Baghdad University
College of Science
Department of Mathematics
Competitive Exam Ph.D (No.3)
2016-2017

Note: **Answer all the questions**.

Group A

Q1) Indicate whether the following statements are true or false .(30 marks)

1- Every invertible element in a ring R is a non –zero divisor.

2- Let L:V→W be a linear transformation , if dimV=dimW and L is onto, then L is one to one.

3- If $S = \{X_1, X_2, \ldots, X_n\}$ is a basis for a vector space V , then every vector in V can be written in one and only one way as a linear combination of vectors in S .

4- The set of rational numbers is not zero set.

5- The interval (0,1) is a compact subset of ℝ.

6- If f is analytic lies on a straight line L in ℂ ,then f is a constant map.

7- Let f be analytic in ℂ and $f(z) = \sum_{n=0}^{\infty} a_n z^n$. If $f(z) = f(-z)$ $\forall\, z \in \mathbb{C}$, then $a_n = 0$, $\forall\, n$ even number.

8- The ring of integer is a local ring.

9- Let A and B be two sets, then P(A)∩P(B)=P(A∩B), where P(A) denotes the power set of A .

10- Every abelian group is cyclic.

Q2:

a) For each n= 0, 1, 2, … define a function P_n on ℝ by $P_n(x) = x^n$. Show that the set $\{P_0, P_1, P_2, \ldots\ldots\}$ is a linearly independent subset of the vector space of continuous functions on ℝ.

b) Let $f: R \to S$ be an epimorphisim of commutative rings. If S is a field , show that $kerf$ is a maximal ideal of R. **(10 marks)**

Q3: a) Compute $\int_c \frac{1}{z}\, dz$ where c is any positively oriented simple closed contour surrounding the origin.

b) Give an example of a function $f: [a, b \to] \mathbb{R}$ which is discontinuous everywhere. **(10 marks)**

Group B

Q1)Choose the proper selection for the following statements: (30 marks)

1-Two cards are drawn from a beck of 52 cards without replacement. Let A={The first card is a spade},and B={The second card is a spade}. Then

a. P(A∪B)=3/51 b. P(A∪B)=51/3 c. P(A∪B)=3/5

2- Two machines work independently . Machine A works with probability 0.8. The conditional probability that machine B works given that machine A works is 0.5. The conditional probability that machine A works given that machine B works is

a. 0.22. b. 0.23 c. 0.32

3- In which of the following method, we approximate the curve of solution by the tangent in each interval.
a. Picard's method, b. Euler's method, c. Newton's method
d. Runge Kutta method

4- If the general solution of $t^2\frac{d^2y}{dt^2}-4t\frac{dy}{dt}+6y=0$ is of the form $y=At^2+Bt^3$ for appropriate constants A and B, what is the value of the solution $y(2)$ that satisfies $y(1)=2$ and $y'(1)=-1$.

a. $y(2)=-10$. b. $y(2)=12$. c. $y(2)=-5$. d. $y(2)=-12$.

5- Consider the equation

$$y^{//}+(\frac{x^2\sin x}{e^2\sqrt{\pi}})^8(y^{/})^3+xy=10$$

This equation is
a. An ordinary, linear differential equation of order 2
b. An ordinary, nonlinear differential equation of order 2
c. An ordinary, linear differential equation of order 3
d. An ordinary, nonlinear differential equation of order 3

6- The solution of the differential equation $4y''+9y=0$, where c_1, c_2 are constants, is
a. $y=c_1e^x+c_2e^{-x}$

b. $y = c_1 e^{x} + c_2 e^{-4x}$

c. $y = c_1 \cos(x) + c_2 \sin(x)$

d. $y = c_1 \cos(3x/2) + c_2 \sin(3x/2)$

7- If the exact solution is A and the numerical solution is B , then the absolute error is

a. |A+B| ,b. |A+B|/|A|, c. |A-B|

8- A quantity used to measure the quality of matrix A is called condition number and defined as

a. $K(A)=||A||\ ||A^T||$, b. $K(A)= ||A||\ ||A^{-1}||$

9- Match the following two sets:

Set one: A. Newton-Raphson, B. Runge-Kutta, C. Gauss-Seidel,

D. Simpson's Rule

Set two: 1. Integration, 2. Root finding,

3. Ordinary Differential Equations,

4. Solution of system of Linear Equations

The correct sequence is

a. A2-B3-C4-D1 b. A3-B2-C1-D4

c. A1-B4-C2-D3 d. A4-B1-C2-D3

10- For the equation $x^3 + 3x - 1 = 0$ the root of the equation lies in the interval

a. (1,3) b. (-2,0) c. (0,1) d. (2,4)

Q2) (10 marks)

a) Find the general solution of $y'' - 2y' - 3y = e^{2t}$.

b) Find the area between the curve $y=4-x^2$ and the x-axis.

Q3) (10 marks)

Find numerically the exact value of $\int_0^1 x\, dx$

Group C

Q1) (20 marks)

Consider the ODE

$$y'' - 49\,y = 0, \quad y(0)=1, \quad y'(0)=0$$

a) Solve the given equation by using Laplace transform.

b) Solve the given equation by using series method

c) Match the solution in (a) and (b).