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Solar Magnetism
Lecture#1: Introduction of Stellar Formation
Dr. Huda Sh. Ali

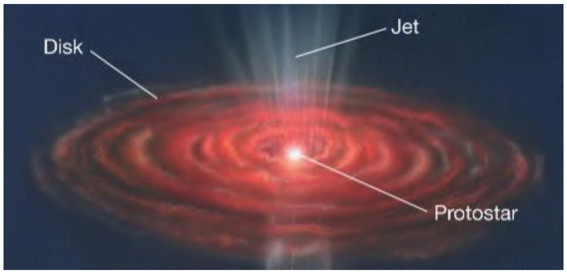
Star formation is the process by which dense regions within molecular clouds in interstellar space, sometimes referred to as "stellar nurseries" or "star-forming regions", fuse to form stars. As a branch of astronomy, star formation includes the study of the interstellar medium (ISM) and giant molecular clouds (GMC) as precursors to the star formation process, and the study of protostars and young stellar objects as its immediate products. It is closely related to planet formation, another branch of astronomy. Star formation theory, as well as accounting for the formation of a single star, must also account for the statistics of binary stars and the initial mass function.

Nebula

- A nebula is a cloud of gas (mostly hydrogen and helium) and dust in space.
- Nebulae are the birthplaces of stars.

Protostar

- A Protostar looks like a star, but its core is not yet hot enough for nuclear fusion to take place (nuclear fusion: the fusion of 2 hydrogen atoms into a helium atom with the liberation of a huge amount of energy. Nuclear fusion occurs only when the initial temperatures are very high – a few million degree Celsius. That is why it is hard to achieve and control).



- The luminosity comes exclusively from the heating of the Protostar as it contracts (because of gravity).
- Protostars are usually surrounded by dust, which blocks the light that they emit, so they are difficult to observe in the visible spectrum.

T Tauri star

- A very young, lightweight star, less than 10 million years old, that it still undergoing gravitational contraction; it represents an intermediate stage between a Protostar and a low-mass main sequence star like the Sun.

Main sequence stars

- Main sequence stars are stars that are fusing hydrogen atoms to form helium atoms in their cores.
- Most of the stars in the universe — about 90 per cent of them — are main sequence stars.
- The sun is a main sequence star.
- Towards the end of its life, a star like the Sun swells up into a red giant, before losing its outer layers as a planetary nebula and finally shrinking to become a white dwarf.

Red dwarf

- The faintest (less than 1/1000th the brightness of the Sun) main sequence stars are called the red dwarfs.
- Because of their low luminosity, they are not visible to the naked eye.
- They are quite small compared to the sun and have a surface temperature of about 4000°C.
- According to some estimates, red dwarfs make up three-quarters of the stars in the Milky Way.
- Proxima Centauri, the nearest star to the Sun, is a red dwarf.

Red giant

- Red giants have diameters between 10 and 100 times that of the Sun.
- They are very bright, although their surface temperature is lower than that of the Sun.
- A red giant is formed during the later stages of the evolution as it runs out of hydrogen fuel at its centre.
- It still fuses hydrogen into helium in a shell surrounding a hot, dense degenerate helium core.
- As the layer surrounding the core contains a bigger volume the fusion of hydrogen to helium around the core releases far more energy and pushes much harder against gravity and expands the volume of the star.
- Red giants are hot enough to turn the helium at their core into heavy elements like carbon (this is how elements were formed one after the other).
- But most stars are not massive enough to create the pressures and heat necessary to burn heavy elements, so fusion and heat production stops.

Degenerate matter

- Fusion in a star's core produces heat and outward pressure, but this pressure is kept in balance by the inward push of gravity generated by a star's mass (gravity is a product of mass).
- When the hydrogen used as fuel vanishes, and fusion slows, gravity causes the star to collapse in on itself. This creates a degenerate star.
- Great densities (degenerate star) are only possible when electrons are displaced from their regular shells and pushed closer to the nucleus, allowing atoms to take up less space. The matter in this state is called 'degenerate matter'.

Red Supergiant

- As the red giant star condenses, it heats up even further, burning the last of its hydrogen and causing the star's outer layers to expand outward.
- At this stage, the star becomes a large red giant. A very large red giant is often called Red Supergiant.

Planetary Nebula

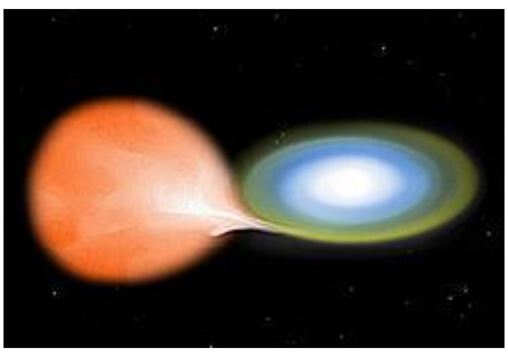
- Planetary nebula is an outer layer of gas and dust (no planets involved!) that are lost when the star changes from a red giant to a white dwarf.
- At the end of its lifetime, the sun will swell up into a red giant, expanding out beyond the orbit of Venus. As it burns through its fuel, it will eventually collapse under the influence of gravity.
- The outer layers will be ejected in a shell of gas that will last a few tens of thousands of years before spreading into the vastness of space.

White dwarf

- A white dwarf is very small, hot star, the last stage in the life cycle of a star like the Sun.
- White dwarfs are the remains of normal stars, whose nuclear energy supplies have been used up.
- White dwarf consists of degenerate matter with a very high density due to gravitational effects, i.e. one spoonful has a mass of several tonnes.

Nova

- Novae occur on the surface of a white dwarf in a binary system.
- If the two stars of the system are sufficiently near to one another, material (hydrogen) can be pulled from the companion star's surface onto the white dwarf.
- When enough material builds up on the surface of the white dwarf, it triggers a nuclear fusion on a white dwarf which causes a sudden brightening of the star.



Supernova

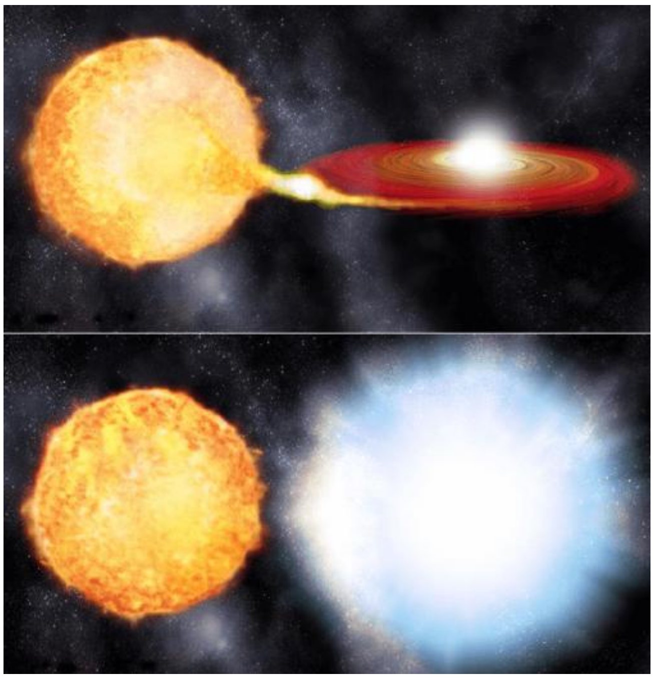
- A supernova is the explosive death of a star and often results in the star obtaining the brightness of 100 million suns for a short time.

- The extremely luminous burst of radiation expels much or all of a star's material at a great velocity, driving a shock wave into the surrounding interstellar medium.
- These shock waves trigger condensation is a nebula paving the way for the birth of a new star — if a star has to be born, a star has to die!
- A great proportion of primary cosmic rays comes from supernovae.

Supernovae can be triggered in one of two ways:

Type I supernova or Type Ia supernova (read as one-a)

- Occurs when there is a sudden re-ignition of nuclear fusion on the surface of a degenerate white dwarf in a binary system.
- A degenerate white dwarf may accumulate sufficient material from a companion star to raise its core temperature, ignite carbon fusion, and trigger runaway nuclear fusion, completely disrupting the star.



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The difference between Nova and Type I supernova

Nova	Type I supernova
In a nova, the system can shine up to a million times brighter than normal.	A supernova is a violent stellar explosion that can shine as brightly as an entire galaxy of billions of normal stars.
As long as it continues to take gas from its companion star, the white dwarf **can produce nova outbursts at regular intervals**.	If enough gas piles up on the surface of the white dwarf, a **runaway thermonuclear explosion blasts the star to bits**.

Type II supernova

- Type II supernova is a supernova that occurs by the gravitational collapse of the core of a massive star (mostly made of iron). E.g. Supernova of a red supergiant.

Importance of supernova: Creating and dispersing new elements

- When a star's core runs out of hydrogen, the star begins to die out. The dying star expands into a red giant, and this now begins to manufacture carbon by fusing helium atoms.
- More massive stars begin a further series of nuclear burning. The elements formed in these stages range from oxygen through to iron.
- During a supernova, the star releases very large amounts of energy as well as neutrons, which allows elements heavier than iron, such as uranium and gold, to be produced.
- In the supernova explosion, all of these elements are expelled out into space, and new stars are born out of this matter (recycling of matter in the universe!).

Black dwarf

- The last stage of stellar evolution is a black dwarf.

- A black dwarf is a white dwarf that has sufficiently cooled that it no longer emits significant heat or light.
- Because the time required for a white dwarf to reach this state is calculated to be longer than the current age of the universe (13.8 billion years), no black dwarfs are expected to exist in the universe yet.

Brown Dwarfs

- Brown dwarfs are objects which are too large to be called planets and too small to be stars.
- Brown dwarfs are thought to form in the same way that stars do – from a collapsing cloud of gas and dust.
- However, as the cloud collapses, the core is not dense enough to trigger nuclear fusion.

Neutron stars

- These stars are composed mainly of neutrons and are produced after a supernova, forcing the protons and electrons to combine to produce a neutron star.
- Neutron stars are very dense. (mass of three times the Sun can be fit in a sphere of just 20km in diameter).
- If its mass is any greater, its gravity will be so strong that it will shrink further to become a black hole.

Black holes

- Black holes are believed to form from massive stars at the end of their lifetimes.

- The gravitational pull in a black hole is so great that nothing can escape from it, not even light.
- The density of matter in a black hole cannot be measured (infinite!).
- Black holes distort the space around them and can suck neighbouring matter into them including stars.
- Gravitational lensing: Light around a massive object, such as a black hole, is bent, causing it to act as a lens for the things that lie behind it.

Lecture #2

2. Basic Equations of Stellar Structure

To understand the H-R diagram we must first understand the basic structure of a star and then how they evolve. Prior to 1905, it was not understood how a star could power itself. That is, the luminosity of the Sun is so large that it could not last for more than about 20 million years on the basis of the gravitational potential energy that it gained by contracting to its present size. Prior to 1905, therefore, astronomers and physicists believed that the Earth could not have been around form more than 20 million years or so and there certainly could not have been life on this planet for longer than that, since life requires sunlight to exist. The geologists and biologists of that era were sure, however, that the slow processes of geologic and biologic evolution required billions of years to occur, not millions, so there was a problem!

A solution become possible in 1905 when Einstein wrote down his famous equation, $E=mc^2$. It became at least theoretically possible that stars could convert some of their own mass into energy and gain the power needed to keep themselves shining for billions of years. It took another 35 years before physicists worked out the details of this process, namely the p-p reactions that turn Hydrogen into Helium, releasing energy in the process. Now we understand a star as basically a nuclear fusion machine that produces energy by fusion in its core to replace the energy it radiates at its surface. The Sun has so much Hydrogen fuel that it can go on doing this for about 10 billion years before significantly depleting the H in its core.

The basic theory of stellar structure assumes spherical symmetry, so that all variables depend on only one thing, the distance (r) from the center of the star. On spherical shells of radius r, all physical variables (e.g. temperature, density, pressure, chemical composition, etc.) are assumed to be uniform. Note that this assumption should be very good for the Sun (and most stars) since it spins very slowly and is spherically symmetric at its surface to such a high degree of accuracy that we cannot measure any departure from a sphere. Some rapidly rotating stars (generally young ones) may not quite fit this assumption and more sophisticated models may be required to explain some of their properties. That is beyond the scope of this course!

We have learned over the years that a star can be well described by only 4 differential equations that are based on certain physical principles, plus 3 "auxiliary" equations that relate some of the variables. The principle variables of stellar structure are pressure (P) temperature (T), density (ρ), luminosity through a shell at r (L_r) and mass interior to r (M_r). Auxiliary parameters include the rate of nuclear energy generation and the opacity of the gas (i.e. how transparent it is to radiation). Free parameters for the models include the chemical composition of the initial gas and the initial mass that came together to form the star. Complicating factors such as the magnetic field are generally ignored in making the models, although it can play a role. Another complicating factor is that stars are often highly convective (i.e. they have streams of hotter gas that moves like a warm ocean current carrying energy from one place to another). Convection and magnetic effects are very hard to model and are generally treated in highly simplified ways, if at all.

Despite the simplifications, the theory of stellar structure has had good success at providing us with a basic understanding of how stars evolve and why the H-R diagram looks the way it does (e.g. what the main sequence is, what red giants are, etc.). In the basic theory there are only four equations and we can derive three of them here. The fourth is a bit more difficult to deal with and is left for an upper level course in stellar structure, although its nature will be described here.

1. Mass interior to r (M_r)

The first equation of stellar structure comes from the principle of conservation of mass. Of course, stars do not precisely conserve their mass – they turn some of it into energy by nuclear fusion. But that amount is so small that we can ignore it and use the principle of mass conservation as if no nuclear processes occurred. Consider a shell inside the star that has radius r and a thickness dr (indicating that it is a very thin shell – of infinitesimally small thickness). Suppose that the density in that shell is ρ(r) and recall that it is uniform everywhere in the shell by the spherical symmetry assumption. The volume (dV) of such a shell is $dV = 4\pi r^2 dr$ (i.e. its surface area times its thickness. The mass of such a shell is $\rho(r)dV = 4\pi r^2 \rho dr$. Now, by the conservation of mass we can write that the mass interior to r must increase by an amount (dM_r) equal to the mass of the shell at r, i.e.

$$dM_r = 4\pi r^2 \rho dr$$

or, as it is usually written,

$$\frac{dM_r}{dr} = 4\pi r^2 \rho.$$

This is the first equation of stellar structure.

2. Conservation of Energy

A very similar equation comes from the principle of conservation of energy. While we can ignore the mass loss that comes from nuclear fusion, we cannot ignore the energy produced, which is enormous. Here we need a parameter that describes how much energy is created in each shell and that parameter is usually called ϵ. Its value depends on the chemical composition in the shell (assumed uniform), as well as the density and temperature. Calculating ϵ is very difficult and involves calculating the rates of all the nuclear reactions that occur within the shell to contribute to the energy. We will discuss some important nuclear processes later. Here we simply wish to write the basic equation. If ϵ is defined as the amount of power produced per gram within the shell then multiplying the mass of the shell by ϵ gives the total addition to the power of the star, which we call dL_r. Hence, conservation of energy requires that

$$dL_r = 4\pi r^2 \rho \epsilon dr$$

or, as it is usually written

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon.$$

This is the second equation of stellar structure.

3. Equation of Hydrostatic Equilibrium

The physical principle of the next equation is the same principle that is used to calculate how the pressure in a lake or ocean increases as one goes deeper into it. Therefore, it is referred to as hydrostatic equilibrium. Physically it states that the pressure in a fluid (including a gas) must increase in order to support the weight of the column of fluid above it. (The same principle is used to calculate air pressure as a function of height above sea level.) The pressure gradient (acting upwards) must be just right to balance the force of gravity acting downwards to keep each layer in a fluid at its static level.

Considering, again, a shell of radius r inside the star, we know that the gravitational force acting on that shell is

$$F_{grav} = \frac{GM_r m_{shell}}{r^2}$$

but plugging in for the mass of the shell, $4\pi r^2 \rho dr$, we have

$$F_{grav} = \frac{GM_r 4\pi r^2 \rho dr}{r^2}$$

Now, the counter-balancing force is the fact that the pressure at r, P(r), must be greater than the pressure at little further out in the star P(r+dr). Since pressure is force per unit area to get the full pressure difference on the shell we must multiply by the surface area of the shell at radius r, which is $4\pi r^2$. So the outward directed force coming from the pressure gradient (i.e. difference in pressure at r and at r+dr) is

$$F_{pressuregradient} = (P(r) - P(r + dr))4\pi r^2$$

Now, setting the pressure gradient force equal to the gravitational force (and recalling that they act in opposite directions!) we have:

$$\frac{P(r) - P(r + dr)}{dr} = \frac{GM_r\rho}{r^2}$$

Taking the limit as dr goes to zero means that the left hand side becomes the negative of the derivative of P, so the equation of hydrostatic equilibrium is:

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$$

This is the third equation of stellar structure

4. Energy Transport Equation

The fourth and final equation of stellar structure is set by how a star transports its energy from the inside, where it created by nuclear fusion, to the outside where it is radiated into space as photons. There are two basic methods for transporting energy in most stars: radiation and convection. Which occurs depends largely on the opacity of the matter. In regions of the star where it is fairly transparent the energy can travel by photons. This proceeds like a random walk. A photon only goes, on average, about 1 cm before it is absorbed and re-emitted by matter. It can take millions of years for a photon to diffuse out of the star by this random walk process. If we were to turn off the nuclear power of the Sun today, it would continue to shine at the current rate for millions of years into the future just on the basis of the photons that are already working their way out!

A star that is getting its energy out by radiation is said to be using radiative transport. Sometimes, however, the energy input by nuclear reactions is so rapid or the material of the star is so opaque that the energy cannot get out by radiation fast enough. In that case the temperature and pressure in the interior of the star tends to build up rapidly and causes the matter in some zone to be over-pressured. That element of a star then becomes buoyant and tends to easily float up towards the surface. Soon, a current can develop with warm material flowing upwards and being replaced by colder material (it's still very hot of course!) flowing downwards. One gets a "convection current" developing, much like boiling water.

This is a very effective heat transport mechanism. The star basically boils. The Sun does this in its outer one-third or so of its mass. Convection is important in mixing a star and also helps generate strong magnetic fields, which erupt at the surface of the sun as sunspots. This drives solar flares and the sunspot cycle and the solar wind.

The equations governing radiative or convective transport are not so simple as the first three equations of stellar structure and we will save them for an upper level course in stellar astrophysics. In some cases, there is a third mechanism of energy transport, namely conduction. This occurs when atoms are locked in place within a solid ...not something that usually occurs in a star. However, as we will see later, very dense cores within stars can become essentially solid, (actually electron-degenerate is the term for this condition) and while the atoms have the freedom of a gas, the electrons are locked in position like a solid and can easily transmit energy via conduction. This is the preferred mechanism of energy transport for very dense, degenerate cores of stars and for white dwarf stars (and neutron stars).

5. The Kelvin Time

Before turning to the important question of nuclear fusion, let us see exactly why it is required for a star to have a long life time (long by astronomical standards – most people would consider 20 million years to be long, but not astronomers!) We can estimate the total gravitational potential energy of the Sun in the following way. It is a sphere and we know that its average density is $<\rho>=\frac{M}{V}=1.4\ gm\ cm^{-3}$. We can calculate the contribution of each shell within it to its gravitational potential energy if we recall that for two masses, M and m separated by a distance r, the gravitational potential energy is given by $\frac{GMm}{r}$. Replacing M with M_r (because it is only the mass interior to r that pulls on the shell at r, and replacing m with the mass of the shell (its density times its volume, as before) and integrating from the center of the star (r=0) to its surface (r=R; we use R as the radius of the star), we have for the total gravitational potential energy of the star (E_G):

$$E_G=\int_0^R \frac{GM_r 4\pi r^2\rho}{r}dr$$

In the case of uniform density, we can take $\rho=<\rho>=constant$ and do the integration explicitly. We find that:

$$E_G=\frac{3}{5}\frac{GM^2}{R}$$

According to the Virial Theorem, as a star forms, one-half of the gravitational potential energy released goes into heating up the star and one-half is available to be radiated, producing its luminosity (L). Putting in solar values of the mass (M), Radius (R) and current luminosity (L) of the Sun, we can calculate the length of time (t_{Kelvin}), known as the Kelvin time, that the Sun could have radiated its current luminosity if it had no energy source other than gravitational contraction. For the Sun, we find:

$$t_{Kelvin} = \frac{3}{10}\frac{GM^2}{RL} \approx 10 \; million \; years.$$

Today we recognize the Kelvin time as the approximate time that it takes the star to reach the main sequence stage, where it turns on its nuclear fusion in its core and stops the gravitational contraction, settling into a stable star (on the main sequence) that can last there for *billions* of years. The time in a star's life before it has turned on its nuclear fusion, is known as its *pre-main sequence* lifetime, and the Kelvin time is a good estimate of the pre-main sequence lifetime of stars. Of course, our estimate above is a little bit off because the Sun is not really of uniform density...its interior is more highly compressed than its exterior, so it does have a little more gravitational potential energy than the simple calculation above would suggest. Its pre-main sequence lifetime is more like 30 million years than 10 million years, but this is a relatively small correction to astronomers (a factor of 3).

Solar Magnetism
Lecture#3: Introduction of Stellar Nucleosynthesis
Dr. Huda Sh. Ali

Stellar nucleosynthesis is the process by which the natural abundances of the chemical elements within stars change due to nuclear fusion reactions in the cores and their overlying mantles. Stars are said to evolve (age) with changes in the abundances of the elements within. Core fusion increases the atomic weight of elements and reduces the number of particles, which would lead to a pressure loss except that gravitation leads to contraction, an increase of temperature, and a balance of forces. A star loses most of its mass when it is ejected late in the star's lifetime, thereby increasing the abundance of elements heavier than helium in the interstellar medium.

Key reactions:

The most important reactions in stellar nucleosynthesis:

1. Hydrogen fusion, which includes:
   - Deuterium fusion
   - The proton–proton chain (P-P chain)
   - The carbon–nitrogen–oxygen cycle (CNO cycle)
2. Helium fusion, which includes:
   - The triple-alpha process
   - The alpha process
3. Fusion of heavier elements:
   - Lithium burning: a process found most commonly in brown dwarfs
   - Carbon-burning process
   - Neon-burning process
   - Oxygen-burning process

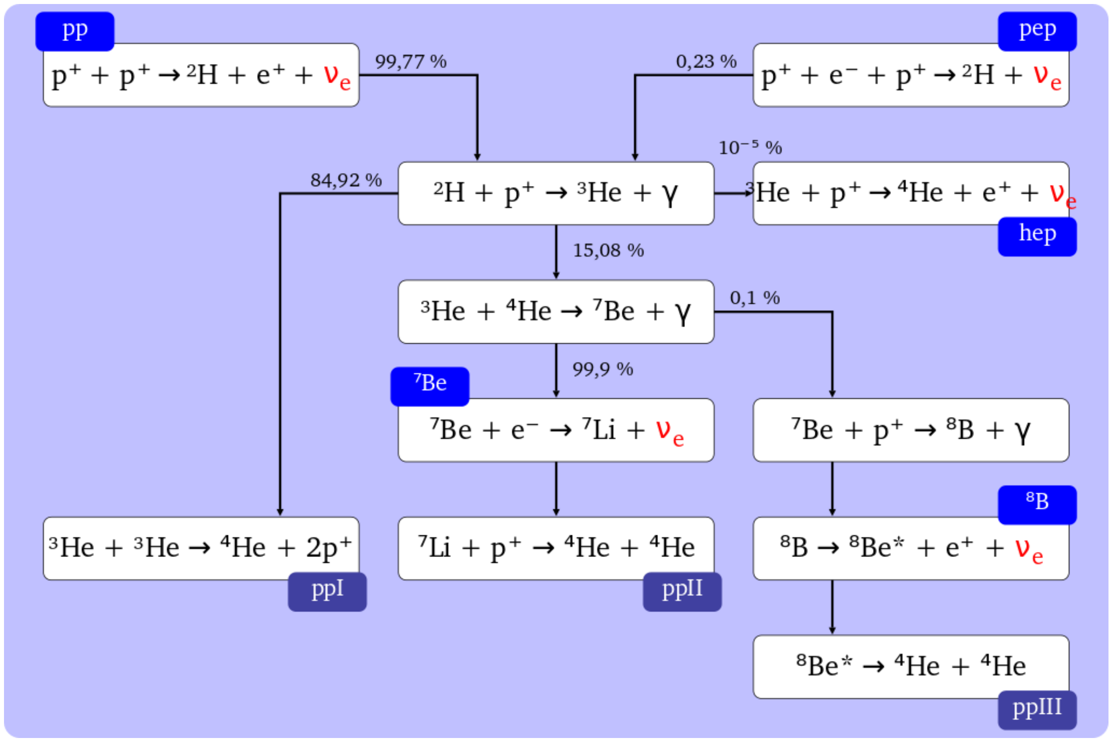
- Silicon-burning process

4. Production of elements heavier than iron:

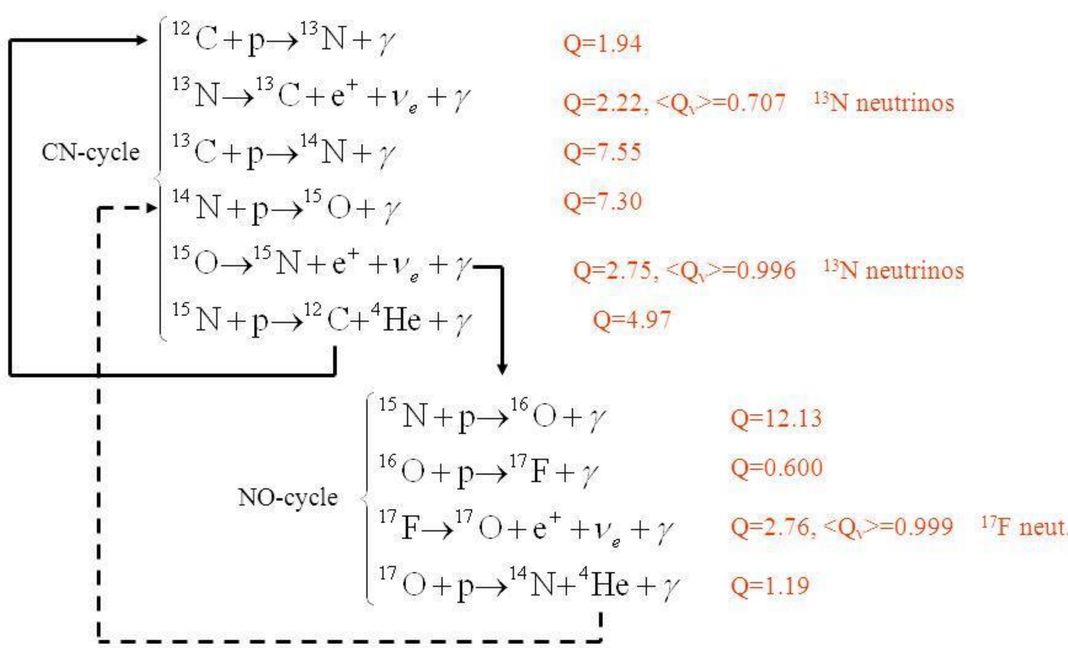
Deuterium fusion is the most easily fused nucleus available to accreting protostars, and such fusion in the center of protostars can proceed when temperatures exceed 10^6 K. The reaction rate is so sensitive to temperature that the temperature does not rise very much above this. The energy generated by fusion drives convection, which carries the heat generated to the surface.

The ***proton–proton chain reaction*** is one of the two sets of fusion reactions by which stars convert hydrogen to helium. It dominates in stars the size of the Sun or smaller.

The other reaction is the ***CNO cycle***, it is dominant source of energy in stars more massive than about 1.3 times the mass of the Sun. Ninety percent of all stars, with the exception of white dwarfs, are fusing hydrogen by these two processes.



Interlude on hydrogen burning – CNO cycle



CNO cycle is regulated by ^{14}N+p reation (slowest)

The **triple-alpha process** is a set of nuclear fusion reactions by which three helium-4 nuclei (alpha particles) are transformed into carbon. When a star runs out of hydrogen to fuse in its core, it begins to collapse until the central temperature rises to 10^8 K, six times hotter than the sun's core. At this temperature and density, alpha particles are able to fuse rapidly enough to produce significant amounts of carbon and restore thermodynamic equilibrium in the core.

$$^4He + {}^4He \rightleftharpoons {}^8Be$$
$$^4He + {}^8Be \rightleftharpoons {}^{12}C^* \rightarrow {}^{12}C + 2\gamma$$

This strong temperature dependence has consequences for the late stage of stellar evolution, the red giant stage.

The **alpha process**, also known as the **alpha ladder**, is one of two classes of nuclear fusion reactions by which stars convert helium into heavier elements, the other being the triple-alpha process. The triple-alpha process consumes only helium, and produces carbon. After enough carbon has accumulated, the reactions below take place, also consuming only helium and the product of the previous reaction.

$$\begin{array}{ll}
{}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{16}_{8}\mathrm{O} + \gamma & E = 7.16\ \mathrm{MeV} \\
{}^{16}_{8}\mathrm{O} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{20}_{10}\mathrm{Ne} + \gamma & E = 4.73\ \mathrm{MeV} \\
{}^{20}_{10}\mathrm{Ne} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{24}_{12}\mathrm{Mg} + \gamma & E = 9.32\ \mathrm{MeV} \\
{}^{24}_{12}\mathrm{Mg} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{28}_{14}\mathrm{Si} + \gamma & E = 9.98\ \mathrm{MeV} \\
{}^{28}_{14}\mathrm{Si} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{32}_{16}\mathrm{S} + \gamma & E = 6.95\ \mathrm{MeV} \\
{}^{32}_{16}\mathrm{S} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{36}_{18}\mathrm{Ar} + \gamma & E = 6.64\ \mathrm{MeV} \\
{}^{36}_{18}\mathrm{Ar} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{40}_{20}\mathrm{Ca} + \gamma & E = 7.04\ \mathrm{MeV} \\
{}^{40}_{20}\mathrm{Ca} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{44}_{22}\mathrm{Ti} + \gamma & E = 5.13\ \mathrm{MeV} \\
{}^{44}_{22}\mathrm{Ti} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{48}_{24}\mathrm{Cr} + \gamma & E = 7.70\ \mathrm{MeV} \\
{}^{48}_{24}\mathrm{Cr} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{52}_{26}\mathrm{Fe} + \gamma & E = 7.94\ \mathrm{MeV} \\
{}^{52}_{26}\mathrm{Fe} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{56}_{28}\mathrm{Ni} + \gamma & E = 8.00\ \mathrm{MeV}
\end{array}$$

E: is the energy produced by the reaction, released primarily as gamma rays (γ).

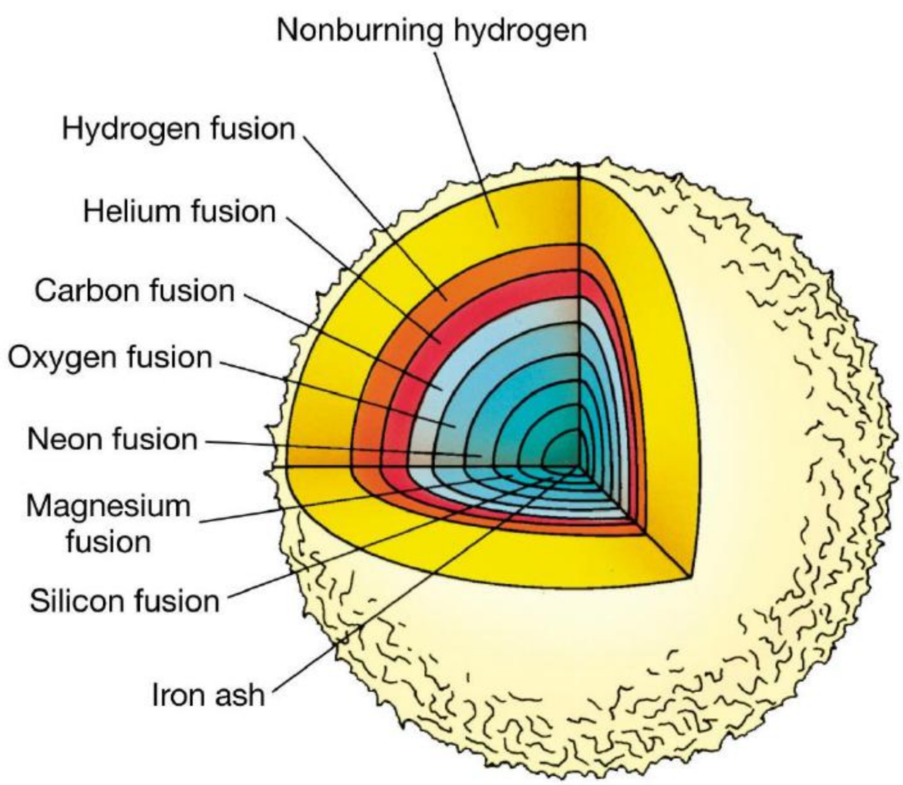
Lithium burning is a nucleosynthetic process in which lithium is depleted in a star. Lithium is generally present in brown dwarfs and not in low-mass stars. Stars, which by definition must achieve the high temperature (2.5×10^6 K) necessary for fusing hydrogen, rapidly deplete their lithium.

The ***carbon-burning process or carbon fusion*** is a set of nuclear fusion reactions that take place in the cores of massive stars (at least 8 $M_\odot$ at birth) that combines carbon into other elements. It requires high temperatures ($>5\times10^8$ K) and densities ($> 3\times10^9$ kg/m^3).

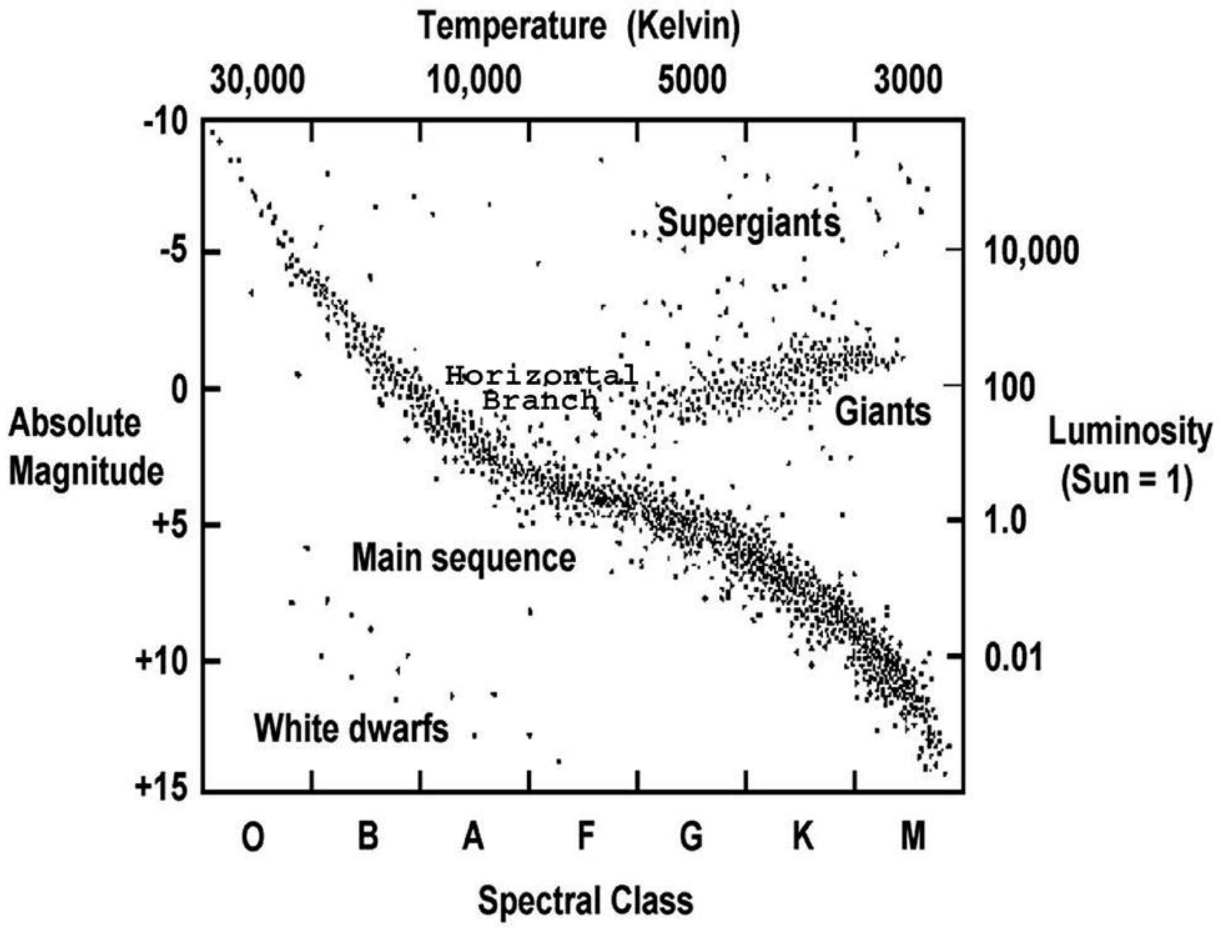
The ***Neon-burning process*** (nuclear decay) is a set of nuclear fusion reactions that take place in massive stars (at least 8 $M_\odot$). Neon burning requires high temperatures and densities (around 1.2×10^9 K and 4×10^9 kg/m^3).

The ***Oxygen-burning process*** is a set of nuclear fusion reactions that take place in massive stars that have used up the lighter elements in their cores. Oxygen-burning is preceded by the neon-burning process and succeeded by the silicon-burning process. As the neon-burning process ends, the core of the star contracts and heats until it reaches the ignition temperature for oxygen burning. Oxygen in the core ignites in the temperature range of $(1.5\text{-}2.6)\times10^9$ K and in the density range of $(2.6\text{-}6.7)\times10^9$g/cm^3. During the oxygen-burning process, proceeding outward, there is an oxygen-burning shell, followed by a neon shell, a carbon shell, a helium shell, and a hydrogen shell. The oxygen-burning process is the last nuclear reaction in the star's core which does not proceed via the alpha process.

In astrophysics ***Silicon burning*** is a very brief sequence of nuclear fusion reactions that occur in massive stars with a minimum of about 8-11 solar masses. Silicon burning is the final stage of fusion for massive stars that have run out of the fuels that power them for their long lives in the *main sequence* on the Hertzsprung-Russell diagram. It follows the previous stages of hydrogen, helium, carbon, neon and oxygen burning processes. Silicon burning begins when gravitational contraction raises the star's core temperature to (2.7-3.5 GK). When a star has completed the silicon-burning phase, no further fusion is possible. The star catastrophically collapses and may explode in what is known as a Type II supernova.



Solar Magnetism
Lecture#4: Hertzsprung-Russell diagram (HR diagram)
Dr. Huda Sh. Ali



The **Hertzsprung-Russell diagram** (**HR diagram**) is one of the most important tools in the study of **stellar evolution**. Developed independently in the early 1900s by Ejnar Hertzsprung and Henry Norris Russell, it plots the temperature of **stars** against their **luminosity** (the theoretical HR diagram), or the colour of stars (or **spectral type**) against their **absolute magnitude** (the observational HR diagram, also known as a colour-magnitude diagram).

Depending on its initial **mass**, every **star** goes through specific evolutionary stages dictated by its internal structure and how it produces energy. Each of these stages corresponds to a change in the temperature and luminosity of the star, which can be seen to move to different regions on the HR diagram as it evolves. This reveals the true power of the HR diagram. **astronomers** can know a star's internal structure and evolutionary stage simply by determining its position in the diagram.

This Hertzsprung-Russell diagram shows a group of stars in various stages of their evolution. By far the most prominent feature is the main sequence, which runs from the upper left (hot, luminous stars) to the bottom right (cool, faint stars) of the diagram. The giant branch is also well populated and there are many white dwarfs. Also plotted are the luminosity classes that distinguish between stars of the same temperature but different luminosity. There are 3 main regions (or evolutionary stages) of the HR diagram:

1. The ***Main Sequence*** stretching from the upper left (hot, luminous stars) to the bottom right (cool, faint stars) dominates the HR diagram. It is here that stars spend about 90% of their lives burning **hydrogen** into **helium** in their cores.
2. ***Red Giant and Super Giant*** stars occupy the region above the main sequence. They have low surface temperatures and high **luminosities** which, according to the Stefan-Boltzmann law, means they also have large radii. Stars enter this evolutionary stage once they have exhausted the hydrogen fuel in their cores and have started to burn helium and other heavier elements.
3. ***White Dwarf*** stars are the final evolutionary stage of low to intermediate mass stars, and are found in the bottom left of the HR diagram. These stars are very hot but have low luminosities due to their small size.

The **Sun** is found on the main sequence with a luminosity of 1 and a temperature of around5,400 **Kelvin**. Astronomers generally use the HR diagram to either summarise the evolution of stars, or to investigate the properties of a collection of stars. In particular, by plotting a HR diagram for either a globular or open cluster of stars, astronomers can estimate the age of the cluster from where stars appear to turnoff the main sequence.

SPECTRAL TYPE:

Based on their spectral features, **stars** are divided into different spectral types according to the **Harvard spectral classification** scheme. These spectral types indicate the temperature of the **star** and form the sequence OBAFGKM (often remembered by the mnemonic 'Oh Be A Fine Girl/Guy, Kiss Me') running from the hottest stars to the coolest. Within each **spectral type** there are significant variations in the strengths of the **absorption lines**, and each type has been divided into 10 sub-classes numbered 0 to 9. Our **Sun**, with a temperature of about 5,700 **Kelvin** has the spectral type **G2**.

EFFECTIVE TEMPERATURE:

Although **stars** are not perfect blackbodies, they can be approximated as such, allowing us to calculate their surface temperature via the Stefan-Boltzmann Law:

$$L = 4\pi R^2 \sigma T_e^4$$

where

$L =$	**luminosity** of the **star**
$R =$	star's radius
$\sigma =$	Stefan-Boltzmann constant
$=$	$5.67 \times 10^{-8} W m^{-2} K^{-4}$
$T_e =$	**effective temperature**

The surface temperature, calculated by assuming a perfect **blackbody** radiating the same amount of energy per unit **area** as the star, is known as the effective temperature of the star.

Energy in the Sun:

The fusion reaction is a very efficient process, releasing a huge amount of energy. This is because a single helium atom contains less mass than two hydrogen atoms. The excess mass is released as energy. Thanks to the pioneering work of Albert Einstein, the formula

$$E = mc^2$$

The equation is called the mass-energy equivalence and tells you that "m" kilograms of matter can, in principle be turned into pure energy. The "c" in the equation is the speed of light. In the case of the sun hydrogen atoms are fused together to produce helium in a process known as the proton-proton (or PP) cycle.

Using the mass-energy equivalence equation we can precisely quantify how much energy is released during a fusion reaction. To do this we note that 4 hydrogen atoms have slightly more mass than 1 helium atom. We can summarize it this way:

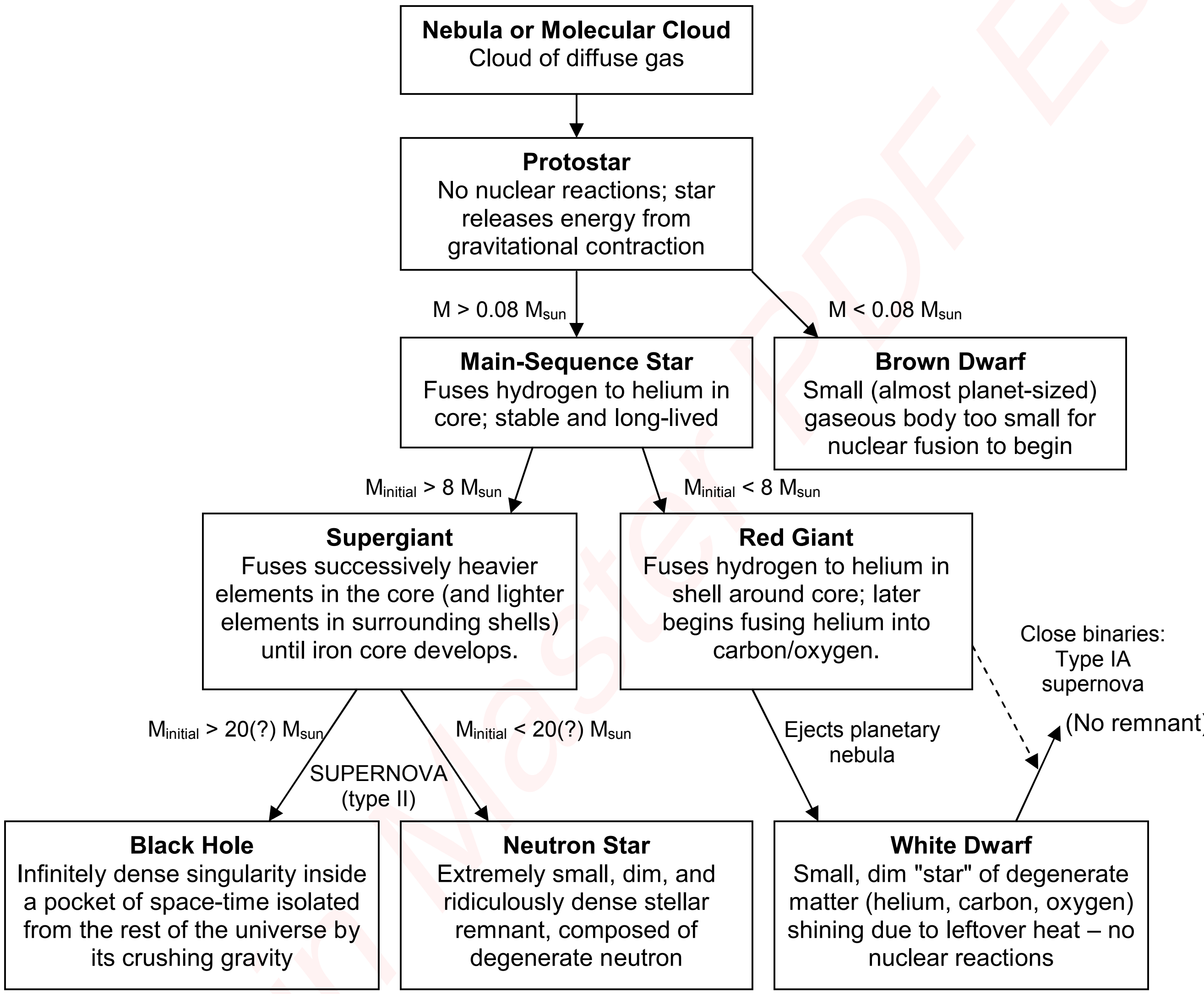
- 4 H nuclei weigh 6.693 x 10^{-27} kg
- 1 He nucleus weighs 6.645 x 10^{-27} kg
- missing mass converted to energy is 0.048 x 10^{-27} kg

This equation tells us exactly how much energy the fusion reaction releases. Fusion will power the star for 90% of its lifetime.

Lecture # 5

Stellar Evolution – Starbirth to Stardeath

The evolution of a star with time is determined almost entirely by its *initial mass*. The major phases of stellar evolution are summed up in the following flowchart:



Original handout by D. Perley

Solar Magnetism
Lecture#1: Solar Structure
Dr. Huda Sh. Ali

The **Sun** is the star at the center of the Solar System. It is a nearly perfect sphere of hot plasma, with internal convective motion that generates a magnetic field via a dynamo process. It is by far the most important source of energy for life on Earth. Its diameter is about 1.39 million km, and accounting for about 99.86% of the total mass of the Solar System. About three quarters of the Sun's mass consists of hydrogen (~73%); the rest is mostly helium (~25%), with much smaller quantities of heavier elements, including oxygen, carbon, neon, and iron.

The Sun is a G-type main-sequence star (G2V) based on its spectral class. As such, it is informally referred to as a yellow dwarf. It formed approximately 4.6 billion years ago from the gravitational collapse of matter within a region of a large molecular cloud. Most of this matter gathered in the center, whereas the rest flattened into an orbiting disk that became the Solar System. The central mass became so hot and dense that it eventually initiated nuclear fusion in its core. It is thought that almost all stars form by this process.

Structure of Sun:

1. Core:

The core of the Sun is considered to extend from the center to about 0.2 to 0.25 of solar radius. It is the hottest part of the Sun and of the Solar System. It has a density of 150 g/cm^3 at the center, and a temperature of, 15 million Kelvin. The core is made of hot, dense plasma (ions and electrons), at a pressure estimated at 26.5 peta pascals (PPa)) at the center. Due to fusion, the composition of the solar plasma drops from 68-70% hydrogen by mass at the outer core, to 33% hydrogen at the core/Sun center.

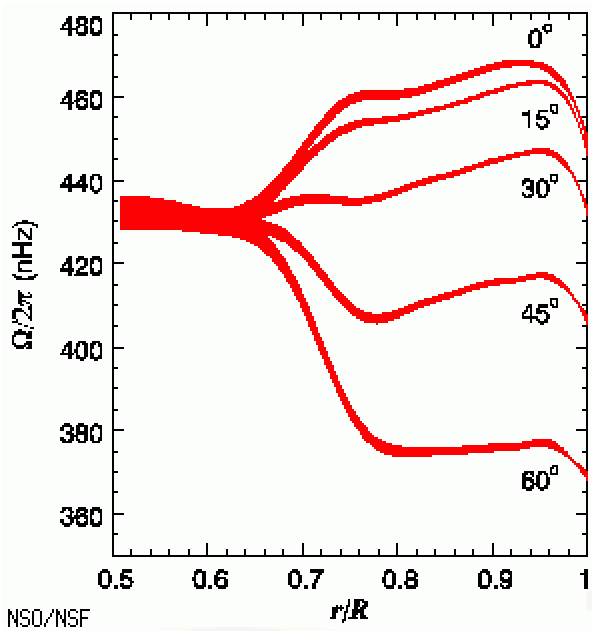
The core inside 0.20-0.25 of the solar radius, contains 34% of the Sun's mass, but only 0.8% of the Sun's volume. Inside 0.24 solar radius, the core generates 99% of the fusion power of the Sun. There are two distinct reactions in which four hydrogen nuclei may eventually result in one helium nucleus: the proton-proton chain reaction – which is responsible for most of the Sun's released energy – and the CNO cycle.

2. Radiative zone:

From the core out to about 0.7 solar radii, thermal radiation is the primary means of energy transfer. The temperature drops from approximately 7 million to 2 million kelvins with increasing distance from the core. This temperature gradient is less than the value of the adiabatic lapse rate and hence cannot drive convection, which explains why the transfer of energy through this zone is by radiation instead of thermal convection. Ions of hydrogen and helium emit photons, which travel only a brief distance before being reabsorbed by other ions. The density drops a hundredfold (from 20 g/cm3 to 0.2 g/cm3) from 0.25 solar radii to the 0.7 radii, the top of the radiative zone.

3. Tachocline:

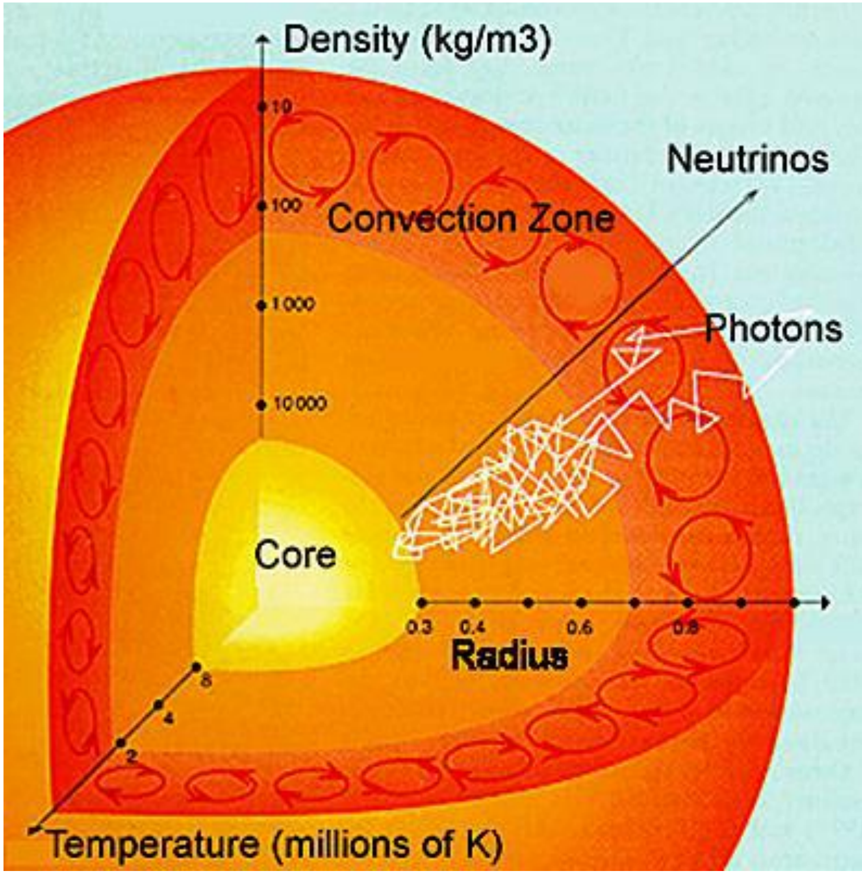
The radiative zone and the convective zone are separated by a transition layer, the tachocline. This is a region where the sharp regime change between the uniform rotation of the radiative zone and the differential rotation of the convection zone results in a large shear between the two—a condition where successive horizontal layers slide past one another. Presently, it is hypothesized that a magnetic dynamo within this layer generates the Sun's magnetic field.



Internal rotation in the Sun, showing differential rotation in the outer convective region and almost uniform rotation in the central radiative region. The transition between these regions is called the tachocline.

4. Convective zone:

The Sun's convection zone extends from 0.7 solar radii (200,000 km) to near the surface. In this layer, the solar plasma is not dense enough or hot enough to transfer the heat energy of the interior outward via radiation. Instead, the density of the plasma is low enough to allow convective currents to develop and move the Sun's energy outward towards its surface. Material heated at the tachocline picks up heat and expands, thereby reducing its density and allowing it to rise. As a result, an orderly motion of the mass develops into thermal cells that carry the majority of the heat outward to the Sun's photosphere above. Once the material diffusively and radiatively cools just beneath the photospheric surface, its density increases, and it sinks to the base of the convection zone, where it again picks up heat from the top of the radiative zone and the convective cycle continues. At the photosphere, the temperature has dropped to 5.700 K and the density to only 0.2 g/m^3


Density (kg/m3)
10
100
1000
10000
Neutrinos
Convection Zone
Photons
Core
0.3
0.4
0.6
0.8
Radius
2
4
8
Temperature (millions of K)

Physical characteristics	
Surface area	6.09×10^{12} km^2 12,000 × Earth
Volume	1.41×10^{18} km^3 1,300,000 × Earth
radius	696,392 km 109 × Earth
Mass	$(1.98855\pm0.00025)\times10^{30}$ kg 333,000 × Earth
Average density	1.408 g/cm^3 0.255 × Earth
Center density	162.2 g/cm^3 12.4 × Earth
Escape velocity (from the surface)	617.7 km/s 55 × Earth
Temperature	Center: 1.57×10^7 K Photosphere (effective): 5,772 K Corona: ≈ 5×10^6 K
Luminosity (L_{sol})	3.828×10^{26} W
Mean radiance (I_{sol})	2.009×10^7 W·m^{-2}·sr^{-1}
Age	≈ 4.6 billion years

Solar and Stellar Magnetism
Lecture#7: Solar Atmosphere Structure
Dr. Huda Sh. Ali

During a total solar eclipse, when the disk of the Sun is covered by that of the Moon, parts of the Sun's surrounding atmosphere can be seen. It is composed of five different parts: the photosphere, the chromosphere, the transition region, the corona and the heliosphere.

1. Photosphere:

The Sun's photosphere has a temperature between 4,230 and 5,730 °C (with an effective temperature of 5,504 °C) and a density of about 2×10^{-4} kg/m^3; other stars may have hotter or cooler photospheres. The Sun's photosphere is composed of convection cells called granules cells of plasma each approximately 1000 km in diameter with hot rising plasma in the center and cooler plasma falling in the narrow spaces between them. Each granule has a lifespan of only about eight minutes, resulting in a continually shifting "boiling" pattern. Grouping the typical granules are supergranules up to 30,000 km in diameter with lifespans of up to 24 hours. Other features can be observed in the photosphere with a simple telescope are sunspots, Pores and faculae.

2. Chromosphere:

Above the temperature minimum layer is a layer about 2,000 km thick, dominated by a spectrum of emission and absorption lines. It is called the *chromosphere* from the Greek root *chroma*, meaning color, because the chromosphere is visible as a colored flash at the beginning and end of total solar eclipses. The temperature of the chromosphere increases gradually with altitude, ranging up to around 20,000 K near the top. In the upper part of the chromosphere helium becomes partially ionized. The interesting feature of note in the chromosphere is *filaments* and *prominences*.

3. Transition Region

Above the chromosphere, in a thin (about 200 km) transition region, the temperature rises rapidly from around 20,000 K in the upper chromosphere to coronal temperatures closer to 1,000,000 K. The temperature increase is facilitated by the full ionization of helium in the transition region, which significantly reduces radiative cooling of the plasma. The transition region does not occur at a well-defined altitude. Rather, it forms a kind of nimbus around chromospheric features such as spicules and filaments, and is in constant, chaotic motion. The transition region is not easily visible from Earth's surface, but is readily observable from space by instruments sensitive to the extreme ultraviolet portion of the spectrum.

4. Corona

The corona is the next layer of the Sun. The low corona, near the surface of the Sun, has a particle density around 10^{15} m^{-3} to 10^{16} m^{-3}. The average temperature of the corona and solar wind is about 1,000,000–2,000,000 K; however, in the hottest regions it is 8,000,000–20,000,000 K. Although no complete theory yet exists to account for the temperature of the corona, at least some of its heat is known to be from magnetic reconnection. The corona is the extended atmosphere of the Sun, which has a volume much larger than the volume enclosed by the Sun's photosphere. The coronal phenomena are *solar wind*, *Coronal loops*, *coronal holes*, *Helmet streamers*, and *Coronal mass ejections (CME).*

5. **Heliosphere**

The heliosphere is the bubble-like region of space dominated by the Sun, which extends far beyond the orbit of Pluto. Plasma "blown" out from the Sun, known as the solar wind, creates and maintains this bubble against the outside pressure of the interstellar medium, the hydrogen and helium gas that permeates the Milky Way Galaxy. For several decades, the solar wind has been thought to form a bow shock at the edge of the heliosphere, where it collides with the surrounding interstellar medium. Moving away from the Sun, the point where the solar wind flow becomes subsonic is the termination shock, the point where the interstellar medium and solar wind pressures balance is the heliopause, and the point where the flow of the interstellar medium becomes subsonic would be the bow shock. This solar bow shock was thought to lie at a distance around 230 AU from the Sun – more than twice the distance of the termination shock as encountered by the Voyager spacecraft. However, data obtained in 2012 from NASA's Interstellar Boundary Explorer (IBEX) indicates the lack of any solar bow shock. Along with corroborating results from the Voyager spacecraft, these findings have motivated some theoretical refinements; current thinking is that formation of a bow shock is prevented, at least in the galactic region through which the Sun is passing, by a combination of the strength of the local interstellar magnetic-field and of the relative velocity of the heliosphere.

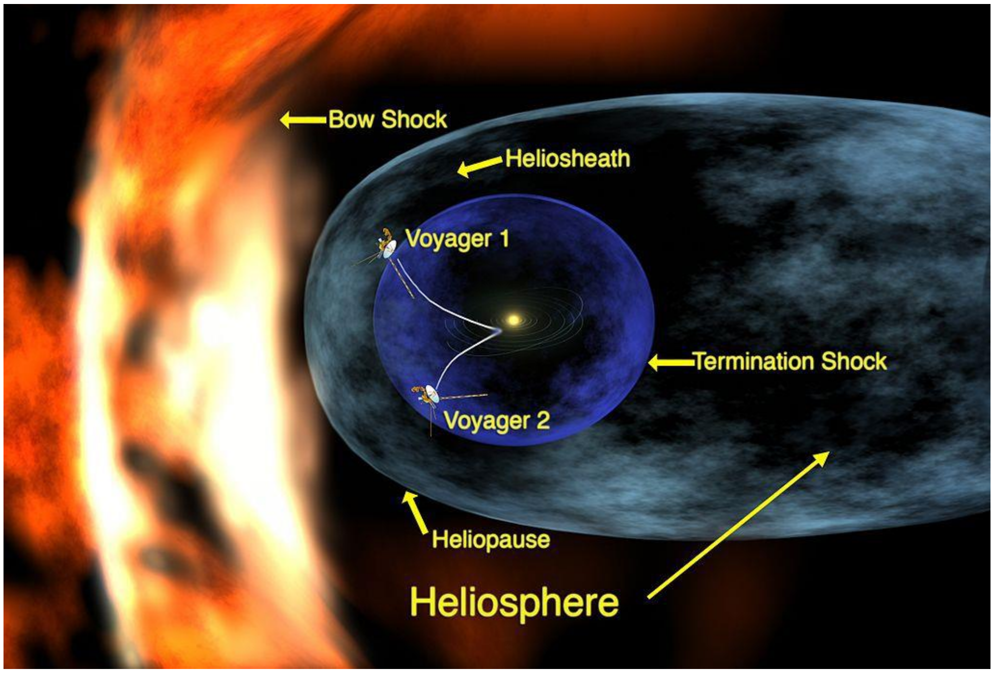


Fig. 1: The Heliosphere.

Structure of Heliospheric:

1. *Heliospheric current sheet:* The heliospheric current sheet is a ripple in the heliosphere created by the rotating magnetic field of the Sun. Extending throughout the heliosphere, it is considered the largest structure in the Solar System.
2. *Outer structure:* The outer structure of the heliosphere is determined by the interactions between the solar wind and the winds of interstellar space. The solar wind streams away from the Sun in all directions at speeds of several hundred km/s in the Earth's vicinity. At some distance from the Sun, well beyond the orbit of Neptune, this supersonic wind must slow down to meet the gases in the interstellar medium. This takes place in several stages:

- The solar wind is traveling at supersonic speeds within the Solar System. At the termination shock, a standing shock wave, the solar wind falls below the speed of sound and becomes subsonic.
- It was previously thought that, once subsonic, the solar wind would be shaped by the ambient flow of the interstellar medium, forming blunt nose on one side and comet-like heliotail behind, a region called the heliosheath.
- The outer surface of the heliosheath, where the heliosphere meets the interstellar medium, is called the heliopause. This is the edge of the entire heliosphere.
- In theory, the heliopause causes turbulence in the interstellar medium as the sun orbits the Galactic Center. Outside the heliopause, would be a turbulent region caused by the pressure of the advancing heliopause against the interstellar medium. However, the velocity of Solar wind relative to the interstellar medium is probably too low for a bow shock.

Heliosphere Observational methods:

Of particular interest is the Earth's interaction with the heliosphere, but its extent and interaction with other bodies in the solar system has also been studied. Some examples of missions that have or continue to collect data related to the heliosphere include:

- Solar Anomalous and Magnetospheric Particle Explorer
- Solar and Heliospheric Observatory
- Solar Dynamics Observatory
- STEREO
- Ulysses (spacecraft)

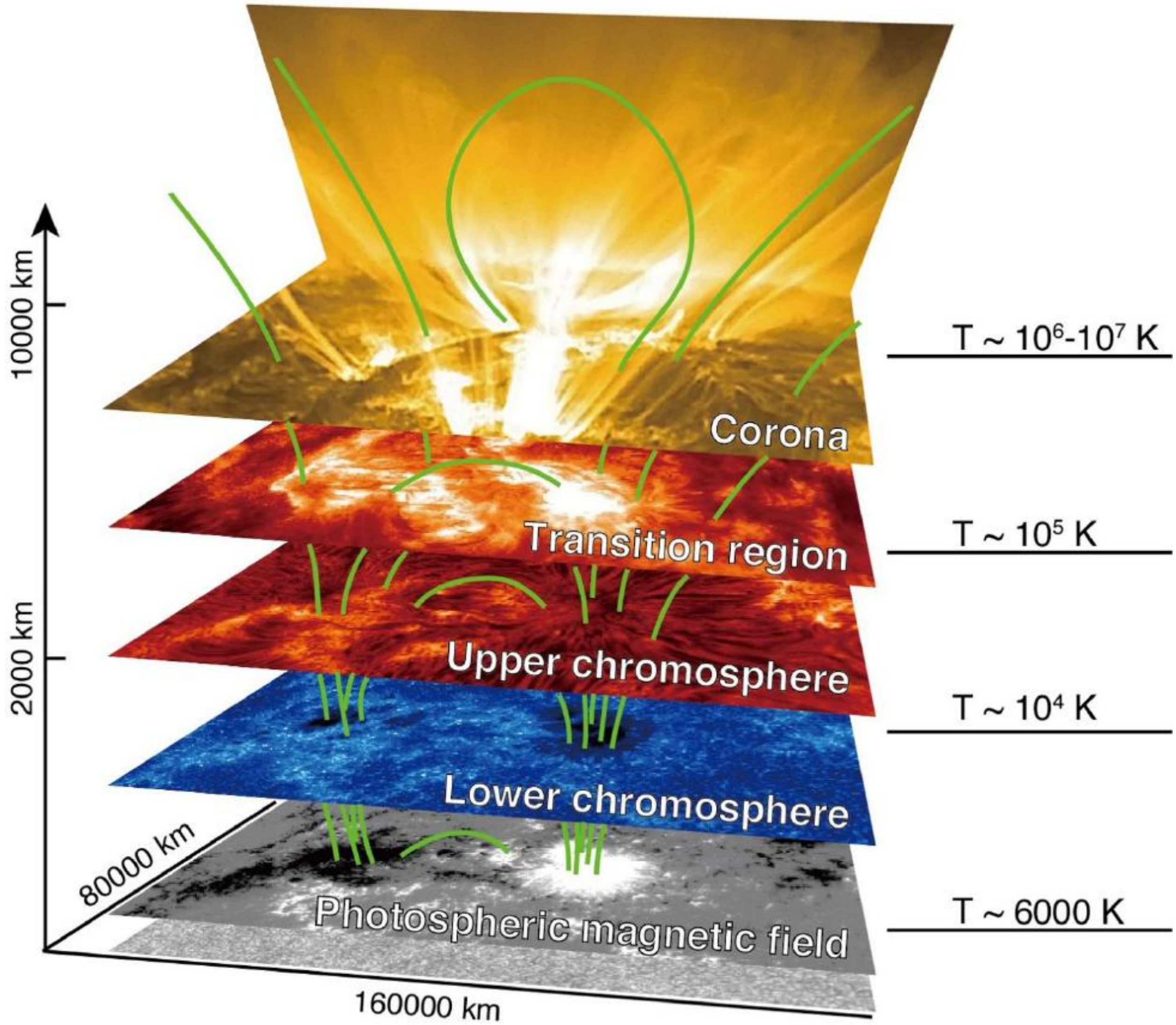


Fig. 2: The structure of the solar atmosphere is connected by magnetic field lines.

Solar phenomena:

Solar phenomena are the natural phenomena occurring within the magnetically heated outer atmospheres in the Sun. These phenomena take many forms, including solar wind, radio wave flux, energy bursts such as solar flares, coronal mass ejection or solar eruptions, coronal heating and sunspots.

These phenomena are generated by a ***helical dynamo*** near the center of the Sun's mass that generates strong magnetic fields and a ***chaotic dynamo*** near the surface that generates smaller magnetic field fluctuations.

The total sum of all solar fluctuations is referred to as solar variation. The collective effect of all solar variations within the Sun's gravitational field is referred to as *space weather*.

Photospheric Features:

Sunspots: Sunspots appear as dark spots on the surface of the Sun. Temperatures in the dark centers of sunspots drop to about 3700 K (compared to 5700 K for the surrounding photosphere). They typically last for several days, although very large ones may live for several weeks. Sunspots are magnetic regions on the Sun with magnetic field strengths thousands of times stronger than the Earth's magnetic field. Sunspots usually come in groups with two sets of spots. One set will have positive or north magnetic field while the other set will have negative or south magnetic field. The field is strongest in the darker parts of the sunspots - the umbra. The field is weaker and more horizontal in the lighter part - the penumbra.

Granules: Granules are small (about 1000 km across) cellular features that cover the entire Sun except for those areas covered by sunspots. These features are the tops of convection cells where hot fluid rises up from the interior in the bright areas, spreads out across the surface, cools and then sinks inward along the dark lanes. Individual granules last for only about 20 minutes. The granulation pattern is continually evolving as old granules are pushed aside by newly emerging ones.

Supergranules: Supergranules are much larger versions of granules (about 35,000 km across) but are best seen in measurements of the "Doppler shift" where light from material moving toward us is shifted to the blue while light from material moving away from us is shifted to the red. These features also cover the entire Sun and are continually evolving. Individual supergranules last for a day or two and have flow speeds of about 0.5 km/s (1000 mph). The fluid flows observed in supergranules carry magnetic field bundles to the edges of the cells where they produce the chromospheric network.

Chromospheric Features:

The chromospheric network: chromospheric network is a web-like pattern most easily seen in the emissions of the red line of hydrogen (H-alpha) and the ultraviolet line of calcium (Ca II K - from calcium atoms with one electron removed). The network outlines the supergranule cells and is due to the presence of bundles of magnetic field lines that are concentrated there by the fluid motions in the supergranules.

Filaments and Plage: Filaments are dark, thread-like features seen in the red light of hydrogen (H-alpha). These are dense, somewhat cooler, clouds of material that are suspended above the solar surface by loops of magnetic field. Plage, the French word for beach, are bright patches surrounding sunspots that are best seen in H-alpha. Plage are also associated with concentrations of magnetic fields and form a part of the network of bright emissions that characterize the chromosphere.

Prominences: Prominences are dense clouds of material suspended above the surface of the Sun by loops of magnetic field. Prominences and filaments are actually the same things except that prominences are seen projecting out above the limb, or edge, of the Sun. Both filaments and prominences can remain in a quiet or quiescent state for days or weeks. However, as the magnetic loops that support them slowly change, filaments and prominences can erupt and rise off of the Sun over the course of a few minutes or hours.

Spicules: Spicules are small, jet-like eruptions seen throughout the chromospheric network. They appear as short dark streaks in the H-alpha image to the left. They last but a few minutes but in the process eject material off of the surface and outward into the hot corona at speeds of 20 to 30 km/s.

Coronal Features:

Helmet streamers: Helmet streamers are large cap-like coronal structures with long pointed peaks that usually overlie sunspots and active regions. We often find a prominence or filament lying at the base of these structures. Helmet streamers are formed by a network of magnetic loops that connect the sunspots in active regions and help suspend the prominence material above the solar surface. The closed magnetic field lines trap the electrically charged coronal gases to form these relatively dense structures. The pointed peaks are formed by the action of the solar wind blowing away from the Sun in the spaces between the streamers.

Coronal loops: Coronal loops are found around sunspots and in active regions. These structures are associated with the closed magnetic field lines that connect magnetic regions on the solar surface. Many coronal loops last for days or weeks but most change quite rapidly. Some loops, however, are associated with solar flares and are visible for much shorter periods. These loops contain denser material than their surroundings.

Coronal holes: Coronal holes are regions where the corona is dark. These features were discovered when X-ray telescopes were first flown above the earth's atmosphere to reveal the structure of the corona across the solar disc. Coronal holes are associated with "open" magnetic field lines and are often found at the Sun's poles. The high-speed solar wind is known to originate in coronal holes.

<u>The solar wind</u>: solar wind is a stream of charged particles released from the corona. This plasma consists of mostly electrons, protons and alpha particles with thermal energies between 1.5 and 10 keV. Embedded within the solar-wind plasma is the interplanetary magnetic field. The solar wind varies in density, temperature and speed over time and over solar latitude and longitude.

<u>Solar flares:</u> Solar flares are tremendous explosions on the surface of the Sun. In a matter of just a few minutes they heat material to many millions of degrees and release as much energy as a billion megatons of TNT. They occur near sunspots, usually along the dividing line (neutral line) between areas of oppositely directed magnetic fields.

<u>Coronal mass ejections:</u> Coronal mass ejections (or CMEs) are huge bubbles of gas threaded with magnetic field lines that are ejected from the Sun over the course of several hours. CMEs most often originate from active regions on the Sun's surface, such as groupings of sunspots associated with frequent flares. Near solar maxima, the Sun produces about three CMEs every day, whereas near solar minima, there is about one CME every five days.

Solar Magnetism
Lecture#8: The Solar Dynamo
Dr. Huda Sh. Ali

The Solar Dynamo:

It is widely believed that the Sun's magnetic field is generated by a magnetic dynamo within the Sun. The fact that the Sun's magnetic field changes dramatically over the course of just a few years, and the fact that it changes in a cyclical manner indicates that the magnetic field continues to be generated within the Sun.

Where is the solar dynamo located?

Today, most solar physicists believe that the solar dynamo is located either at the bottom of the convection zone or in a thin region called the overshoot zone (tachocline). The overshoot zone is located between the convection zone and the radiative zone.

How does the solar dynamo work?

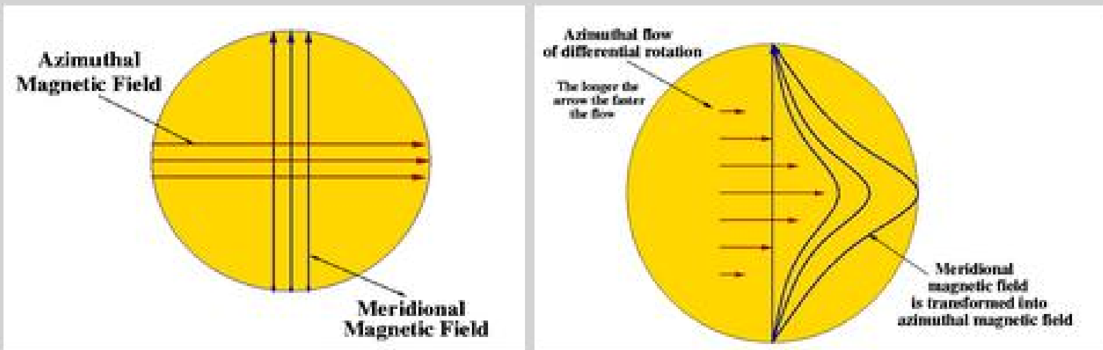
In order to understand how the Sun generates magnetic fields you have basically only need to remember three facts. *Firstly*, the convection zone consists of a plasma, i.e. a gas that contains electrically charged particles. *Secondly*, the plasma in the convection zone is continuously moving around. Since the plasma is moving, the charged particles are moving and we obtain electrical currents. However, electrical currents generate magnetic fields (*Ampere's law*), as we mentioned above. These magnetic fields in turn generate electric currents (*Faraday's law*) and therefore we obtain the following loop: electric current - magnetic field - electric current - magnetic field - electric current - magnetic field etc, etc. As long as this loop is not interrupted the Sun will always produce magnetic fields.

Thirty, we need very complicated flows in order to generate any magnetic fields whatsoever. Another important ingredient for dynamo action is differential rotation, i.e. the fact that the Sun rotates faster at the equator than at the poles. In other words the rotation rate of the Sun varies with latitudes.

Let us look at magnetic fields and the solar dynamo in a different way. Imagine that magnetic fields behave like rubber bands. Thus, like rubber bands, they can be made stronger by stretching them, twisting them, and folding them back on themselves. This stretching, twisting, and folding is done by the motions of the plasma within the solar convection zone. Now, in order for the magnetic field strength to increase the stretching, twisting and folding has to be done in exactly the right way. The motions of the plasma have to transform the meridional magnetic field into an azimuthal magnetic field. A meridional magnetic field is basically a field that points from the north to south or south to north, while an azimuthal magnetic field points from east to west or vice versa. Once that has been done the flow of the plasma has to do the opposite, i.e. transform the azimuthal magnetic field into a meridional. So we get another loop: meridional magnetic field - azimuthal magnetic field - meridional magnetic field - azimuthal magnetic field… etc. Again the Sun will produce magnetic field as long as this cycle is not interrupted.

Let us know describe how the flow of the plasma achieves this loop. Firstly, the solar differential rotation stretches the magnetic field and winds it around the Sun. This stretching transforms a takes a meridional magnetic field and stretches it into an azimuthal magnetic field. This only happens because the Sun rotates faster at the equator than at the pole. If the Sun would rotate at the same rate everywhere nothing would happen to the magnetic field and the dynamo would not work. The effect of stretching the magnetic field by differential rotation is often referred to as the ***omega-effect.***

Now that we stretched the meridional magnetic field into an azimuthal magnetic field we need to do the opposite. This is done by the ***alpha-effect*** which is due to the interaction of convection and rotation. The alpha-effect basically takes the azimuthal magnetic field generated by the omega-effect and transforms

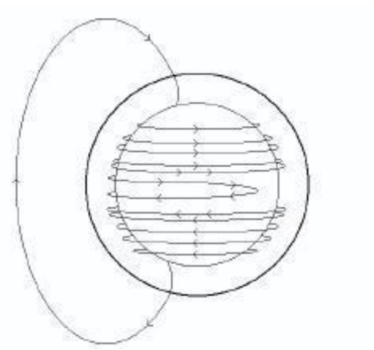


Azimuthal fields
Meridional Fields.

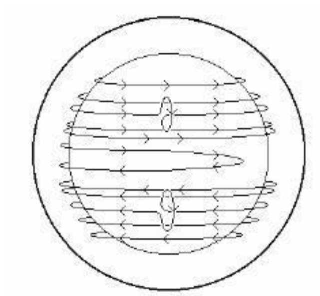
The omega-effect.

it back into meridional flow. This complicated stretching, twisting and folding requires a complicated flow.

- ***The Omega Effect:*** Magnetic fields within the Sun are stretched out and wound around the Sun by differential rotation - the change in rotation rate as a function of latitude and radius within the Sun. The Sun's differential rotation with latitude can take a north-south oriented magnetic field line and wrap it once around the Sun in about 8 months.



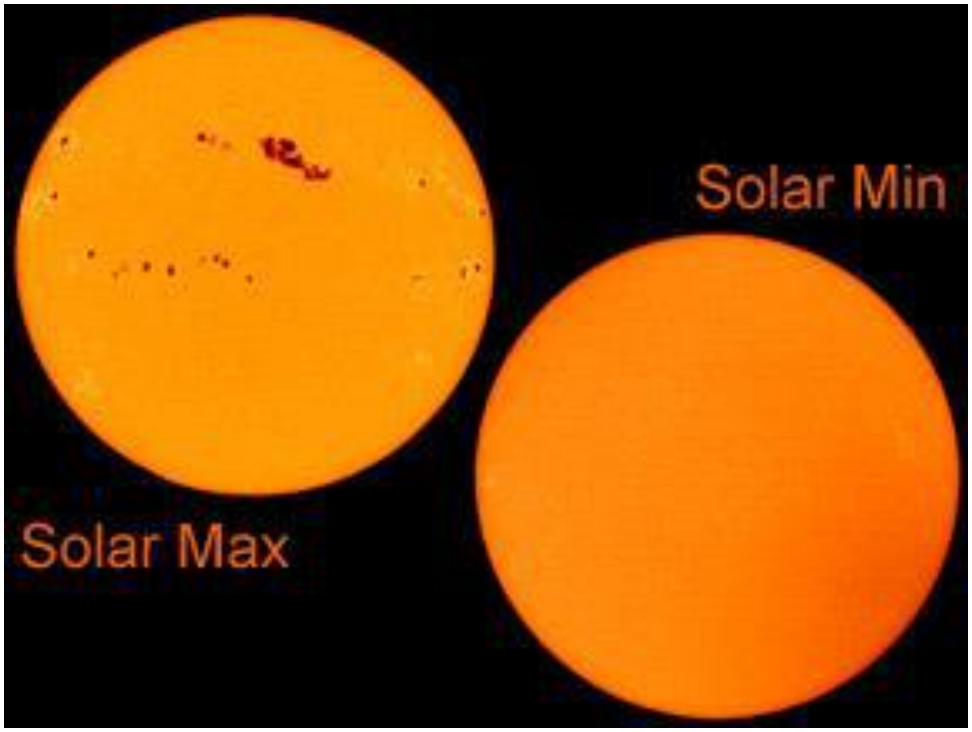
- ***The Alpha Effec***t: Twisting of the magnetic field lines is caused by the effects of the Sun's rotation. Early models of the Sun's dynamo assumed that the twisting is produced by the effects of the Sun's rotation on very large convective flows that carry heat to the Sun's surface.

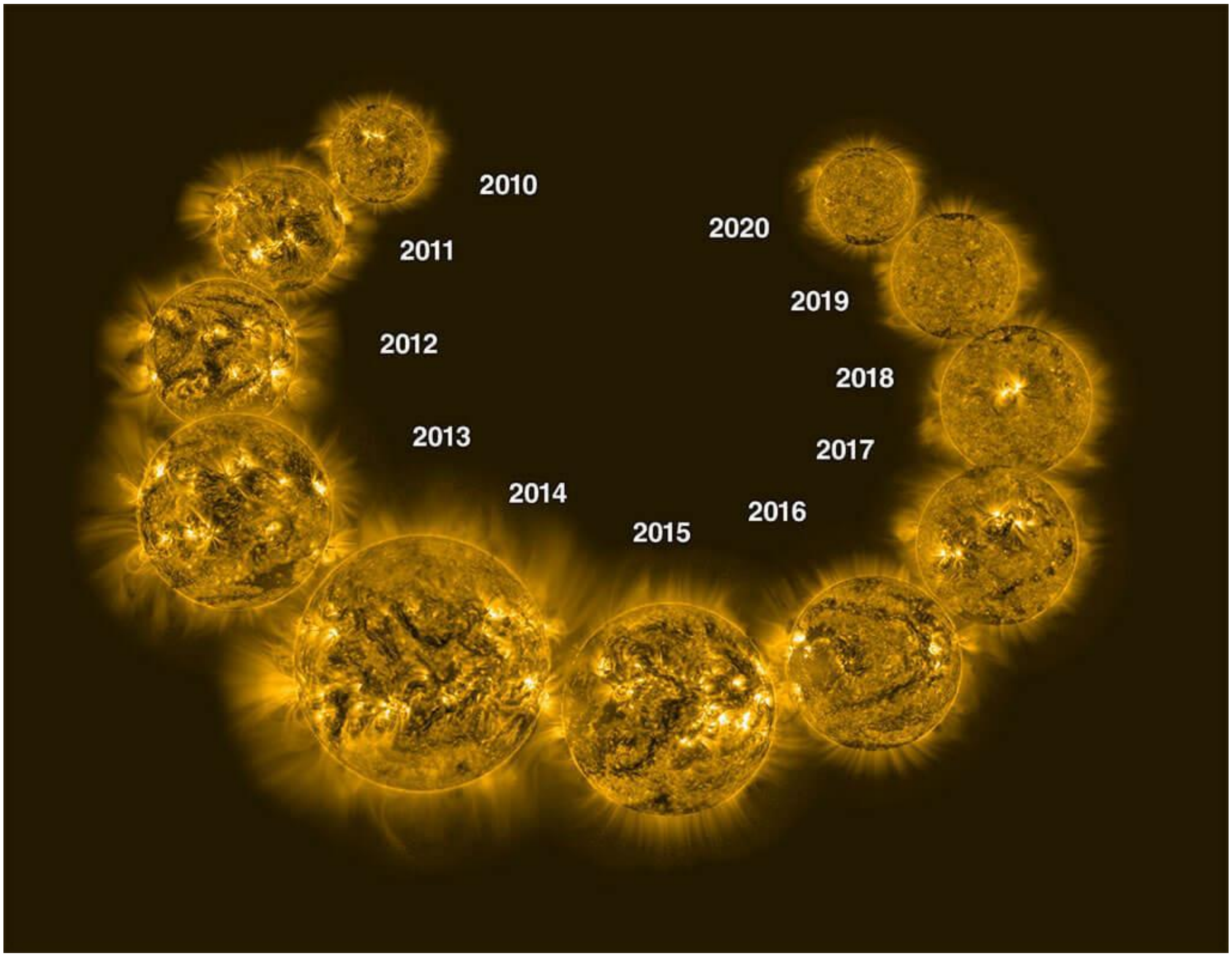


Solar Magnetism
Lecture#9: Solar Cycle
Dr. Huda Sh. Ali

Solar Cycle:

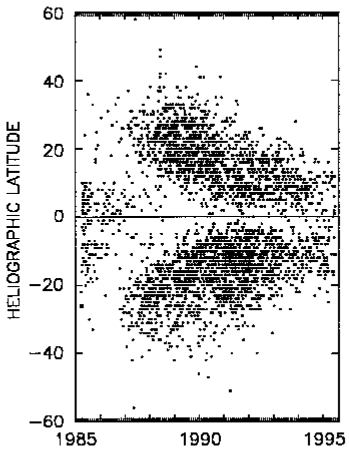
The solar cycle or solar magnetic activity cycle is the nearly periodic 11-year change in the Sun's activity (including changes in the levels of solar radiation and ejection of solar material) and appearance (changes in the number of sunspots, flares, and other manifestations). The part of the cycle with low sunspot activity is referred to as "solar minimum" while the portion of the cycle with high activity is known as "solar maximum". A peak in the sunspot count is called "solar maximum" (or "solar max"). The time when few sunspots appear is called a "solar minimum" (or "solar min"), as shown in this figure.





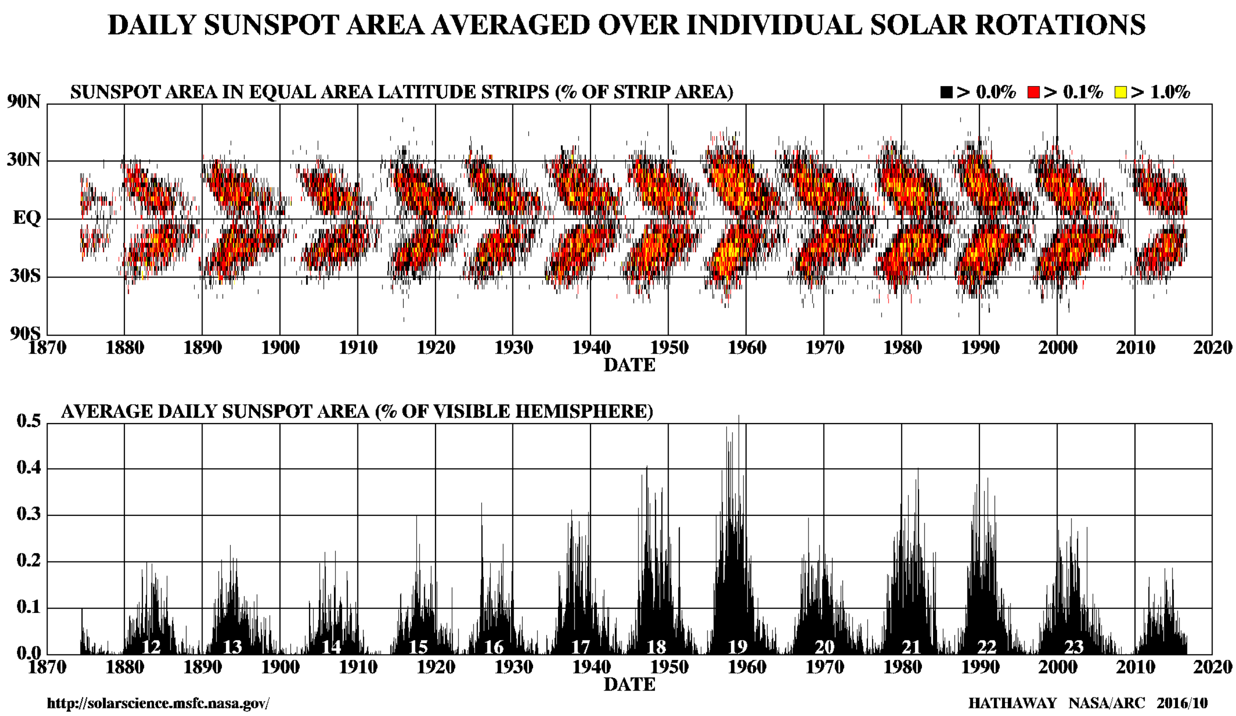
Evolution of the Sun in extreme ultraviolet light from 2010 through 2020, as seen from the telescope aboard Europe's PROBA2 spacecraft. Credit: Dan Seaton/European Space Agency (Collage by NOAA/JPL-Caltech)

By studying the sun's magnetic field, modern astronomers have discovered that the cycle covers twenty-two years, with each eleven-year cycle of sunspots followed by a reversal of the direction of the Sun's magnetic field. According to Fisher, "the overall magnetic field structure changes in a way that is very interesting. It turns out that if the magnetic fields primarily point from west to east in the Northern Hemisphere (of the sun), they point from east to west in the Southern Hemisphere. In the next eleven-year cycle, the fields are reversed. So the cycle is really twenty-two years."



Sunspots appear mostly in the low latitudes near the solar equator. In fact they almost never appear closer than 5 or further than 40 degrees latitude, north or south. As each sunspot cycle progresses, the sunspots gradually start to appear closer and closer to the equator. The sunspot locations for the most recent 11-year cycle are shown in this "butterfly" diagram." The locations "migrated" toward the equator (0 latitude) from both hemispheres throughout this half of the cycle.

The Butterfly Diagram:



Throughout the solar_cycle, the latitude of sunspot occurrence varies with an interesting pattern. The plot on the left shows the latitude of sunspot occurrence versus time (in years). Sunspots are typically confined to an equatorial belt between -35 degrees south and +35 degrees north latitude. At the beginning of a new solar cycle, sunspots tend to form at high latitudes, but as the cycle reaches a maximum (large numbers of sunspots) the spots form at lower latitudes. Near the minimum of the cycle, sunspots appear even closer to the equator, and as a new cycle starts again, sunspots again appear at high latitudes. This recurrent behavior of sunspots gives rise to the ``butterfly'' pattern shown, and was first discovered by Edward Maunder in 1904. The reason for this sunspot migration pattern is unknown. Understanding this pattern could tell us something about how the Sun's internal magnetic field is generated.

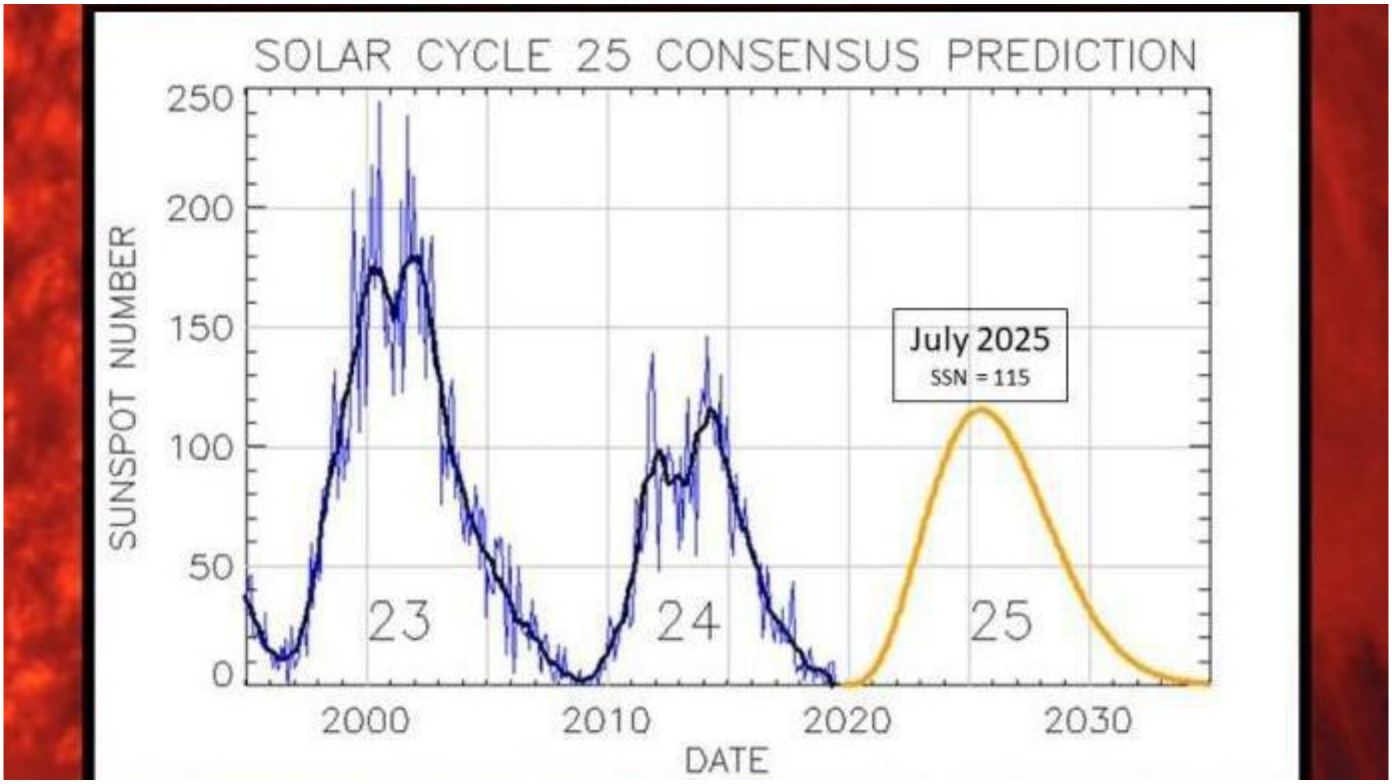
Cycle 25:

The latest solar cycle, number 24, was determined to have ended in December 2019 when the average number of sunspots from this cycle reached a minimum and the first sunspots of the new cycle began to emerge.

A new solar cycle is considered to start when new spots emerging at mid-latitudes on the Sun’s surface are opposite in magnetic polarity than the sunspots from the previous cycle. But because sunspot numbers fluctuate day-by-day and week-by-week, scientists use a rolling average meaning it takes a few months for clear patterns in activity to become clear.

Predicting just how active the Sun will get at the peak of a cycle is a notoriously difficult task. Just like weather on Earth, long term solar forecasts are difficult to gather, although we know there are general seasons of behaviour.

Although the consensus on Solar Cycle 25 is that it will be similar to the last, this prediction comes with more uncertainty than most as solar cycle 25 comes after a general decline in peak solar activity. At this stage, the next solar cycle could continue the downward trend towards cycles with weaker than average activity, or it could mark the beginning of a series of more active cycles.



[Solar cycle 25 prediction, NOAA]

Solar Magnetism
Lecture#10: bipolar magnetic fields
Dr. Huda Sh. Ali

A solar magnetic field is a magnetic field generated by the motion of conductive plasma inside a star. This motion is created through convection, which is a form of energy transport involving the physical movement of material. A localized magnetic field exerts a force on the plasma, effectively increasing the pressure without a comparable gain in density. As a result, the magnetized region rises relative to the remainder of the plasma, until it reaches the star's photosphere. This creates a localized region of the Sun’s surface and atmosphere that displays some or all of these phenomena is often called an “active region” or Bipolar Magnetic Fields.

These bipolar magnetic fields represent the locations of positive or negative polarity where magnetic field lines emerge from or re-enter the solar surface, respectively. Each endpoint of an emerged loop represents a separate photospheric element of opposite polarity. These bipolar magnetic fields are concentrations of strong magnetic fields and, although often complex in structure, they contain on average equal quantities of positive and negative magnetic flux.

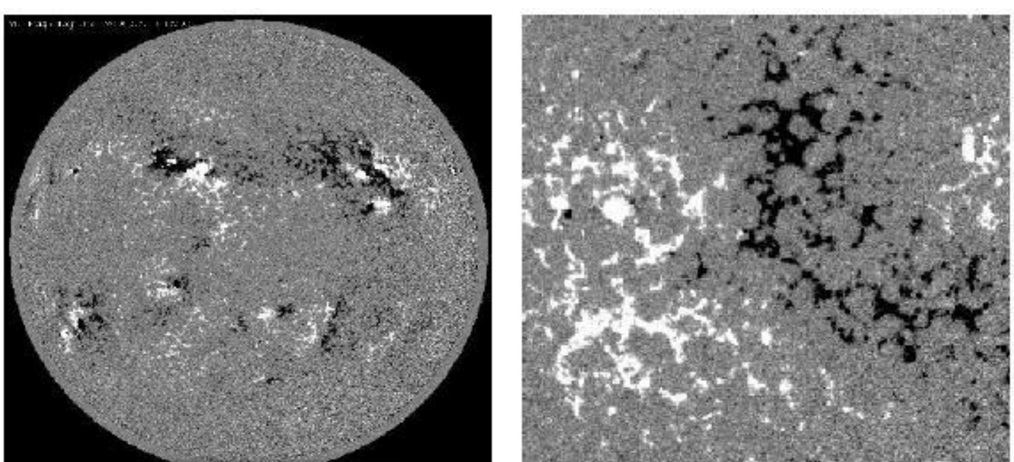


Figure 1:

Figure (1), shows the black and white indicate positive and negative magnetic polarity, respectively, while grey signifies low magnetic flux levels.

Bipolar active regions exhibit a number of systematic properties, providing information on the magnetic structure of the source region near the bottom of the convection zone from where the surface flux merges. Thus bipolar active region properties are:

a. generally orientated roughly in the East-West direction,
b. systematically tilted in latitude,
c. appear only in low and middle latitudes (below 45 degrees),
d. Show different proper motions of the two opposite-polarity patches, with respect to the surrounding plasma.

During and shortly after their emergence at the surface, these properties are displayed by the bipolar regions in a statistical sense; smaller regions show a larger scatter. To define bipolar magnetic regions numerically, consider the particular initial condition of a magnetic bipole with half-separation ρ_0 and tilt angle δ_0 (the angle between the bipolar spots is equal to one half of the latitude). This is given by the magnetic field:

$$B(x, y, 0) = B_0 e^{1/2} \frac{x'}{\rho_0} e^{-\xi} \qquad (1)$$

Where

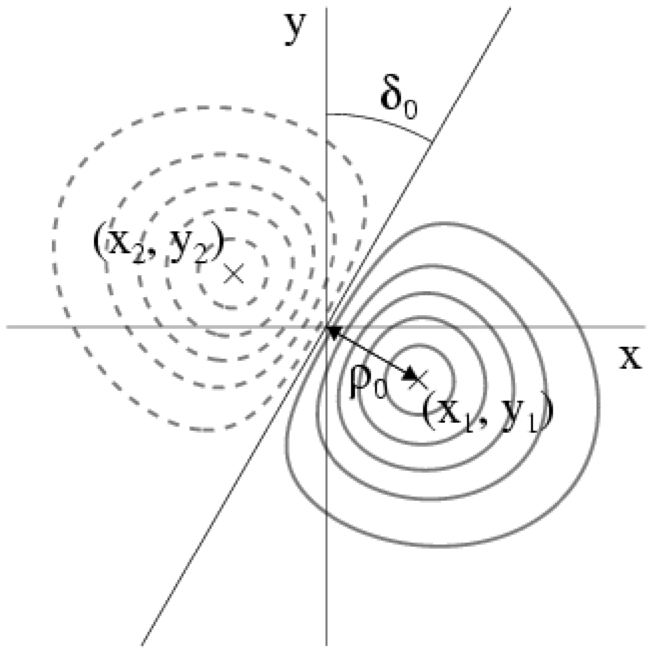
$$\xi = \frac{(x')^2/2 + (y')^2}{\rho_0^2} \qquad (2)$$

And the tilted coordinates (x_0, y_0) are given in terms of the untilted (x, y) as:

$$x' = (x - x_0)\cos\delta_0 - (y - y_0)\sin\delta_0 \qquad (3)$$

$$y' = (x - x_0)\sin\delta_0 - (y - y_0)\,cos\,\delta_0 \qquad (4)$$

Here (x_0, y_0) is the location of the bipole center. Shows figure (3).



This figure presents example illustrate feature the simulation using the initial condition of a single magnetic bipole, before considering simulations of the transport flux, with tilt angle of this bipole which equal :

$$\tan\delta(t) = \frac{y_2(t) - y_1(t)}{x_1(t) - x_2(t)} \qquad (5)$$

Solar Magnetism
Lecture#11: Differential Rotation & Meridional Flow
Dr. Huda Sh. Ali
2018-2017

Motions of magnetic flux on the surface of the Sun are characterized by three primary modes of transport:

1- Supergranular Flows.
2- Differential Rotation.
3- Meridional Flow.

Supergranulation: Convection is a product of the large temperature gradient in the Sun's convection zone. This temperature gradient causes the plasma to rise and fall like boiling water. The size of these convective cells span two to three orders of magnitude: from *granules* with diameters of 1000 km, to *supergranules* with diameters of 30,000 km, and to *giant cells* with diameters of 200,000 km. These cells also have a range of lifetimes. Granules have lifetimes of only 10 minutes, while supergranules have lifetimes of many hours. The flows within these cells varies as well. Granules have internal velocities of 3000 m s^{-1}, whereas supergranules have internal velocities of 500 m s^{-1}.

Differential Rotation: is seen when different parts of a rotating object move with different angular velocities at different latitudes. This indicates that the object is not solid (fluid objects). On the Sun, the solar rotation varies with latitude because the Sun is composed of a gaseous plasma. The rate of rotation is observed to be fastest at the equator (latitude $\varphi = 0$ °), and to decrease as latitude increases. At the equator the solar rotation period is 24.47 days and almost 38 days at the poles (figure 1).

The differential rotation rate ($u_\emptyset$), is approximated by:

$$u_\emptyset = \Omega(\theta)\, R_\odot \sin\theta$$

Where (Ω) is the angular velocity of differential rotation, given by:

$$\Omega(\theta) = 0.18 - 2.3cos^2\theta - 1.62cos^4\theta$$

Figure (2) shows a plot of the rate of differential rotation. It can be seen that at higher latitudes the magnitude of differential rotation is much greater than at lower latitudes. Closer to the equator, the magnitude of differential rotation is much slower. This profile produces the observed differential rotation on the sun. The rate of differential rotation at the polar region is approximately 30 days, while at the equator it is 26 days, this gives a timescale for differential rotation of $T_{dr} = \frac{2\pi}{\Omega\left(\frac{\pi}{2}\right)-\Omega(0)} \approx$ 0.25 years.

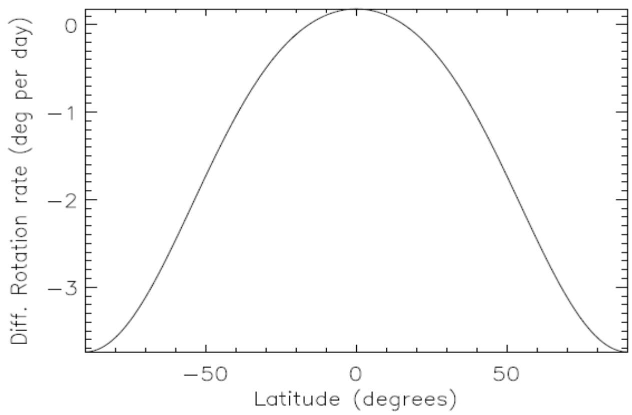


Figure 1: the rate of differential rotation versus latitude.

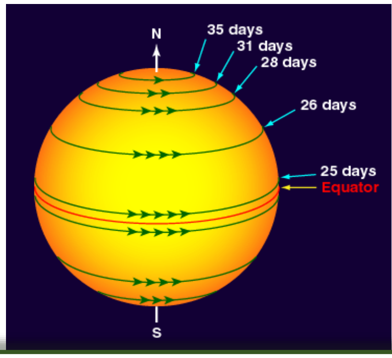


Figure 2: Surface Differential Rotation.

Q1: explain figure 2??

The Sun is gaseous and rotates differentially, and these facts radically affect the character of solar magnetism. As illustrated in Figure 3, because the Sun rotates more rapidly at the equator than at the poles, the differential rotation distorts the solar magnetic field, wrapping it around the solar equator, eventually causing the original north-south magnetic field to reorient itself in an east-west direction. Convection then causes the magnetized gas to upwell toward the surface, twisting and tangling the magnetic field pattern. In some places, the field becomes kinked like a knot in a garden hose, causing it to increase in strength. Occasionally, the field strength becomes so great that it overwhelms the Sun's gravitational field and a "tube" of field lines bursts out of the surface and loops through the lower atmosphere, forming a sunspot pair. The general east-west organization of the underlying solar field accounts for the observed polarities of the pairs in each hemisphere, figure 3.

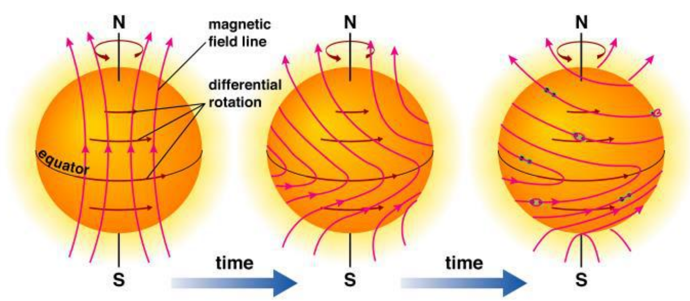


Figure 3: Differential Rotation versus latitude of the Sun.

Meridional Flow: the flow of material along meridian lines from the equator toward the poles at the surface and from the poles to the equator below the surface, must also play an important role in the Sun's magnetic dynamo. At the surface this flow is a slow 20 m/s) but the return flow toward the equator inside the Sun where the density is much higher must be much slower still (1 to 2 m/s), Figure 4. This slow return flow would carry material from the mid-latitudes to the equator in about 11 years.

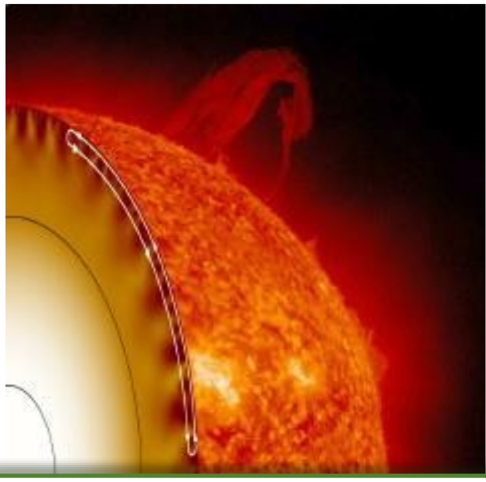


Figure 4: Shallow Meridional Flow

Meridional circulation distribution on the other hand can be given:

$$u_\theta = C\, cos\left[\frac{\pi(\theta_{max} + \theta_{min} - 2\theta)}{2(\theta_{max} - \theta_{min})}\right] cos\theta$$

Where $\theta_{min} = 0\ and\ \ \theta_{max} = {}^{\pi}/_{2}$, C is constant $\cong 16\ ms^{-1}$.

Figure 5, shows a plot of the profile of meridional flow on the sun versus latitude. The magnitude of the flow rate increases where move away from the equator.

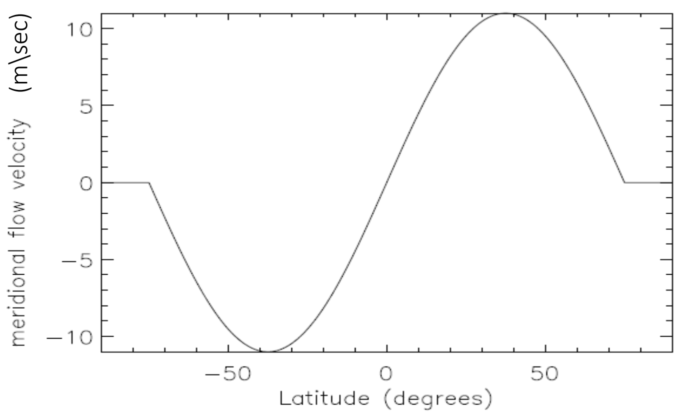


Figure 5: meridional flow rate versus latitude.

Q2: Why do we need such meridional flows and differential rotation for the solar dynamo to work?

Q3: explain figures 1, 3, and 5??

Solar Magnetism
Lecture#12: The Solar Magnetic Flux Transport Model
Dr. Huda Sh. Ali

Flux transport model describe the evolution of the flux distribution at the solar surface as a result of bipolar magnetic regions and the transport of the corresponding radial magnetic flux by the horizontal flows due to convection, differential rotation and meridional circulation. The standard equation of magnetic flux transport of the large-scale magnetic field on the solar surface is described by the induction equation for the radial magnetic field component $B_r(\theta, \emptyset, t)$. In spherical coordinates is written as:

$$\frac{\partial B_r}{\partial t} = \frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\left(-u(\theta)B_r + D\frac{\partial B_r}{\partial\theta}\right)\right) - \Omega(\theta)\frac{\partial B_r}{\partial\emptyset} + \frac{D}{sin^2\theta}\frac{\partial^2 B_r}{\partial\emptyset^2} + S(\theta, \emptyset, t) \quad (1)$$

Where B_r is the radial magnetic flux, θ is the colatitude, $\emptyset$ is the longitude, $\Omega(\theta)$ and $u(\theta)$ represent the surface flows of angular velocity of the photospheric plasma and meridional flow, respectively, D is the isotropic diffusion coefficient representing superganular diffusion, and finally $S(\theta,\ \varphi,\ t)$ is an additional source term added to represent the emergence of new magnetic flux.

The approximate the spherical domain by Cartesian coordinates providing that consider only a localized region, such as that occupied by a single active region. Equation (1) is then replaced by the form:

$$\frac{\partial B_r}{\partial t} = -\nabla.(vB_r) + D\nabla^2 B_r \qquad (2)$$

In this equation v is the plasma velocity vector, D is the diffusion coefficient, ∇ represents gradient, $\nabla.$ represents divergence, and ∇^2 Laplace operator.

The transport refers to processes which move substances through the hydrosphere and atmosphere by physical means. There are two primary modes of transport in fluid mechanics are: advection (transport associated with the flow of a fluid) and diffusion (transport associated with random motions within a fluid). In equation (2) notice a combination of the diffusion and advection equations, which describes flux distribution at the solar surface, where the right-hand side of the equation is the sum of two contributions.

The first $-\nabla.(vB_r)$: describes advection, and

The second $D\nabla^2 B_r$: describes diffusion.

By numerical analytical solutions to equation (2), in Cartesian planar surface for 2D, we get:

$$\frac{\partial B_z}{\partial t} = -v_x \frac{\partial B_z}{\partial x} - v_y \frac{\partial B_z}{\partial y} + D\left(\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2}\right) \qquad (3)$$

Where B_z is magnetic flux through z surface, (x, y) plane represents the photosphere, $v_x(x,\ y)$ component represents differential rotation, and $v_y(x,\ y)$ component represents meridional flow which equal. Else, one of the key ingredients in models of the type considered here is the ***Diffusion Coefficient***. The diffusion coefficient in the range of ($770 - 1540\ km^2 sec^{-1}$).

Numerical Solution:

by using Taylor's series, which illustrate here:

$$f(x+\Delta x) = f(x) + (\Delta x)\frac{\partial f}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 f}{\partial x^2} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 f}{\partial x^3} + \cdots \qquad (1)$$

$$= f(x) + \sum_{n=1}^{\infty} \frac{(\Delta x)^n}{n!}\frac{\partial^n f}{\partial x^n} \qquad (2)$$

Solving for $\partial f / \partial x$, one obtains:

$$\frac{\partial f}{\partial x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} - \frac{\Delta x}{2!}\frac{\partial^2 f}{\partial x^2} - \frac{(\Delta x)^2}{3!}\frac{\partial^3 f}{\partial x^3} + \cdots \tag{3}$$

Summing all terms with factors of Δx and higher and representing them as O(Δx) (that is read as terms of order Δx) yields:

$$\frac{\partial f}{\partial x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} + O(\Delta x) \tag{4}$$

Thus far, there are three approximations for 1st order- 1D are:

(Where the subscript index i, j, k is used to represent the discrete point in the x, y, z – directions).

- Forward Difference Approximation,

$$\frac{\partial f}{\partial x}\Big|_i = \frac{f_{i+1} - f_i}{\Delta x} + O(\Delta x) \tag{5}$$

- Backward Difference Approximation,

$$\frac{\partial f}{\partial x}\Big|_i = \frac{f_i - f_{i-1}}{\Delta x} + O(\Delta x) \tag{6}$$

- Central Difference Approximation .

$$\frac{\partial f}{\partial x}\Big|_i = \frac{f_{i+1} - f_{i-1}}{2\Delta x} + O(\Delta x) \tag{7}$$

<u>For 1st order- 2D are:</u>

- Forward Difference Approximation,

$$\frac{\partial f}{\partial x \partial y}\Big|_{ij} = \frac{f_{i+1,j} - f_{ij}}{\Delta x} + \frac{f_{i,j+1} - f_{ij}}{\Delta y} + O(\Delta x) + O(\Delta y) \tag{8}$$

- Backward Difference Approximation,

$$\frac{\partial f}{\partial x \partial y}\Big|_{ij} = \frac{f_{ij} - f_{i-1,j}}{\Delta x} + \frac{f_{ij} - f_{i,j-1}}{\Delta y} + O(\Delta x) + O(\Delta y) \tag{9}$$

- Central Difference Approximation,

$$\frac{\partial f}{\partial x \partial y}|_{ij} = \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x} + \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta y} + O(\Delta x)^2 + O(\Delta y)^2 \qquad (10)$$

For 1st order- 3D are:

- Forward Difference Approximation,

$$\frac{\partial f}{\partial x \partial y \partial z}|_{ijk} = \frac{f_{i+1,j,k} - f_{ijk}}{\Delta x} + \frac{f_{i,j+1,k} - f_{ijk}}{\Delta y} + \frac{f_{ij,k+1} - f_{ijk}}{\Delta z} + O(\Delta x) + O(\Delta y) + O(\Delta z) \qquad (11)$$

- Backward Difference Approximation,

$$\frac{\partial f}{\partial x \partial y \partial z}|_{ijk} = \frac{f_{ijk} - f_{i-1,jk}}{\Delta x} + \frac{f_{i,jk} - f_{i,j-1,k}}{\Delta y} + \frac{f_{ijk} - f_{ijk-1}}{\Delta z} + O(\Delta x) + O(\Delta y) + O(\Delta z) \qquad (12)$$

- Central Difference Approximation,

$$\frac{\partial f}{\partial x \partial y \partial z}|_{ijk} = \frac{f_{i+1,jk} - f_{i-1,jk}}{2\Delta x} + \frac{f_{i,j+1,k} - f_{i,j-1,k}}{2\Delta y} + \frac{f_{ijk+1} - f_{ijk-1}}{2\Delta z} + O(\Delta x)^2 + O(\Delta y)^2 + O(\Delta z)^2 \qquad (13)$$

For 2nd order- 1D are:

- Central Difference Approximation,

$$\frac{\partial^2 f}{\partial x^2}|_{i} = \frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta x^2} + O(\Delta x)^2 \qquad (14)$$

For 2nd order- 2D are:

- Central Difference Approximation,

$$\frac{\partial^2 f}{\partial x^2 \partial y^2}|_{ij} = \frac{f_{i+1,j} - 2f_{ij} + f_{i-1,j}}{\Delta x^2} + \frac{f_{i,j+1} - 2f_{ij} + f_{i,j-1}}{\Delta y^2} + O(\Delta x)^2 + O(\Delta y)^2 \qquad (15)$$

For 2nd order- 3D are:

- Central Difference Approximation,

$$\frac{\partial^{2} f}{\partial x^{2} \partial y^{2} \partial z^{2}}|_{ijk} = \frac{f_{i+1,jk} - 2f_{ijk} + f_{i-1,jk}}{\Delta x^{2}} + \frac{f_{i,j+1,k} - 2f_{ijk} + f_{i,j-1,k}}{\Delta y^{2}} + \frac{f_{ijk+1} - 2f_{ijk} + f_{i,jk-1}}{\Delta z^{2}} + O(\Delta x)^{2} + O(\Delta y)^{2} + O(\Delta z)^{2} \qquad (16)$$

Explicit Methods:

The explicit method which is obtained by approximating the time derivative at time level $t_n$by means of a forward differencing scheme, is:

$$\frac{\partial f}{\partial t}(t_n) = \frac{f^{n+1} - f^{n}}{\Delta t_n} \qquad (17)$$

For example, when a first-order backward difference approximation for the time derivative and a second-order central difference approximation for spatial derivative is used, the discretized equation takes the form:

$$\frac{u_{ij}^{n+1} - u_{ij}^{n}}{\Delta t} = \propto (\frac{u_{i+1j}^{n} - 2u_{ij}^{n} + u_{i-1j}^{n}}{\Delta x^{2}} + \frac{u_{ij+1}^{n} - 2u_{ij}^{n} + u_{ij-1}^{n}}{\Delta y^{2}}) \qquad (18)$$

In this equation, u_{i}^{n+1} is the only unknown, and it can be computed from the following:

$$u_{ij}^{n+1} = u_{ij}^{n} + \frac{\propto \Delta t}{\Delta x^{2}}\left(u_{i+1j}^{n} - 2u_{ij}^{n} + u_{i-1j}^{n}\right) + \frac{\propto \Delta t}{\Delta y^{2}}\left(u_{ij+1}^{n} - 2u_{ij}^{n} + u_{ij-1}^{n}\right) \qquad (19)$$

$$u_{ij}^{n+1} = u_{ij}^{n}(1 - \frac{2\propto\Delta t}{\Delta x^{2}} - \frac{2\propto\Delta t}{\Delta y^{2}}) + \frac{\propto\Delta t}{\Delta x^{2}}\left(u_{i+1j}^{n} + u_{i-1j}^{n}\right) + \frac{\propto\Delta t}{\Delta y^{2}}\left(u_{ij+1}^{n} + u_{ij-1}^{n}\right) \qquad (20)$$

Note that the value of the dependent variable at time level *n* is known from a previous solution or given as initial data,(figure (1)).

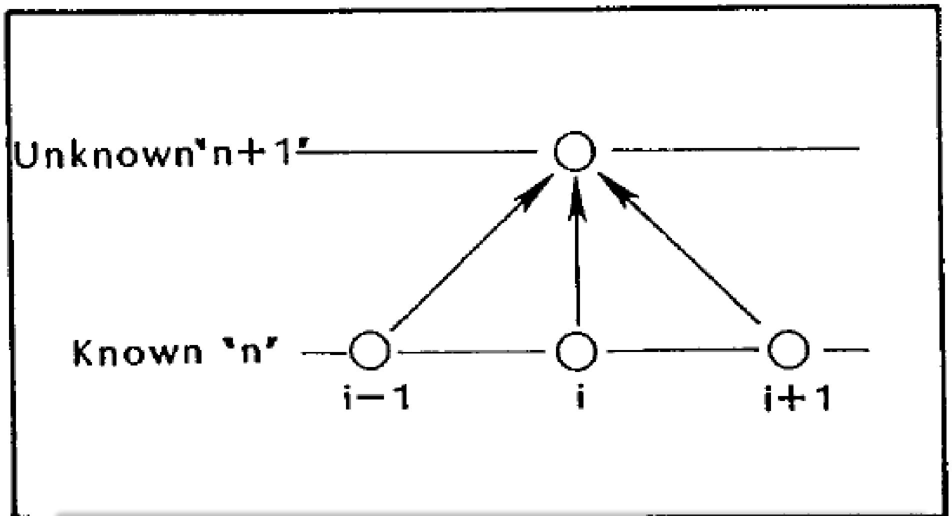


***Figure 1*:** Procedure for explicit method.

The Cartesian form of equation (3) for surface flux transport is solved numerically in more details.

First term *in forward time*, is:

$$\frac{\partial B_r}{\partial t} = \frac{B_{ij}^{n+1} - B_{ij}^{n}}{\Delta t} \tag{21}$$

The advection term, *Central Difference Approximation for 1st order- 2D is*:

$$-\nabla.(vB_r) = -v_x \frac{\partial B_z}{\partial x} - v_y \frac{\partial B_z}{\partial y}$$

$$= -vx\left(\frac{B_{i+1,j}^{n} - B_{i-1,j}^{n}}{2\Delta x}\right) - vy\left(\frac{B_{i,j+1}^{n} - B_{i,j-1}^{n}}{2\Delta y}\right) \tag{22}$$

The diffusion term, *Central Difference Approximation for 2nd order- 2D, is:*

$$D\nabla^2 B_r = D\left(\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2}\right)$$

$$= D\left(\frac{B_{i+1,j}^{n} - 2B_{ij}^{n} + B_{i-1,j}^{n}}{\Delta x^2}\right) + D\left(\frac{B_{i,j+1}^{n} - 2B_{ij}^{n} + B_{i,j-1}^{n}}{\Delta y^2}\right) \tag{23}$$

The equation (3) rearranged as:

$$\frac{B_{ij}^{n+1}-B_{ij}^{n}}{\Delta t}=-v_x\left(\frac{B_{i+1,j}^{n}-B_{i-1,j}^{n}}{2\Delta x}\right)-v_y\left(\frac{B_{i,j+1}^{n}-B_{i,j-1}^{n}}{2\Delta y}\right)+D\left(\frac{B_{i+1,j}^{n}-2B_{ij}^{n}+B_{i-1,j}^{n}}{\Delta x^2}\right)+D\left(\frac{B_{i,j+1}^{n}-2B_{ij}^{n}+B_{i,j-1}^{n}}{\Delta y^2}\right) \qquad (24)$$

$$B_{ij}^{n+1}-B_{ij}^{n}=-\frac{v_x\Delta t}{2\Delta x}\left(B_{i+1,j}^{n}-B_{i-1,j}^{n}\right)-\frac{v_y\Delta t}{2\Delta y}\left(B_{i,j+1}^{n}-B_{i,j-1}^{n}\right)+\frac{D\Delta t}{\Delta x^2}\left(B_{i+1,j}^{n}-2B_{ij}^{n}+B_{i-1,j}^{n}\right)+\frac{D\Delta t}{\Delta y^2}\left(B_{i,j+1}^{n}-2B_{ij}^{n}+B_{i,j-1}^{n}\right) \qquad (25)$$

$$B_{ij}^{n+1}=B_{ij}^{n}\left(1-\frac{2D\ \Delta t}{\Delta x^2}-\frac{2D\ \Delta t}{\Delta y^2}\right)+B_{i+1,j}^{n}\left(\frac{v_x\ \Delta t}{2\Delta x}-\frac{D\ \Delta t}{\Delta x^2}\right)+B_{i-1,j}^{n}\left(-\frac{v_x\ \Delta t}{2\Delta x}-\frac{D\ \Delta t}{\Delta x^2}\right)+B_{i,j+1}^{n}\left(\frac{v_y\ \Delta t}{2\Delta y}-\frac{D\ \Delta t}{\Delta y^2}\right)+B_{i,j-1}^{n}\left(-\frac{v_y\ \Delta t}{2\Delta y}-\frac{D\ \Delta t}{\Delta y^2}\right) \qquad (26)$$

Stability criteria of the explicit method is

$$\Delta t\le\frac{\Delta x+\Delta y}{v}$$

And

$$\Delta t\le\frac{\Delta x^2+\Delta y^2}{2D}$$

Where the subscript i and j represents x and y coordinates in 2D, and $\Delta x, \Delta y$ is the grid spacing, v is the velocity, Δt is the time step, and D is the diffusivity.

Solar Magnetism
Lecture# 13: Coronal Magnetic Field Models
Dr. Huda Sh. Ali

Coronal Magnetic Field Models

The magnetic fields observed in the solar photosphere extend out into the corona, where they structure the plasma, store free magnetic energy and produce a wide variety of phenomena. While the distribution and strength of magnetic fields are routinely measured in the photosphere, the same is not true for the corona, where the low densities mean that such measurements are very rare. To understand the nature of coronal magnetic fields, theoretical models that use the photospheric observations as a lower boundary condition are required. In this section, survey the variety of techniques that have been developed to model the coronal magnetic field based on the input of photospheric magnetic fields.

This techniques divided into five categories are:

1. Potential field source surface model,
2. Linear force-free field model
3. Nonlinear force-free field model
4. Magneto hydrostatic (MHS) model,
5. Magneto hydrodynamic (MHD) model.

All of these models output only (or primarily) the magnetic field. The fifthly category are full MHD models, which aim to self-consistently describe both the magnetic field and other plasma properties.

1. Potential Field Source Surface Model

Potential Field Source Surface (PFSS) Model provide an approximate description of the solar coronal magnetic field based on observed photospheric fields (magnetograms). A potential (or current-free) field obeys the equation:

$$\nabla \times \mathrm{B} = 0 \tag{27}$$

We can define a magnetic potential field $B(r)$ by a scalar potential$\emptyset(\mathrm{r})$:

$$B(r) = \nabla\emptyset(\mathrm{r}) \tag{28}$$

Or expressed in Cartesian coordinates,

$$\left(B_x, B_y, B_z\right) = \left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right)\emptyset(x, y, z) \tag{29}$$

The simplest representation of a magnetic potential field that fulfills Maxwell divergence-free condition *($\nabla \cdot B = 0$)* is a magnetic charge that is buried below the solar surface (to avoid magnetic monopoles in the corona), which predicts a magnetic field $B(x)$ that points away from the buried unipolar charge and whose field strength falls off with the square of the distance r ,

$$B(x) = B_o\left(\frac{d_o}{r}\right)^2 \frac{\boldsymbol{r}}{r} \tag{30}$$

Where B_o is the magnetic field strength at the solar surface directly above the buried magnetic charge, $\boldsymbol{r_o} = (x_o, y_o, z_o)$ is the subphotospheric position of the buried charge, $d_o = \sqrt{1 - x_o^2 - y_o^2 - z_o^2}$ is the depth of the magnetic charge, and $\boldsymbol{r} = [(x - x_o), (y - y_o), (z - z_o)]$ is the solar corona from the location $\boldsymbol{r_o}$ of the buried charge.

In order to obtain the Cartesian coordinates (B_x, B_y, B_z) of the magnetic field vector $B_j(x)$, we can rewrite equation (30) as,

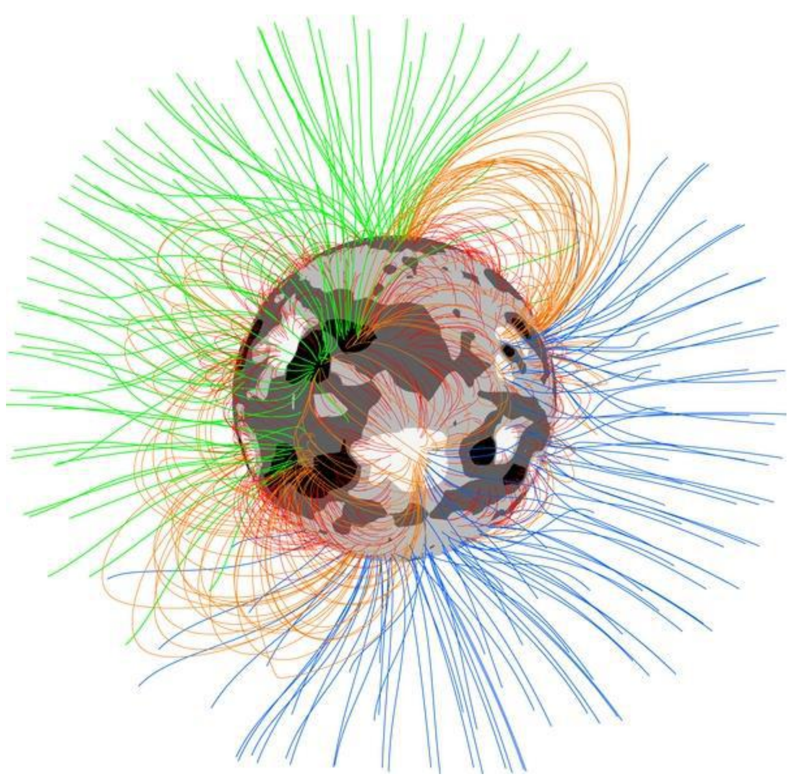
$$B_x(x, y, z) = B_0\left(\frac{d_0}{r}\right)^2 \frac{(x - x_0)}{r} \tag{31}$$

$$B_y(x, y, z) = B_0\left(\frac{d_0}{r}\right)^2 \frac{(y - y_0)}{r} \tag{32}$$

$$B_z(x, y, z) = B_0 \left(\frac{d_0}{r}\right)^2 \frac{(z - z_0)}{r} \qquad (33)$$

This way can parameterize a 3D magnetic field *B(x)*. The formal boundary conditions for the potential model are the prescription of the normal component of the magnetic field at the photosphere. Observations determine the line-of-sight component of the magnetic field more accurately than the transverse component, and early instruments provided only the line-of-sight component. An equivalent formulation of the boundary value problem based on a prescription of B at the photosphere is more useful in practice.

Potential field models have been used to model active regions, to model the global coronal magnetic field, and often used to determine the structure of the coronal magnetic at the Earth in order to predict the regions of fast and slow solar wind.



Solar Magnetism
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2. Force-Free Field Model

A force-free magnetic field is a magnetic field that arises when the plasma pressure is so small, relative to the magnetic pressure, that the plasma pressure may be ignored, and so only the magnetic pressure is considered. For a force free field, the *electric current density* is either zero or parallel to the magnetic field. The name "force-free" comes from being able to neglect the force from the plasma. In the absence of non-magnetic forces, the magnetic force must be self-balancing, i.e.

$$j \times B = 0 \qquad (34)$$

This equation is the defining equation for a force-free magnetic field. A consequence of equation (34) is that electric currents are constrained to flow along magnetic field lines. The magnetic field and current also satisfy the magnetohydrostatic Maxwell equations:

$$\nabla \times B = u_o j \qquad (35)$$

Where μ_o the magnetic permeability of free space, and

$$\nabla . B = 0 \qquad (36)$$

For the coronal volume with B specified at the lower boundary. Non-trivial solutions of (34) and (36) require the field and current to be parallel, i.e.

$$u_o j = \alpha B \qquad (37)$$

Which mean,

$$(\nabla \times \mathrm{B}) = \mu_o \mathrm{j} = \alpha \mathrm{B} \qquad (38)$$

Where α is a scalar function as a function of position (and may additionally depend on time).

The equation (38) becomes,

$$\nabla \times B = \alpha B \tag{39}$$

And α which equal:

$$\alpha = \frac{1}{B}(\nabla \times B) \tag{40}$$

This models used to describe the slowly evolving structure of the solar coronal field has great mathematical convenience but its physical significance remains conjectural.

3 Nonlinear Force-Free Field Model

The coronal magnetic field is often modeled as a nonlinear force-free field (NFFF), and to concentrate on how to use the photospheric $(B_x, B_y, and\ B_z)$ to derive the coronal magnetic field, used to approximate the normal electric current distribution by (for the transverse photospheric magnetic field (B_x, B_y))

$$\mu_o j = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \tag{41}$$

And from this one gets the distribution of α on the photosphere by (from equation (38))

$$\alpha = \mu_o \frac{j}{B} \tag{42}$$

Physically α represents the constant ratio of the electric current density to the magnetic field along a field line, the scalar function α is constant on every field line, but will usually change from one field line to another.

Where the photospheric boundary condition at height $z = 0$ is given by the measured magnetic field components (B_x, B_y, B_z), which also define the α parameter at the photospheric boundary (e.g., from the z-component of equation,

$$\alpha(x, y, z = 0) = \frac{1}{B_z}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) \tag{43}$$

Using the $x\ and\ y - component$ of (equation (38)), and α can be written as a function of horizontal derivatives,

$$\frac{\partial B_y}{\partial z} = -\alpha B_x + \frac{\partial B_z}{\partial y} \tag{44}$$

$$\frac{\partial B_x}{\partial z} = \alpha B_y + \frac{\partial B_z}{\partial x} \tag{45}$$

$$\frac{\partial B_z}{\partial z} = -\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y} \tag{46}$$

$$\frac{\partial \alpha}{\partial z} = -\frac{1}{B_z}\left(B_x \frac{\partial \alpha}{\partial x} + B_y \frac{\partial \alpha}{\partial y}\right) \tag{47}$$

The right hand sides of equations (44)-(47) depend only on variables at the current height z. As show here, there are three different assumptions on the nature of the α parameter can be made.

If $\alpha = 0$, the field is potential, that is, the lowest order approximation for a description of realistic solar magnetic field. If α has the same value throughout the field domain, the resulting subclass of force-free field is called a constant- α or linear field, since the field components satisfy a linear differential equation. And for a general class of field where $\alpha = f(r)$, is a variable value is called nonlinear force-free (NLFF) field, where depending on position (r).

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4. Magnetohydrostatic model

The starting point of the investigation is the set of magnetohydrostatic (MHS) equations including a pressure gradient and an external gravitational field, but without an energy equation or an equation of state:

$$j \times B - \nabla p - \rho \nabla \psi = 0 \qquad (48)$$

$$\nabla \times B = u_o j \quad and \quad \nabla . B = 0$$

Where the coronal application is $\psi = -GM/r$ is taken to be the gravitational potential, p is the pressure, ρ is the mass density, $G = 6.673 \times 10^{-11} Nm^2 kg^{-2}$) is the gravitational constant, and $M = 1.989 \times 10^{30}\ kg$ is the solar mass.

The solutions that have been used to extrapolate photospheric data have a current density of the form:

$$j = \alpha B + \xi(r) \nabla (r.B) \qquad (49)$$

Where

$$\xi(r) = \frac{1}{r^2} - \frac{1}{(r+a)^2} \qquad (50)$$

r and a are constant parameters. Thus, the current comprises a field-aligned part and a part perpendicular to gravity. This model used for resistive diffusion of a force-free magnetic field in a compressible plasma is analyzed. Such a model has been suggested for describing the behavior of the sunspot, solar wind, corona loop, corona hole, and solar flares.

5. Magnetohydrodynemacis (MHD) model

Magnetohydrodynamics (**MHD)** is the study of the magnetic properties of electrically conducting fluids. Examples of such magnetofluids include plasmas, liquid metals, salt water and electrolytes. The word "magnetohydrodynamics" is derived from magneto- meaning magnetic field, hydro- meaning water, and -dynamics meaning movement. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. The set of equations that describe MHD are a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. Which include, simplified Maxwell's equations, Newton's Law of Motion, an energy equation, Ohms law, an equation of state and the equation of mass continuity describe the behavior of the magnetic field and plasma. Using these equations, and eliminating the electric field between Maxwell's equation and Ohm's law, gives rise to the Magnetic Induction Equation, which relates the plasma velocity (v) to the magnetic field (B). Thus, a full set of ideal MHD equations includes]:

Momentum Equation:

$$\rho\frac{\partial v}{\partial t}+(\rho v.\nabla)v=-\nabla(p+p_{w)}+\rho g+\frac{1}{\mu_o}(\nabla\times B)\times B \qquad (51)$$

Ampere's law:

$$\nabla\times B=\frac{4\pi}{c}j \qquad (52)$$

Faraday's law:

$$\nabla\times E=-\frac{1}{c}\frac{\partial B}{\partial t} \qquad (53)$$

Ohm's law:

$$j\eta = E + \frac{v \times B}{c} \qquad (54)$$

Adiabatic Energy Equation:

$$\frac{\partial T}{\partial t} + \mathrm{v}.\nabla\mathrm{T} = (\gamma - 1)\left(-\mathrm{T}\nabla.\mathrm{v} + \frac{m_p}{k\rho}\mathrm{S}\right) \qquad (55)$$

$$where \quad S = -\nabla.q - n_e n_p Q(t) + H_{ch}$$

Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho v) = 0 \qquad (56)$$

where B, J, E, ρ, v, p, T, g, η, v, $Q(t)$, n_e, n_p, γ = 5/3,H_{ch}, q, p_w, and m_p are the magnetic field, electric current density, electric field, plasma mass density, velocity, plasma pressure = $(n_e + n_p)kT$, temperature, gravitational acceleration which equal($-g_o\,\hat{\boldsymbol{r}}/r^2$), resistivity, kinematic viscosity, radiative losses, electron and proton number densities (which are equal for a hydrogen plasma), polytropic index, coronal heating term, heat flux, wave pressure, and m_p is the proton mass respectively.

The significant advance has been made in the construction of realistic 3D global MHD models. Such models are required to give a self-consistent description of the interaction between the magnetic field and the plasma in the Sun's atmosphere. They allow for comparison with observed plasma emission, and enable more consistent modeling of the solar wind, so that the simulation domain may extend far out into the heliosphere. On the other hand, additional boundary conditions are required, and the problem of non-uniqueness of solutions found in nonlinear force-free field models continues to apply here. While these models are sometimes used to simulate eruptive phenomena such as coronal mass ejections, see table (1.1).

Table 1.1: Coronal Magnetic Field Models			
Models	Mathematics	Observations Needed	Validity
Potential field	$\nabla \times B = 0$ $\nabla . B = 0$	Line Of Sight (LOS) magnetogram	(Global) current free regions, quiet sun
Linear force-free	$\nabla \times B = \alpha B$ $\nabla . B = 0$	LOS magnetogram + observations of plasma structures	Local in active regions, Low beta plasma
Nonlinear force-free	$\nabla \times B = \alpha(r)B$ $\nabla . B = 0$	Vectormagnetogram (3 times more data) + LOS magnetogram	Active regions, low beta plasma in low corona
Magnetohydrostatic	$\nabla \times B = u_o j$ $\nabla . B = 0$	Vectormagnetogram	sunspot, solar wind, corona loop, corona hole, and solar flares
Magnetohydrodynamic	$\mu_o(\rho \nabla v + \nabla p) =$ $(\nabla \times B) \times B$ $\nabla . B = 0$	Vectormagnetogram +Tomographic Inversion of density	Helmet streamer, finite beta plasma, full solar corona