Department of Mathematics
College of Science
University of Baghdad
Test of New Applicants for Graduate Studies
MSc. of Pure Mathematics 2017-2018

Note: Answer all the questions.

Q1. For each of the following statement specify whether the statement is true or false.

1. If $\sum_{n=1}^{\infty} a_n$ is convergent, then so is $\sum_{n=1}^{\infty} {a_n}^2$.
2. The limit $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.
3. The following compound statement is tautology $(p \wedge \sim q) \wedge (\sim p \vee q)$.
4. If A is $n\times n$ matrix, then the rank of A is equal n if and only if $\det(A) \neq 0$.
5. Every subgroup of cyclic group is cyclic.
6. Every subring is an ideal.
7. The field of rational numbers is complete.
8. If $f:(a,b) \to R$ is a differentiable function, then f is continuous
9. If $f(z) = \frac{z}{\bar{z}}$ then $\lim_{z\to 0} f(z)$ does not exist.
10. $\lim_{z\to -1} \frac{iz+3}{z+1} = \infty$.
11. The set of all rational numbers Q is closed set in R with Euclidean topology.
12. Each singleton set in R with Euclidean topology is closed in R.

Q2. a) Prove that no group of order 20 is simple?

b) Show that $Cent\ R$ is a subring of R?

Q3. a) Prove that S= $\{1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots, \frac{1}{n}, \ldots\ldots\}$ is not compact ?

b) Give an example of a set which contains infinite numbers of cluster points. Explain your example?

Q4. a) Let $f(z) = \frac{2z-1}{z+2}$ be defined for any complex number $z \neq -2$. Using definition to prove that the function $f(z)$ is differentiable on its domain and compute its derivative.

b) Use Cauchy Residue Theorem to evaluate the integral $\oint_C \frac{1}{1+z^2} dz$; $C: |z| = 3$.

Q5. a) Show that if $X = \{a, b\}$ and $T = \{X, \varphi, \{a\}, \{b\}\}$, then (X, T) is topological space?

b) Let X be any set with discrete topology and let $Y \subseteq X$, then T^* the subspace topology is discrete topology on Y?

Good luck