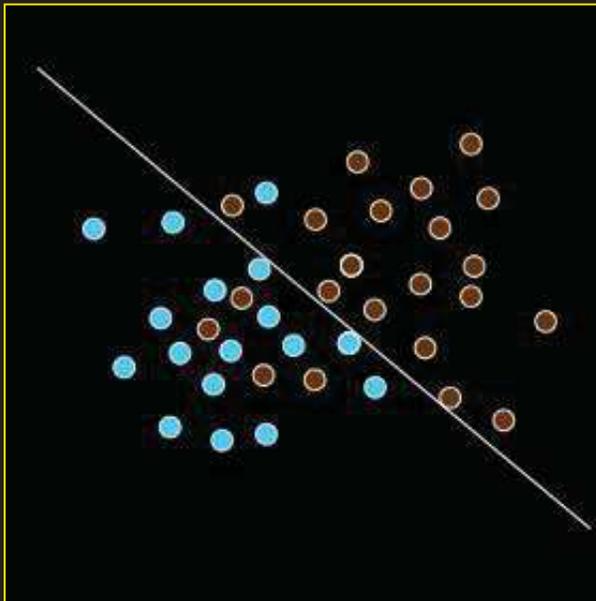
1. neural network (multi-layer perceptron),
2. support vector machines,
3. k-nearest neighbors,
4. Gaussian mixture model,
5. Gaussian, naive Bayes,
6. decision tree and RBF classifiers.



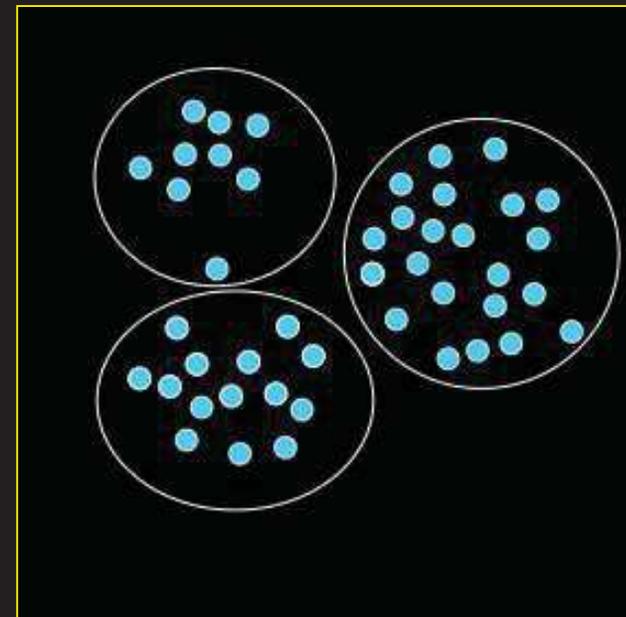
# *1. An overview on classification*

## Supervised vs. unsupervised classification

**Unsupervised:-**  
First classify, then  
apply knowledge



**Supervised:-**  
First apply knowledge,  
then classify





# *1. An overview on classification*

## Supervised vs. unsupervised classification

### Supervised Classification

- Classes are predetermined
- Training sites are established, used to train the classifier
- Class statistics are entered into a variety of classifiers to generate a map

### Unsupervised Classification

- Patterns are discovered in the data (no training)
- Classes are identified after classification
- Can be used in combination with supervised classification (apply supervised classification to unsupervised output)



## 2. *Supervised classification*

### Many techniques exist:

- Standard:  
Minimum distance, Maximum likelihood, Parallellipiped.
- Decision trees
- Spectral mixture analysis

### General procedure

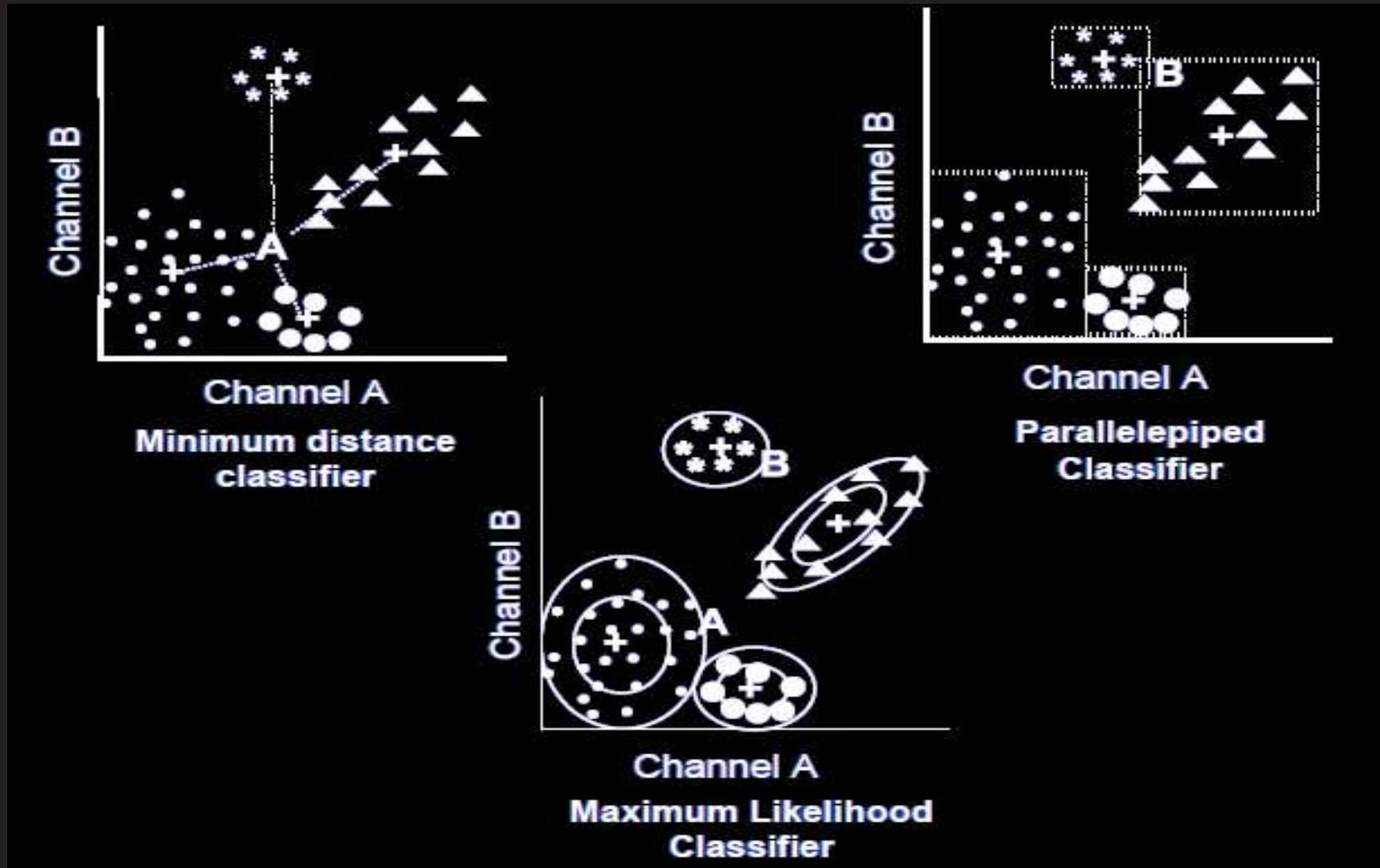
1. Define classes according to your objectives
2. Locate training areas / training sites
3. Extract training statistics
4. Train model
5. Classify
6. Assess accuracy (Training statistics or image)  
- Error matrix, kappa
7. Final product



## 2. Supervised classification

### Classification algorithms: Example

Algorithms: Some basic concepts





## 2. *Supervised classification*

### Classification algorithms: **Example**

#### Algorithms: Some basic concepts

- To understand these algorithms, let's review some basic math concepts
- These terms are central to *Linear Algebra*, which is the foundation of remote sensing
- Scalar value (one number) - e.g. 45
- Vector (1-dimensional row/column) – e.g.  $A = [45 \ 32 \ 61]$
- Array/matrix (2+-dimensional arrangement of vectors)

$$B = \begin{bmatrix} 44 & 31 & 47 \\ 7 & 34 & 9 \\ 29 & 10 & 12 \end{bmatrix}$$

- Summation :  $\Sigma$  (Sigma):  $\sum_{j=1}^n A_j = 45 + 32 + 61 = 138$
- Indexing –  $B_{i,j} = B_{\text{row,col}}$  e.g.)  $B_{3,2} = 10$



## 2. Supervised classification

### Classification algorithms: Example

You are given a three band image and there are two classes that you are trying to identify in the image. The mean vectors ( $\mu_c$ ) for the two classes are:

$$\mu_1 = [71 \ 53 \ 62]$$

$$\mu_2 = [23 \ 19 \ 30]$$

and the standard deviations for each are:

$$\sigma_1 = [3 \ 4 \ 3]$$

$$\sigma_2 = [20 \ 17 \ 25]$$

The program is given pixel

$$BV_{ij} = [63 \ 46 \ 53]$$

Which class will the pixel be assigned to, using these two methods?



## 2. Supervised classification

### Classification algorithms:

#### Example

$$\mu_1 = [71 \ 53 \ 62]$$

$$\mu_2 = [23 \ 19 \ 30]$$

and the standard deviations for each are:

$$\sigma_1 = [3 \ 4 \ 3]$$

$$\sigma_2 = [20 \ 17 \ 25]$$

The program is given pixel

$$BV_{ij} = [63 \ 46 \ 53]$$

### 1. Minimum Distance to the Mean:

$$\text{Dist } \mu_1 = \text{sqrt}((63-71)^2 + (46-53)^2 + (53-62)^2) = 13.92$$

$$\text{Dist } \mu_2 = \text{sqrt}((63-23)^2 + (46-19)^2 + (53-30)^2) = 53.46$$

Pixel will be assigned to **Class 1**.

$$\text{Dist} = \sqrt{\left(BV_{ijk} - \mu_{ck}\right)^2 + \left(BV_{ijl} - \mu_{cl}\right)^2}$$



## 2. Supervised classification

### Classification algorithms: Example

#### 2. Parallelepiped

$$\mu_{ck} - \sigma_{ck} \leq BV_{ijk} \leq \mu_{ck} + \sigma_{ck}$$

allowing up to 2 standard deviations:

Class 1:

band1: 68 < 63 < 74 - No

band2: 2 < 46 < 36 - No

band3: 5 < 53 < 55 - Yes

Class 2:

band1: 3 < 63 < 43 - No

band2: 2 < 46 < 36 - No

band3: 27 < 53 < 55 - Yes

$$\mu_1 = [71 \ 53 \ 62]$$

$$\mu_2 = [23 \ 19 \ 30]$$

and the standard deviations for each are:

$$\sigma_1 = [3 \ 4 \ 3]$$

$$\sigma_2 = [20 \ 17 \ 25]$$

The program is given pixel

$$BV_{ij} = [63 \ 46 \ 53]$$

**Pixel will not be assigned to any class.**



## *3. Unsupervised classification*

### *Contents*

*We'll look at clustering methods:*

1. Introduction
2. Clustering by Thresholding method
3. Clustering by Max-Min method
4. K-mean clustering method
5. ISO-DATA clustering method



### *3. Unsupervised classification*

#### *1- Introduction*

- In Unsupervised Classification, the identities of pattern types to be specified as classes within a features space are not generally known a prior.
- The computer is instructed to group patterns with similar feature points into unique clusters according to some statistically determined criteria.
- Then re-labeled and combine the patterns clusters into discrete  $c$  clustering classes.

### *3. Unsupervised classification*



## *Abstract*

*Clustering* is a basic tool used in data analysis, pattern recognition and data mining for finding unknown groups in data. In this course, we learn algorithms for finding the location of clusters, and cluster validity measures to recognize how many clusters there are. Feature selection, data normalization and outlier removal are also considered.

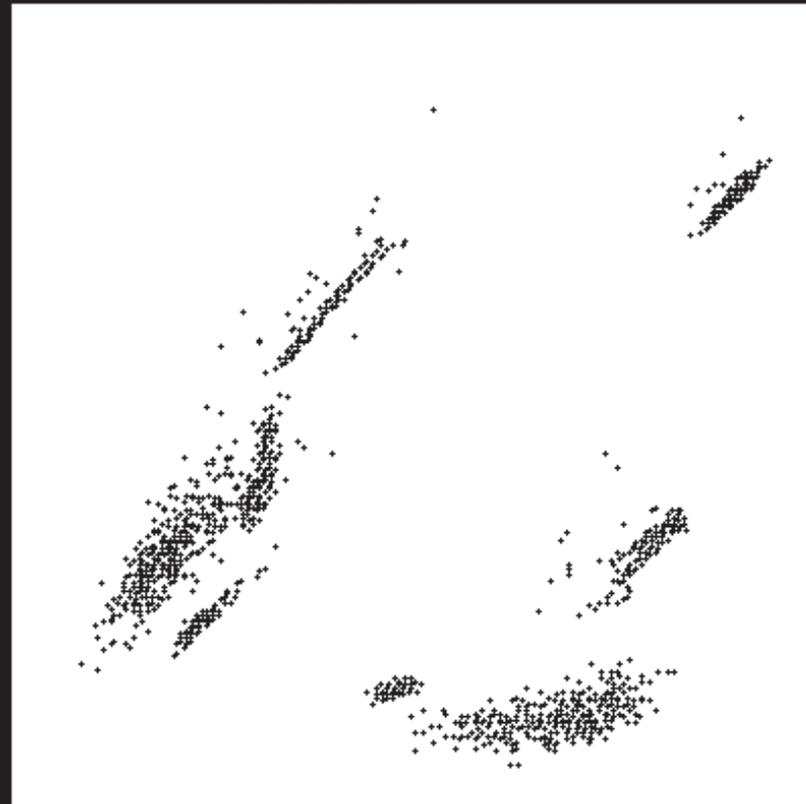
### *3. Unsupervised classification*

## *Example of dataset*

*Sources of **RGB**  
vectors*



***Red Green** plot of the  
vectors*





### *3. Unsupervised classification*

#### *Example of dataset*

##### *RGB color values*

<b>R</b>	<b>G</b>	<b>B</b>
26	20	45
28	5	46
28	12	44
23	13	46
31	4	51
•	•	•
•	•	•



### *3. Unsupervised classification*

#### *Subproblems of clustering*

1. Where are the clusters?  
(Algorithmic problem)
2. How many clusters  
(Methodological problem: which criterion?)
3. Selection of attributes  
(Application related problem)
4. Preprocessing the data  
(Practical problems: normalization, outliers)



### *3. Unsupervised classification*

#### *1. Clustering by Thresholding*

It consist of the following steps:

1. **Input**: pattern array; Threshold Value
2. Find **mean** of all input patterns
3. Choose the **closest** pattern ( $P$  closest to the mean & set  $C_1 = P_{\text{closest}}$ )
4. Loop over all patterns:
  - Measure the dist. btw each  $P_i$  and  $C_1$
  - If **dist.  $\geq$  thr**, then set  $C_2 = P_i$  : exit loop
  - else continue to loop
  - end loop
5. Repeat step 3 but check the dist. btw  $C_1$  &  $C_2$
6. Repeat step 4 reaching the end of loop.



### *3. Unsupervised classification*

#### *1. Clustering by Thresholding*

Example:

Cluster the following patterns by thresholding?

$$p_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, p_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, p_3 = \begin{pmatrix} 7 \\ 6 \end{pmatrix}, p_4 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, p_5 = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$$
$$p_6 = \begin{pmatrix} 7 \\ 7 \end{pmatrix}, p_7 = \begin{pmatrix} 8 \\ 6 \end{pmatrix}, p_8 = \begin{pmatrix} 3 \\ 9 \end{pmatrix}, p_9 = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$$



### 3. Unsupervised classification

#### 1. Clustering by Thresholding

Solution:

Let the threshold = 4

1- We calculate the **Mean**

$$\begin{aligned}\text{Mean} &= \frac{1}{9} * \begin{pmatrix} 3 + 2 + 7 + 4 + 1 + 7 + 8 + 3 + 4 \\ 2 + 1 + 6 + 3 + 9 + 7 + 6 + 9 + 8 \end{pmatrix} \\ &= \frac{1}{9} * \begin{pmatrix} 39 \\ 51 \end{pmatrix} = \begin{pmatrix} 4\frac{1}{3} \\ 5\frac{2}{3} \end{pmatrix}\end{aligned}$$

2- We calculate the **minimum** distance

$$\text{Dist. (M, p1)} = \left| 4\frac{1}{3} - 3 \right| + \left| 5\frac{2}{3} - 2 \right| = 5$$



### 3. Unsupervised classification

#### 1. Clustering by Thresholding

$$\text{Dist. (M, p2)} = \left| 4\frac{1}{3} - 2 \right| + \left| 5\frac{2}{3} - 1 \right| = 7$$

$$\text{Dist. (M, p3)} = \left| 4\frac{1}{3} - 7 \right| + \left| 5\frac{2}{3} - 6 \right| = 5$$

$$\text{Dist. (M, p4)} = \left| 4\frac{1}{3} - 4 \right| + \left| 5\frac{2}{3} - 3 \right| = 3 \leftarrow \text{Min. distance}$$

$$\text{Dist. (M, p5)} = \left| 4\frac{1}{3} - 1 \right| + \left| 5\frac{2}{3} - 9 \right| = 8$$

$$\text{Dist. (M, p6)} = \left| 4\frac{1}{3} - 7 \right| + \left| 5\frac{2}{3} - 7 \right| = 6$$

$$\text{Dist. (M, p7)} = \left| 4\frac{1}{3} - 8 \right| + \left| 5\frac{2}{3} - 6 \right| = 6$$

$$\text{Dist. (M, p8)} = \left| 4\frac{1}{3} - 3 \right| + \left| 5\frac{2}{3} - 9 \right| = 6$$

$$\text{Dist. (M, p9)} = \left| 4\frac{1}{3} - 4 \right| + \left| 5\frac{2}{3} - 8 \right| = 4$$



### *3. Unsupervised classification*

#### *1. Clustering by Thresholding*

Since p4 has the **minimum** distance then:

$$C1 = p4 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

3- We calculate the distance between **C1** and  
Patterns

$$\text{Dist. } (C1, p1) = |4 - 3| + |2 - 3| = 2 < \text{thr.}$$

$$\text{Dist. } (C1, p2) = |2 - 4| + |1 - 3| = 4 \geq \text{thr.}$$

$$** C2 = p2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$



### 3. Unsupervised classification

#### 1. Clustering by Thresholding

4- We calculate the distance between **C1, C2** and Patterns

$$\text{Dist. (C1, p3)} = |7 - 4| + |6 - 3| = 6 > \text{thr.}$$

$$\text{Dist. (C2, p3)} = |7 - 2| + |6 - 1| = 10 > \text{thr.}$$

$$** \text{ C3} = \text{p3} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$$

5- We calculate the distance between **C1, C2, C3** and Patterns

$$\text{Dist. (C1, p5)} = |7 - 4| + |9 - 3| = 9 > \text{thr.}$$

$$\text{Dist. (C2, p5)} = |1 - 2| + |9 - 1| = 9 > \text{thr.}$$

$$\text{Dist. (C3, p5)} = |1 - 7| + |9 - 6| = 9 > \text{thr.}$$

$$** \text{ C4} = \text{p5} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$$



### 3. Unsupervised classification

#### 1. Clustering by Thresholding

6- We calculate the distance between **C1,C2,C3,C4** and **P6**

$$\text{Dist. (C1, p6)} = |7 - 4| + |7 - 3| = 7 > \text{thr.}$$

$$\text{Dist. (C2, p6)} = |7 - 2| + |7 - 1| = 11 > \text{thr.}$$

$$\text{Dist. (C3, p6)} = |7 - 7| + |7 - 6| = 1 < \text{thr.}$$

$$\text{Dist. (C4, p6)} = |7 - 1| + |7 - 9| = 8 > \text{thr.}$$

**\*\* No class is identified**

7- We calculate the distance between **C1,C2,C3,C4** and **P7**

$$\text{Dist. (C1, p7)} = |8 - 4| + |6 - 3| = 7 > \text{thr.}$$

$$\text{Dist. (C2, p7)} = |8 - 2| + |6 - 1| = 11 > \text{thr.}$$

$$\text{Dist. (C3, p7)} = |8 - 7| + |6 - 6| = 1 < \text{thr.}$$

$$\text{Dist. (C4, p7)} = |8 - 1| + |6 - 9| = 11 > \text{thr.}$$

**\*\* No Class identifies**



### 3. Unsupervised classification

#### 1. Clustering by Thresholding

8- We calculate the distance between C1,C2,C3,C4 and P8

$$\text{Dist. (C1, p8)} = |4 - 3| + |3 - 9| = 7 > \text{thr.}$$

$$\text{Dist. (C2, p8)} = |2 - 3| + |1 - 9| = 9 > \text{thr.}$$

$$\text{Dist. (C3, p8)} = |7 - 3| + |6 - 9| = 7 > \text{thr.}$$

$$\text{Dist. (C4, p8)} = |7 - 1| + |7 - 9| = 2 < \text{thr.}$$

\*\* No class is identified

9- We calculate the distance between C1,C2,C3,C4 and P9

$$\text{Dist. (C1, p9)} = |4 - 4| + |3 - 8| = 5 > \text{thr.}$$

$$\text{Dist. (C2, p9)} = |2 - 4| + |1 - 8| = 9 > \text{thr.}$$

$$\text{Dist. (C3, p9)} = |7 - 4| + |6 - 8| = 5 > \text{thr.}$$

$$\text{Dist. (C4, p9)} = |1 - 4| + |9 - 8| = 4 > \text{thr.}$$

\*\* C5 = p9 =  $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$



### *3. Unsupervised classification*

#### *Clustering by Thresholding*

**C1**, P4, P1

**C2**, P2

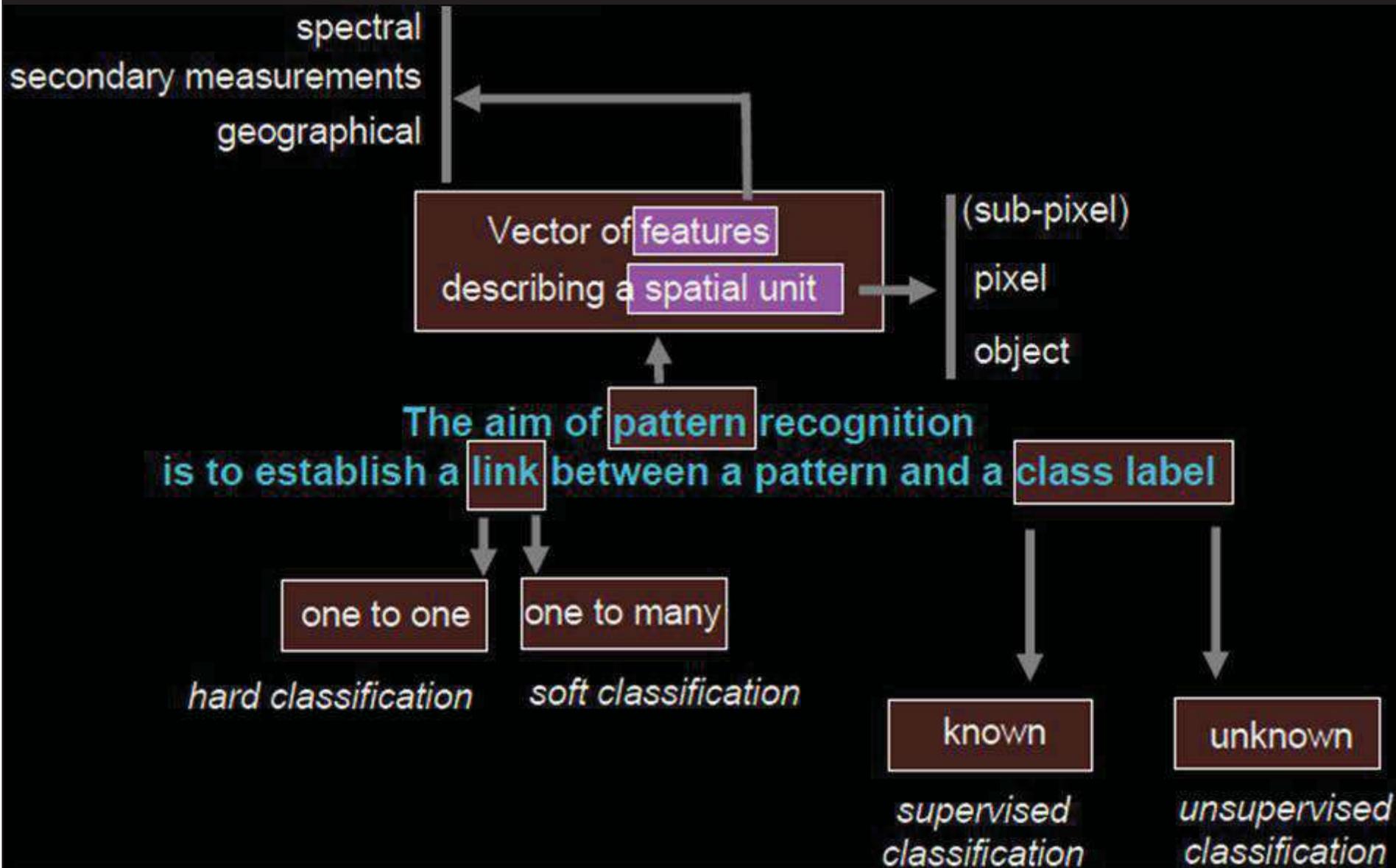
**C3**, P3, P6, P7

**C4**, P5, P8

**C5**, P9



# SUMMARY





Thank you

Any Questions ?



**END**

**Of Lecture**