Department of Mathematics
College of Science
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Test of New Applicants for Graduate Studies
Phd. of Mathematics 2017-2018

Note: Answer all the questions.

Q1. For each of the following statement specify whether the statement is true or false.

1. If $\sum_{n=1}^{\infty} a_n$ is convergent, then so is $\sum_{n=1}^{\infty} {a_n}^2$.
2. The limit $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.
3. The following compound statement is tautology $(p \wedge \sim q) \wedge (\sim p \vee q)$.
4. If A is $n \times n$ matrix, then the rank of A is equal n if and only if $\det(A) \neq 0$.
5. If A is an event of a sample space with $P(A) = p(\bar{A})$, then $P(A) = 0.5$.
6. Every subgroup of cyclic group is cyclic.
7. The field of rational numbers is complete.
8. Every subring is an ideal.
9. The Trapezoidal rule to approximate an integral function use an interpolation polynomial of degree two.
10. If $f(z) = \frac{z}{\bar{z}}$ then $\lim_{z\to 0} f(z)$ does not exist.
11. If $y_1(t)$ and $y_2(t)$ are two solutions of equation $\ddot{y} + p(t)\dot{y} + q(t)y = 0$, then the formula $y(t) = c_1 y_1(t) + c_2 y_2(t)$ gives all solutions to the given equation.
12. The set of all rational numbers Q is closed set in R with Euclidean topology.

Q2. a) Prove that no group of order 20 is simple?

b) Find the bound of the number of iterations needed to achieve a approximation with accuracy 10^{-3} to the solution of $x^3 - x - 1 = 0$ lying in the interval [1,4]?

Q3. a) Prove that S= $\{1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots, \frac{1}{n}, \ldots\ldots\}$ is not compact ?

b) Use Cauchy Residue Theorem to evaluate the integral $\oint_C \frac{1}{1+z^2} dz$; $C: |z| = 3$?

Q4. a) Show that if $X = \{a, b\}$ and $T = \{X, \varphi, \{a\}, \{b\}\}$, then (X, T) is topological space?

b) Use the Wronskian to show that the following set of functions $\{e^x, e^{2x}, e^{3x}\}$ are linearly independent?

Q5. a) Solve the initial value problem $\frac{d^2y}{dx^2} - y = e^x; y(0) = 0; \dot{y}(0) = \frac{1}{2}$?

b) If $X_1, X_2, \ldots, X_n$ are independent random variables then prove that the moment generating function of their sum is $M_{X_1}(t).M_{X_2}(t) \ldots M_{X_n}(t)$?

Good luck