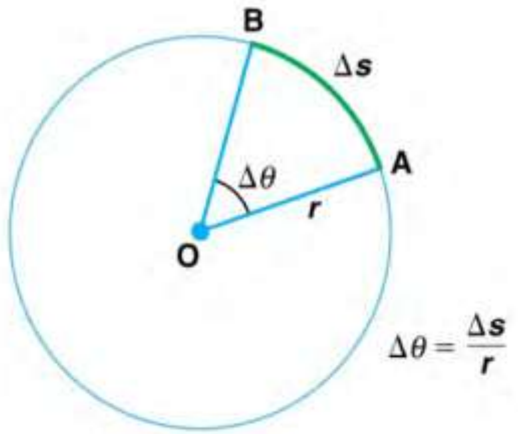
Rotational Motion

Rotation Angle

When objects rotate about some axis—for example, when the CD (compact disc) rotates about its center—each point in the object. follows a circular arc. Consider a line from the center of the CD to its edge. Each **pit** used to record sound along this line moves through the same angle in the same amount of time. The rotation angle is the amount of rotation and is analogous to linear distance. We define the **rotation angle** $\Delta\theta$ to be the ratio of the arc length to the radius of curvature:

$$\Delta\theta = \frac{\Delta s}{r}.$$



The radius of a circle is rotated through an angle $\Delta\theta$. The arc length Δs is described on the circumference.

$$\Delta\theta = \frac{2\pi r}{r} = 2\pi.$$

This result is the basis for defining the units used to measure rotation angles, $\Delta\theta$ to be **radians** (rad), defined so that 2 π rad = 1 revolution.

Angular Velocity

How fast is an object rotating? We define angular velocity (ω) as the rate of change of an angle. In symbols, this is

$$v = \frac{\Delta s}{\Delta t}.$$

where an angular rotation $\Delta\ \theta$ takes place in a time $\Delta \mathbf{t}$. The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are radians per second (rad/s).

Angular velocity (ω) is analogous to linear velocity v . To get the precise relationship between angular and linear velocity, we again consider a pit on the rotating CD. This pit moves an arc length Δs in a time Δt , and so

$$\omega = \frac{\Delta\theta}{\Delta t},$$

it has a linear velocity

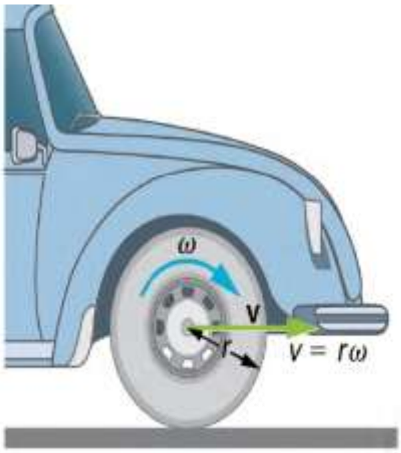
From $\Delta\theta = \frac{\Delta s}{r}$ we see that $\Delta s = r\Delta\theta$. Substituting this into the expression for v gives

$$v = \frac{r\Delta\theta}{\Delta t} = r\omega.$$

We write this relationship in two different ways and gain two different insights:

$$v = r\omega \text{ or } \omega = \frac{v}{r}.$$

The first relationship in $v = r\omega$ or $w = \frac{v}{r}$ states that the linear velocity v is proportional to the distance from the center of rotation, thus, it is largest for a point on the rim (largest r), as you might expect. We can also call this linear speed v of a point on the rim the *tangential speed*. The second relationship in $v = r\omega$ or $w = \frac{v}{r}$ can be illustrated by considering the tire of a moving car. Note that the speed of a point on the rim of the tire is the same as the speed v of the car.



A car moving at a velocity v to the right has a tire rotating with an angular velocity ω .The speed of the tread of the tire relative to the axle is v , the same as if the car were jacked up. Thus the car moves forward at linear velocity $v = r\omega$, where r is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

Angular velocity is not constant when a skater pulls in her arms, when a child starts up a merry-go-round from rest, or when a computer's hard disk slows to a halt when switched off. In all these cases, there is an angular acceleration, in which (ω) changes. The faster the change occurs, the greater the angular acceleration. Angular acceleration α is defined as the rate of change of angular velocity. In equation form, angular acceleration is expressed as follows:

$$\alpha = \frac{\Delta\omega}{\Delta t},$$

where $\Delta\omega$ is the change in angular velocity and Δt is the change in time. The units of angular acceleration are (rad/s)/s , or rad/s^2 . If ω increases, then α is positive. If ω decreases, then α is negative.

Q\ Calculate the angular velocity of a 0.300 m radius car tire when the car travels at 15.0 m/s (about 54 km/h).

Because the linear speed of the tire rim is the same as the speed of the car, we have v = 15.0 m/s. The radius of the tire is given to be r = 0.300 m. Knowing v and r , we can use the second relationship in v = r ω , $w = \frac{v}{r}$ to calculate the angular velocity.

Solution

To calculate the angular velocity, we will use the following relationship:

$$w = \frac{v}{r}$$

Substituting the known

$$\omega = \frac{15.0\ \text{m/s}}{0.300\ \text{m}} = 50.0\ \text{rad/s}.$$

Q\ Suppose a teenager puts her bicycle on its back and starts the rear wheel spinning from rest to a final angular velocity of 250 rpm in 5.00 s. (a) Calculate the angular acceleration in rad/s^2 . (b) If she now slams on the brakes, causing an angular acceleration of – 87.3 rad/s^2 , how long does it take the wheel to stop?

Solution for (a)

The angular acceleration can be found directly from its definition in

$$\alpha = \frac{\Delta\omega}{\Delta t},$$

because the final angular velocity and time are given. We see that $\Delta\ \omega$ is 250 rpm and Δ t is 5.00 s.

Entering known information into the definition of angular acceleration, we get

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{250 \text{ rpm}}{5.00 \text{ s}}.$$

Because $\Delta\omega$ is in revolutions per minute (rpm) and we want the standard units of rad/s^2 for angular acceleration, we need to convert $\Delta\omega$ from rpm to rad/s:

$$\Delta\omega = 250\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 26.2\frac{\text{rad}}{\text{s}}.$$

Entering this quantity into the expression for α, we get

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{26.2 \text{ rad/s}}{5.00 \text{ s}} = 5.24 \text{ rad/s}^2.$$

Solution for (b)

In this part, we know the angular acceleration and the initial angular velocity. We can find the stoppage time by using the definition of angular acceleration and solving for Δt, yielding

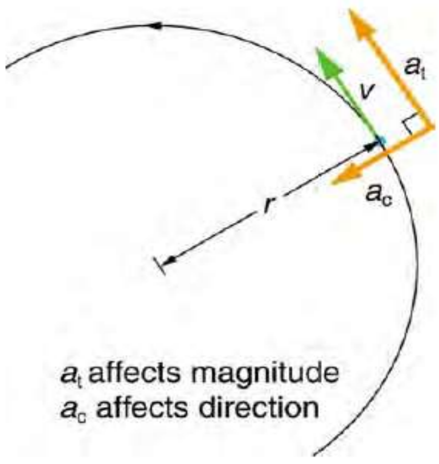
$$\Delta t = \frac{\Delta\omega}{\alpha}.$$

Here the angular velocity decreases from 26.2 rad/s (250 rpm) to zero, so that $\Delta\omega$ is $-$ 26.2 rad/s , and α is given to be $-$ 87.3 rad/s^2 .

Thus,

$$\Delta t = \frac{-26.2 \text{ rad/s}}{-87.3 \text{ rad/s}^2} = 0.300 \text{ s.}$$

Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction. We know from Uniform Circular Motion and Gravitation that in circular motion centripetal acceleration, a_c , refers to changes in the direction of the velocity but not its magnitude. An object undergoing circular motion experiences centripetal acceleration. Thus, a_t and a_c are perpendicular and independent of one another. Tangential acceleration a_t is directly related to the angular acceleration α and is linked to an increase or decrease in the velocity, but not its direction.



Centripetal acceleration a_c occurs as the direction of velocity changes; it is perpendicular to the circular motion. Centripetal and tangential acceleration are thus perpendicular to each other.

Now we can find the exact relationship between linear acceleration at and angular acceleration α . Because linear acceleration is proportional to a change in the magnitude of the velocity, it is defined (as it was in One-Dimensional Kinematics) to be

$$a_t = \frac{\Delta v}{\Delta t}.$$

For circular motion, note that v = rω , so that

$$a_t = \frac{\Delta(r\omega)}{\Delta t}.$$

The radius r is constant for circular motion, and so $\Delta(r\omega) = r(\Delta\omega)$. Thus,

$$a_t = r\frac{\Delta\omega}{\Delta t}.$$

By definition, $\alpha = \frac{\Delta\omega}{\Delta t}$. Thus,

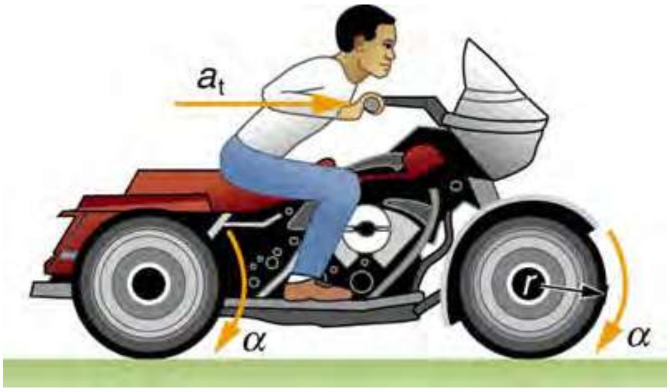
$$a_t = r\alpha,$$

or

$$\alpha = \frac{a_t}{r}.$$

These equations mean that linear acceleration and angular acceleration are directly proportional. The greater the angular acceleration is, the larger the linear (tangential) acceleration is, and vice versa. For example, the greater the angular acceleration of a car's drive wheels, the greater the acceleration of the car. The radius also matters. For example, the smaller a wheel, the smaller its linear acceleration for a given angular acceleration α .

Q\ A powerful motorcycle can accelerate from 0 to 30.0 m/s (about 108 m/h) in 4.20 s. What is the angular acceleration of its 0.320-m-radius wheels?



We are given information about the linear velocities of the motorcycle. Thus, we can find its linear acceleration a_t . Then, the expression $\alpha = \frac{\alpha_t}{r}$ can be used to find the angular acceleration

$$a_t = \frac{\Delta v}{\Delta t} = \frac{30.0 \text{ m/s}}{4.20 \text{ s}} = 7.14 \text{ m/s}^2.$$

We also know the radius of the wheels. Entering the values for a_t and r into $\alpha = \frac{a_t}{r}$, we get

$$\alpha = \frac{a_t}{r} = \frac{7.14 \text{ m/s}^2}{0.320 \text{ m}} = 22.3 \text{ rad/s}^2.$$

Kinematics of Rotational Motion

The **kinematics of rotational motion** describes the relationships among rotation angle, angular velocity, angular acceleration, and time. Let us start by finding an equation relating ω , α , and t . To determine this equation, we recall a familiar kinematic equation for translational, or straight-line, motion:

$$v = v_0 + at \text{ (constant } a\text{)}$$

Note that in rotational motion $a = a_t$, and we shall use the symbol a for tangential or linear acceleration from now on. As in linear kinematics, we assume a is constant, which means that angular acceleration α is also a constant, because $a = r\alpha$. Now, let us substitute $v = r\omega$ and $a = r\alpha$ into the linear equation above:

$$r\omega = r\omega_0 + r\alpha t.$$

The radius r cancels in the equation, yielding

$$\omega = \omega_0 + at \text{ (constant a)},$$

where ω_0 is the initial angular velocity. This last equation is a kinematic relationship among ω , α , and t −that is, it describes their relationship without reference to forces or masses that may affect rotation. It is also precisely analogous in form to its translational counterpart.

Rotational Kinematic Equations

Rotational	Translational	
$\theta = \bar{\omega} t$	$x = \bar{v} t$	
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$	(constant α , a)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$	(constant α , a)
$\omega^2 = \omega_0{}^2 + 2\alpha\theta$	$v^2 = v_0{}^2 + 2ax$	(constant α , a)

In these equations, the subscript 0 denotes initial values (θ_0 , x_0 , and t_0 are initial values), and the average angular velocity $\bar{\omega}$ - and average velocity v - are defined as follows:

$$\bar{\omega} = \frac{\omega_0 + \omega}{2} \text{ and } \bar{v} = \frac{v_0 + v}{2}.$$

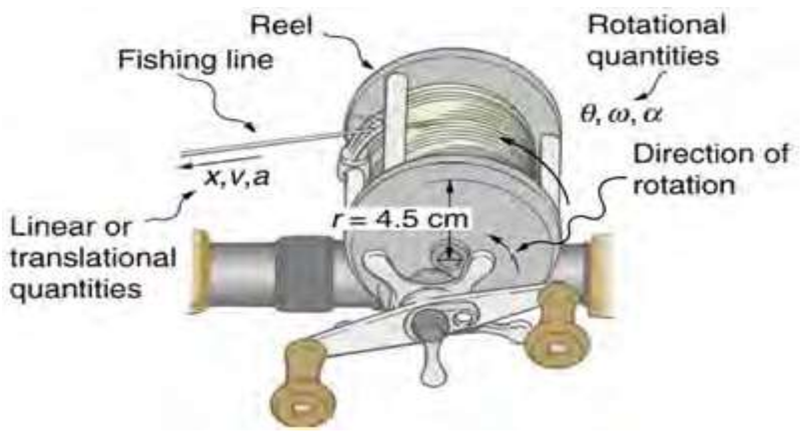
Q\ A deep-sea fisherman hooks a big fish that swims away from the boat pulling the fishing line from his fishing reel. The whole system is initially at rest and the fishing line unwinds from the reel at a radius of 4.50 cm from its axis of rotation. The reel is given an angular acceleration of 110 rad/s^2 for 2.00 s as seen in Figure

(a) What is the final angular velocity of the reel?

(b) At what speed is fishing line leaving the reel after 2.00 s elapses?

(c) How many revolutions does the reel make?

(d) How many meters of fishing line come off the reel in this time?



Solution for (a)

Here α and t are given and ω needs to be determined. The most straightforward equation to use is $\omega = \omega_0 + \alpha t$ because the unknown is already on one side and all other terms are known. That equation states that

$$\omega = \omega_0 + \alpha t.$$

We are also given that $\omega_0 = 0$ (it starts from rest), so that

$$\omega = 0 + \left(110 \text{ rad/s}^2\right)(2.00\text{s}) = 220 \text{ rad/s}.$$

Solution for (b)

Now that ω is known, the speed v can most easily be found using the relationship

$$v = r\omega,$$

where the radius r of the reel is given to be 4.50 cm; thus,

$$v = (0.0450 \text{ m})(220 \text{ rad/s}) = 9.90 \text{ m/s}.$$

Note again that radians must always be used in any calculation relating linear and angular quantities. Also, because radians are dimensionless, we have $\text{m} \times \text{rad} = \text{m}$.

Solution for (c)

Here, we are asked to find the number of revolutions. Because 1 rev = 2π rad , we can find the number of revolutions by finding θ in radians. We are given α and t , and we know ω 0 is zero, so that θ can be obtained using

$$\begin{aligned} \theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ &= 0 + (0.500)\left(110 \text{ rad/s}^2\right)(2.00 \text{ s})^2 = 220 \text{ r} \end{aligned}$$

Converting radians to revolutions gives

$$\theta = (220 \text{ rad})\frac{1 \text{ rev}}{2\pi \text{ rad}} = 35.0 \text{ rev}.$$

Solution for (d)

The number of meters of fishing line is x , which can be obtained through its relationship with θ :

$x = r\theta = (0.0450 \text{ m})(220 \text{ rad}) = 9.90 \text{ m}$.

Q\ Now let us consider what happens if the fisherman applies a brake to the spinning reel, achieving an angular acceleration of – 300 rad/s^2 . How long does it take the reel to come to a stop?

Solution:

We are asked to find the time t for the reel to come to a stop. The initial and final conditions are different from those in the previous problem, which involved the same fishing reel. Now we see that the initial angular velocity is ω_0 = 220 rad/s and the final angular velocity ω is zero.

The angular acceleration is given to be α = −300 rad/s^2 . Examining the available equations, we see all quantities but t are known in $\omega = \omega_0 + \alpha t$, making it easiest to use this equation.

$$\omega = \omega_0 + \alpha t.$$

We solve the equation algebraically for *t*, and then substitute the known values as usual, yielding

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 220 \text{ rad/s}}{-300 \text{ rad/s}^2} = 0.733 \text{ s}.$$

Heat , Temperature, Heat transfer

As molecules of all materials are moving so they have kinetic energy. The average kinetic energy of an ideal gas can be shown to be directly proportional with temperature. The same thing is for liquids and solids .The movement of gas molecules are more free than liquid and liquid molecules are more free than solid, an increase of temp. of any material means an increase in the energy of molecules of that material.

In order to increase the temp. of a gas it is necessary to increase the average kinetic energy of its molecules by putting the gas in contact with a flame ,the energy transferred from the flame to the gas causing temp. rise is called **(heat).**

Temperature is a measure of the *average* kinetic energy of all the molecules in a substance. Because it is an average of the kinetic energy of all the molecules, the temperature of a substance does not depend on the total number of molecules in a substance. On the other hand, heat is a function of the number of molecules in a substance, so there is a **difference between heat and temperature.** Heat is a form of energy, while temperature is a measure of the degree of hotness of a substance, and is measured by a thermometer.

Kinetic energy (K) is equal to ½ of the mass (m) of a molecule (or a body of some type) times the square of its velocity (v). $K = \frac{1}{2} mv^2$. Therefore the kinetic energy of a molecule is dependent on both the mass of the molecule and its velocity.

Temperature measurement devices:-

- Glass liquid thermometer
- Thermistor
- Thermocouple

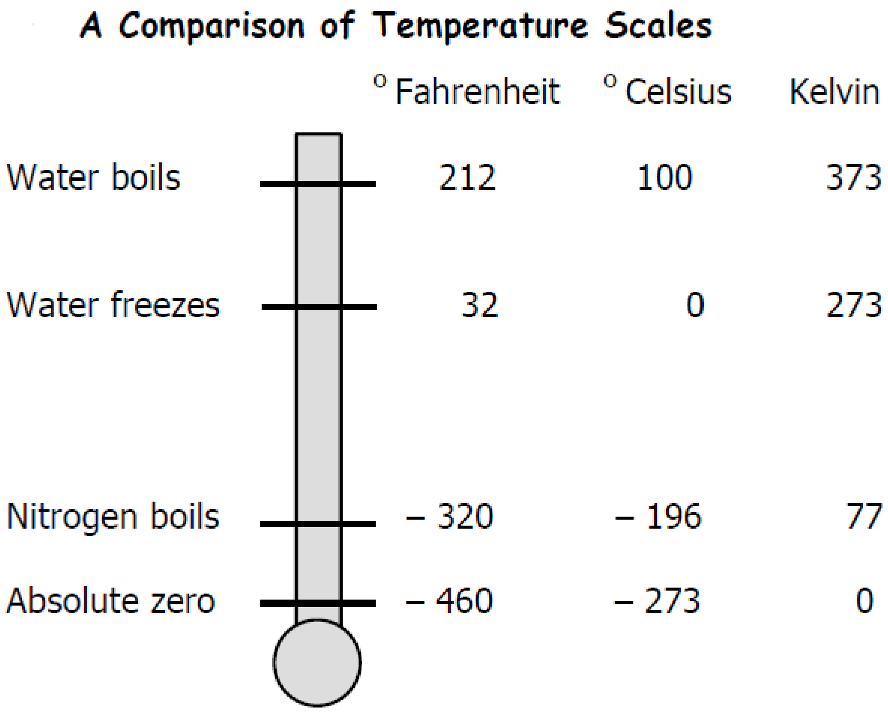
Thermometry and temperature scales

Temperature is difficult to measure directly, so we usually measure it indirectly by measuring one of many physical properties that change with temp.

1-***Fahrenheit scale (°F):*** in this scale the freezing temp. is 32°F and boiling point is 212°F ,and normal body temp. is about 98.6°F .

2-***The Celsius (°C):***the freezing point is 0°C and the boiling point is 100°C, in between is divided into 100 division.

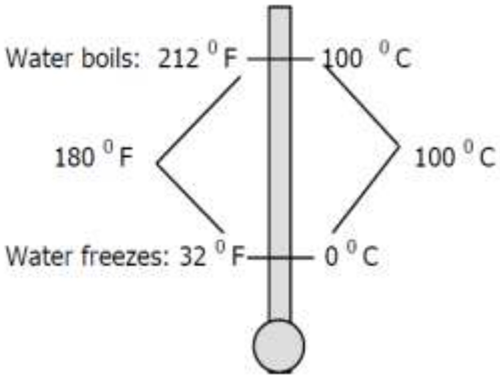
3-***The Kalvin scale(°K):***or the absolute scale this scale has the same divisions as the Celsius but takes the 0° K at the absolute zero which is = -273.15°C.



3. To change °C to °F

[°C= (°F-32) 5/9] or [°F=°C (9/5)+32]

Also °C=°K-273 or °K=°C+273



$$\frac{180\ ^0F}{100\ ^0C} = \frac{9\ ^0F}{5\ ^0C}$$

Q1\ How many Celsius degrees correspond to 50 Fahrenheit degrees?

Q2\ What Fahrenheit temperature corresponds to 32 ºC?

Types of thermometers

Glass-liquid thermometer

It used to know the temp. of the body. This thermometer composed of

- Capillary tube with alcohol or mercury.
- This capillary tube is surrounded by magnifying glass in front.
- An opaque white backing on the back.

The principle behind this thermometer is that an increase in the temp. of different material causes them to expand different amounts.

In this thermometer, a temp. increase causes alcohol or mercury to expand more than the glass. This produce on increase in the level of the liquid.

Note:-

If the liquid expanded the same amount as the glass the level of the liquid remain constant with temp.

Q:- why we are used the mercury instead of alcohol in the thermometer?

We used the mercury instead of alcohol in the thermometer because:

- It has clear color, which can be seen easily.
- Mercury has low adhesion force with the wall of the glass and has high adhesive force.

When the thermometer is taken from the mouth, it is show the maximum temp. it reached under the tongue, in order to return the mercury to the bulb , it is important to snap the thermometer.

Q: It is difficult to measure body temperature with the house thermometer because:

- It is difficult to place the house thermometer under the tongue.
- The hose thermometer would give a low reading because the temp. would fall when the thermometer is removed from the mouth.

Thermal Energy

Thermal energy is the total internal energy of the atoms or molecules of a substance. Heat is thermal energy that is being transferred between two places and is measured in the same units as energy. However, there are units that are commonly used for thermal energy. In the British system of units, thermal energy is normally measured in British Thermal Units (BTU). In scientific work, the usual unit of thermal energy is the calorie (c) or the kilocalorie (C), which is 1000 calories. When describing the energy content of food, the “calories" .

The units are joule or calorie1cal= 4.184J or 1Kcal=4184J

The power is defined as energy or work per unit time=J/s=watt.

Q\ How many joules of energy does a 350 "calorie" doughnut contain?

The 350 "calories" are actually 350 kilocalories = 350,000 calories.

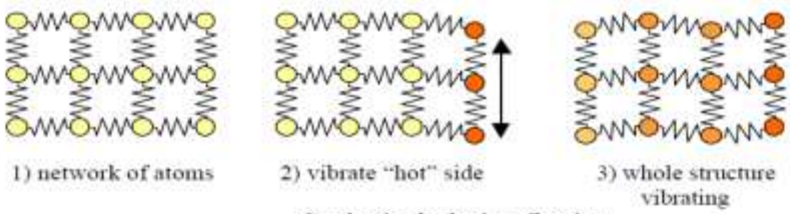
$$350{,}000 \text{ calories} \times \frac{4.19 \text{ joules}}{\text{calorie}} = 1{,}466{,}500 \text{ joules}$$

Transfer of Thermal Energy as Heat

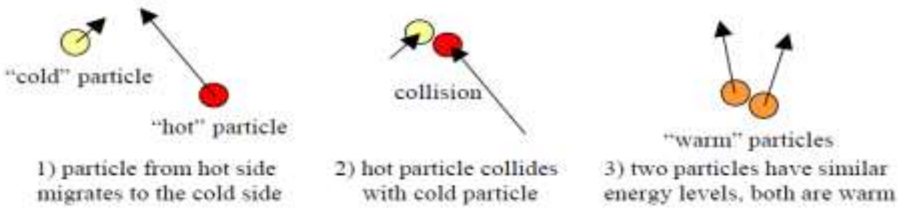
There are three modes of heat transfer: conduction, convection, and radiation. Any energy exchange between bodies occurs through one of these modes or a combination of them. Conduction is the transfer of heat through solids or stationery fluids. Convection uses the movement of fluids to transfer heat. Radiation does not require a medium for transferring heat; this mode uses the electromagnetic radiation emitted by an object for exchanging heat.

1- **Conduction**

Conduction is at transfer through solids or stationery fluids. When you touch a hot object, the heat you feel is transferred through your skin by conduction. Two mechanisms explain how heat is transferred by conduction: lattice vibration and particle collision. Conduction through solids occurs by a combination of the two mechanisms; heat is conducted through stationery fluids primarily by molecular collisions.



Conduction by lattice vibration

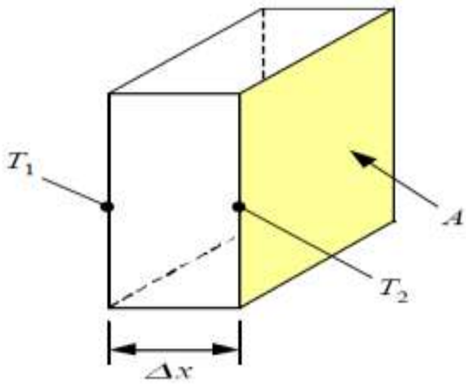


Conduction by particle collision

Conductivity is measured in watts per meter per Kelvin (W/mK). The rate of heat transfer by conduction is given by:

$$q_{conduction} = -kA\frac{\Delta T}{\Delta x}$$

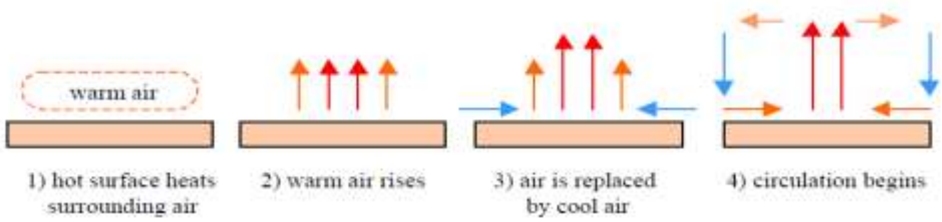
where A is the cross-sectional area through which the heat is conducting, ΔT is the temperature difference between the two surfaces separated by a distance Δx . In heat transfer, a positive q means that heat is flowing into the body, and a negative q represents heat leaving the body.



Heat transfer by conduction

2- Convection

Convection uses the motion of fluids to transfer heat. In a typical convective heat transfer, a hot surface heats the surrounding fluid, which is then carried away by fluid movement such as wind. The warm fluid is replaced by cooler fluid, which can draw more heat away from the surface. Since the heated fluid is constantly replaced by cooler fluid, the rate of heat transfer is enhanced.



Natural convection

Heat losses by convection (H_c)

$$\mathbf{H_c = K_c A_c (T_s - T_a)}$$

Where

H_c is the amount of heat gained or lost be convection, A_c is the effective surface area, T_s is the skin temp., T_a is the environment temp. or air temp.

K_c is a constant that depends on the movement of the air,for a resting body and no apparent wind K_c s about 2.3kcal/m^{2}hr°C.

When the air is moving K_c increases according to the equation:-

$$\mathbf{K_c = 10.45 - v + 10\sqrt{v}}$$

where v is the wind speed in m/sec

This equation is valid for speeds between 2.23m/sec (5mph) and 20m/sec(45mph) (1 mile=1.6 km).

The equivalent temp. due to moving air is called the wind chillfactorand is determined by the actual temp. and wind speed. For example for a windy day speed 10 m/sec an-20°C has the same cooling effect on the body as -40°C on a calm day.

Q\ **1- Calculate the convective heat loss per hour for a nude standing in a $5\frac{m}{\sec}$ wind. Assume $T_s=33C^0$, $T_a=10C^0$, and $A_c=1.2m^2$.**

2- If the wind speed were $2.23\frac{m}{\sec}$, find the still air temperature that would produce the same heat loss (the wind chill equivalent temperature).

Solution

1- $H_c=K_c\,A_c(T_s-T_a)$

$\mathrm{Kc} = 10.45 - \mathrm{v} + 10\sqrt{\mathrm{v}} \Longrightarrow Kc = 10.45 - 5 + 10\sqrt{5} \Longrightarrow Kc = 27.810\frac{Kcal}{m^2hrC^0}$

$H_c=27.810\times1.2\times(33-10)=767.574$ Kcal\hr

2- **$H_c=K_c\,A_c(T_s-T_a)$**

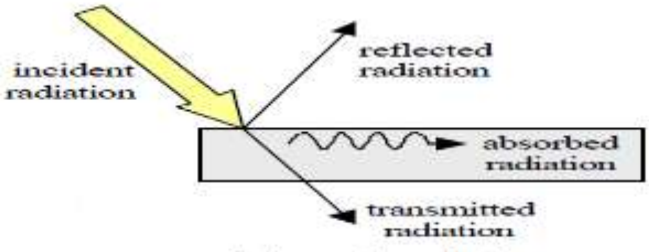
$\mathrm{Kc} = 10.45 - \mathrm{v} + 10\sqrt{\mathrm{v}} \Longrightarrow Kc = 10.45 - 2.23 + 10\sqrt{2.23} \quad Kc = 23.15\frac{Kcal}{m^2hrC^0}$

$767.574=23.15\times1.2\times(33-T_a)$

$T_a=5.36C^0$

3- <u>Radiation</u>

Radiative heat transfer does not require a medium to pass through; thus, it is the only form of heat transfer present in vacuum. It uses electromagnetic radiation (photons), which travels at the speed of light and is emitted by any matter with temperature above 0 degrees Kelvin (-273 °C). Radiative heat transfer occurs when the emitted radiation strikes another body and is absorbed.



Interaction between a surface and incident radiation

Heat radiation power can be measured by:

$W = e\,\sigma T^4$

Where

T: is the absolute temp. of the body

e: is the emissivity depends upon the emitter material and its temp. for radiation from body e is almost 1.

σ: is the Stefan –Boltzmann constant=5.7×10^{-12} W/cm^2 °K^4

Example:

a. what is the power radiated per square centimeters from skin at a temp. of 306°K.?

$W = e\,\sigma T^4 = (5.7\times10^{-12})\,(306)^4 = 0.05$ W/cm^2

b. what is the power radiated from a nude body 1.75m^2(1.75×10^4 cm^2) in area?

$W = (0.05)\,(1.75\times10^4$ cm$^2) = 875$ W

Kinetic Theory of Gases

One of the main subjects in thermodynamics is the physics of gases. A gas consists of atoms (either individually or bound together as molecules) that fill their container's volume and exert pressure on the container's walls. e can usually assign a temperature to such a contained gas. These three variables associated with a gas—volume, pressure, and temperature—are all a consequence of the motion of the atoms. The volume is a result of the freedom the atoms have to spread throughout the container, the pressure is a result of the collisions of the atoms with the container's walls, and the temperature has to do with the kinetic energy of the atoms.

Avogadro's Number

When our thinking is slanted toward atoms and molecules, it makes sense to measure the sizes of our samples in moles. If we do so, we can be certain that we are comparing samples that contain the same number of atoms or molecules. The mole is one of the seven SI base units and is defined as follows:

$$N_A = 6.02 \times 10^{23}\ \text{mol}^{-1} \qquad \text{(Avogadro's number)},$$

where mol^{-1} represents the inverse mole or "per mole," and mol is the abbreviation for mole. The number N_A is called Avogadro's number after Italian scientist Amedeo Avogadro (1776–1856), who suggested that all gases occupying the same volume under the same conditions of temperature and pressure contain the same number of atoms or molecules.

The number of moles n contained in a sample of any substance is equal to the ratio of the number of molecules Nin the sample to the number of molecules N_A in 1 mol:

$$n = \frac{N}{N_A}.$$

(Caution: The three symbols in this equation can easily be confused with one another, so you should sort them with their meanings now, before you end in “N-confusion.”) We can find the number of moles n in a sample from the mass M_{sam} of the sample and either the molar mass M (the mass of 1 mol) or the molecular mass m (the mass of one molecule):

$$n = \frac{M_{sam}}{M} = \frac{M_{sam}}{mN_A}.$$

we used the fact that the mass M of 1 mol is the product of the mass m of one molecule and the number of molecules N_A in 1 mol:

$$M = mN_A.$$

Ideal Gases

To explain the macroscopic properties of a gas—such as its pressure and its temperature—in terms of the behavior of the molecules that make it up. However, there is an immediate problem: which gas? Should it be hydrogen, oxygen, or methane, or perhaps uranium hexafluoride? They are all different. Experimenters have found, though, that if we confine 1 mol samples of various gases in boxes of identical volume and hold the gases at the same temperature, then their measured pressures are almost the same, and at lower densities the differences tend to disappear. Further experiments show that, at low enough densities, all real gases tend to obey the relation

$$pV = nRT \quad \text{(ideal gas law)},$$

in which p is the absolute (not gauge) pressure, n is the number of moles of gas present, and T is the temperature in kelvins. The symbol R is a constant called the gas constant that has the same value for all gases—namely,

$$R = 8.31 \text{ J/mol} \cdot \text{K}.$$

Equation above is called the ideal gas law. Provided the gas density is low, this law holds for any single gas or for any mixture of different gases. (For a mixture, n is the total number of moles in the mixture.)
We can rewrite Eq. above in an alternative form, in terms of a constant called the Boltzmann constant k, which is defined as

$$k = \frac{R}{N_A} = \frac{8.31 \text{ J/mol} \cdot \text{K}}{6.02 \times 10^{23} \text{ mol}^{-1}} = 1.38 \times 10^{-23} \text{ J/K}.$$

This allows us to write R =*k*N*A*. Then, with , ($n = N/N_A$), we see that

$$nR = Nk.$$

The second expression for the ideal gas law:

$$pV = NkT \quad \text{(ideal gas law)}.$$

"What is an ideal gas, and what is so 'ideal' about it?"The answer lies in the simplicity of the law that governs its macroscopic properties. Using this law—as you will see - we can deduce many properties of the ideal gas in a simple way. Although there is no such thing in nature as a truly ideal gas, all real gases approach the ideal state at low enough densities—

that is, under conditions in which their molecules are far enough apart that they do not interact with one another. Thus, the ideal gas concept allows us to gain useful insights into the limiting behavior of real gases.

Q\ A cylinder contains 12 L of oxygen at 20°C and 15 atm. The temperature is raised to 35°C, and the volume is reduced to 8.5 L. What is the final pressure of the gas in atmospheres? Assume that the gas is ideal.

Solution:

Because the gas is ideal, we can use the ideal gas law to relate its parameters, both in the initial state *i* and in the final state *f*.

Calculations:

$$p_iV_i = nRT_i \quad \text{and} \quad p_fV_f = nRT_f.$$

Dividing the second equation by the first equation and solving for p_f yields

$$p_f = \frac{p_iT_fV_i}{T_iV_f}.$$

Note here that if we converted the given initial and final volumes from liters to the proper units of cubic meters, the multiplying conversion factors would cancel out . The same would be true for conversion factors that convert the pressures from atmospheres to the proper pascals. However, to convert the given temperatures to kelvins requires the addition of an amount that would not cancel and thus must be included. Hence, we must write

$$T_i = (273 + 20)\ \text{K} = 293\ \text{K}$$

and $$T_f = (273 + 35)\ \text{K} = 308\ \text{K}.$$

Inserting the given data

$$p_f = \frac{(15\ \text{atm})(308\ \text{K})(12\ \text{L})}{(293\ \text{K})(8.5\ \text{L})} = 22\ \text{atm}. \qquad \text{(Answer)}$$

Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

We have developed macroscopic definitions of pressure and temperature. Pressure is the force divided by the area on which the force is exerted, and temperature is measured with a thermometer. We gain a better understanding of pressure and temperature from the kinetic theory of gases, which assumes that atoms and molecules are in continuous random motion.

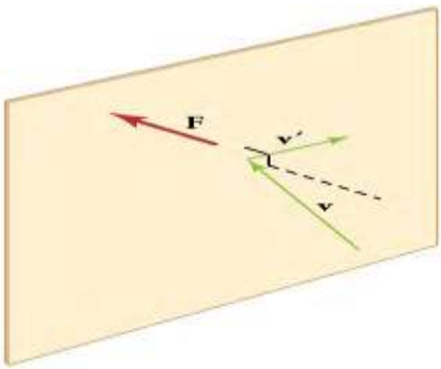


Figure shows: shows an elastic collision of a gas molecule with the wall of a container, so that it exerts a force on the wall (by Newton's third law).

$$PV = \frac{1}{3}Nm\overline{v^2},$$

where P is the pressure (average force per unit area), V is the volume of gas in the container, N is the number of molecules in the container, m is the mass of a molecule, and v^2 is the average of the molecular speed squared.

What can we learn from this atomic and molecular version of the ideal gas law? We can derive a relationship between temperature and the average translational kinetic energy of molecules in a gas. Recall the previous expression of the ideal gas law:

$$PV = NkT.$$

Equating the right-hand side of this equation with the right-hand side of $PV = \frac{1}{3}Nmv^{-2}$ gives

$$\frac{1}{3}Nm\overline{v^2} = NkT.$$

We can get the average kinetic energy of a molecule, $\frac{1}{2}mv^2$ from the left-hand side of the equation by canceling N and multiplying by 3/2. This calculation produces the result that the average kinetic energy of a molecule is directly related to absolute temperature.

$$\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

The average translational kinetic energy of a molecule, KE , is called thermal energy. The equation

$$\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

interpretation of temperature, and it has been found to be valid for gases and reasonably accurate in liquids and solids. It is another definition of temperature based on an expression of the molecular energy.

It is sometimes useful to rearrange

$$\overline{KE} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

and solve for the average speed of molecules in a gas in terms of temperature,

$$\sqrt{\overline{v^2}} = v_{rms} = \sqrt{\frac{3kT}{m}},$$

where v_{rms} stands for root-mean-square (rms) speed.

Q\ (a) What is the average kinetic energy of a gas molecule at 20.0ºC (room temperature)? (b) Find the rms speed of a nitrogen molecule (N_2) at this temperature.

Solution:

The known in the equation for the average kinetic energy is the temperature.

$$\overline{KE} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

Before substituting values into this equation, we must convert the given temperature to kelvins. This conversion gives

$$T = (20.0 + 273)\text{ K} = 293\text{ K}.$$

The temperature alone is sufficient to find the average translational kinetic energy. Substituting the temperature into the translational kinetic energy equation gives

$$\overline{\mathrm{KE}} = \frac{3}{2}kT = \frac{3}{2}\left(1.38\times10^{-23}\ \mathrm{J/K}\right)(293\ \mathrm{K}) = 6.07\times10^{-21}\ \mathrm{J}.$$

Solution (b):

Finding the rms speed of a nitrogen molecule involves a straightforward calculation using the equation

$$\sqrt{\overline{v^2}} = v_{\mathrm{rms}} = \sqrt{\frac{3kT}{m}},$$

but we must first find the mass of a nitrogen molecule. Using the molecular mass of nitrogen N2 from the periodic table,

$$m = \frac{2(14.0067)\times10^{-3}\ \mathrm{kg/mol}}{6.02\times10^{23}\ \mathrm{mol}^{-1}} = 4.65\times10^{-26}\ \mathrm{kg}.$$

Substituting this mass and the value for k into the equation for v_{rms} yields

$$v_{\mathrm{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\ \mathrm{J/K}\right)(293\ \mathrm{K})}{4.65\times10^{-26}\ \mathrm{kg}}} = 511\ \mathrm{m/s}.$$

Q\ Suppose your bicycle tire is fully inflated, with an absolute pressure of 7.00×10^5 Pa (a gauge pressure of just under 90.0 lb/in2) at a temperature of 18.0ºC . What is the pressure after its temperature has risen to 35.0ºC ? Assume that there are no appreciable leaks or changes in volume.

Solution:

We know the initial pressure P_0 = 7.00×105 Pa , the initial temperature T_0 = 18.0°C , and the final temperature T_f = 35.0°C . We must find the final pressure P_f . How can we use the equation PV = NkT ? At first, it may seem that not enough information is given, because the volume V and number of atoms N are not specified. What we can do is use the equation twice: $P_0 V_0 = NkT_0$ and $P_f V_f = NkT_f$. If we divide $P_f V_f$ by $P_0 V_0$ we can come up with an equation that allows us to solve for P_f .

$$\frac{P_f V_f}{P_0 V_0} = \frac{N_f k T_f}{N_0 k T_0}$$

Since the volume is constant, V_f and V_0 are the same and they cancel out. The same is true for N_f and N_0 , and k , which is a constant. Therefore,

$$\frac{P_f}{P_0} = \frac{T_f}{T_0}.$$

We can then rearrange this to solve for P_f :

$$P_f = P_0 \frac{T_f}{T_0},$$

where the temperature must be in units of kelvins, because T_0 and T_f are absolute temperatures.

1. Convert temperatures from Celsius to Kelvin.

$$T_0 = (18.0 + 273)\text{K} = 291 \text{ K}$$
$$T_f = (35.0 + 273)\text{K} = 308 \text{ K}$$

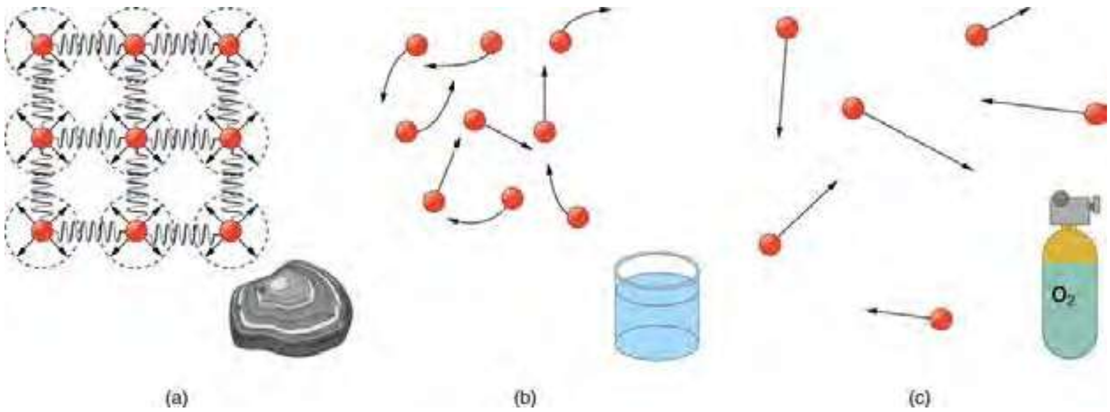
2. Substitute the known values into the equation.

$$P_f = P_0 \frac{T_f}{T_0} = 7.00\times10^5 \text{ Pa}\left(\frac{308 \text{ K}}{291 \text{ K}}\right) = 7.41\times10^5 \text{ Pa}$$

Fluid Statics, Fluid dynamics and its biological and medical applications

What Is a Fluid?

Matter most commonly exists as a solid, liquid, or gas; these states are known as the three common *phases of matter*. Solids have a definite shape and a specific volume, liquids have a definite volume but their shape changes depending on the container in which they are held, and gases have neither a definite shape nor a specific volume as their molecules move to fill the container in which they are held.



Atoms in solids are in close contact, with forces between them that allow the atoms to vibrate but not to change positions with neighboring atoms. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken). Thus a solid resists all types of stress. A solid cannot be easily deformed because the atoms that make up the solid are not able to move about freely. Solids also resist compression, because their atoms form part of a lattice structure in which the atoms are a relatively fixed distance apart. Under compression, the atoms would be forced into one another. Most of the examples we have studied so far have involved solid objects which deform very little when stressed.

When a liquid is placed in a container with no lid on, it remains in the container (providing the container has no holes below the surface of the liquid!).

Because the atoms are closely packed, liquids, like solids, resist compression.

Atoms in gases are separated by distances that are large compared with the size of the atoms. The forces between gas atoms are therefore very weak, except when the atoms collide with one another. Gases thus not only flow (and are therefore considered to be fluids) but they are relatively easy to compress because there is much space and little force between atoms. When placed in an open container gases, unlike liquids, will escape. The major distinction is that gases are easily compressed, whereas liquids are not. We shall generally refer to both gases and liquids simply as fluids, and make a distinction between them only when they behave differently.

Density

Density is the mass per unit volume of a substance or object. In equation form, density is defined as

$$\rho = \frac{m}{V},$$

where ρ is the symbol for density, m is the mass, and V is the volume occupied by the substance. The SI unit of density is kg/m^3.

Q\ A reservoir has a surface area of 50.0 km^2 and an average depth of 40.0 m. What mass of water is held behind the dam?

Solving equation $\rho = m / V$ for m gives $m = \rho V$.

The volume V of the reservoir is its surface area A times its average depth h :

$$\begin{aligned} V &= Ah = (50.0\ \text{km}^2)(40.0\ \text{m}) \\ &= \left[(50.0\ \text{km}^2)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)^2\right](40.0\ \text{m}) = 2.00\times10^9\ \text{m}^3 \end{aligned}$$

The density of water ρ is 1.000×10^3 kg/m^3 . Substituting V and ρ into the expression for mass gives

$$\begin{aligned} m &= (1.00\times10^3\ \text{kg/m}^3)(2.00\times10^9\ \text{m}^3) \\ &= 2.00\times10^{12}\ \text{kg.} \end{aligned}$$

Pressure

You have no doubt heard the word pressure being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids. Pressure P is defined as

$$P = \frac{F}{A}$$

where F is a force applied to an area A that is perpendicular to the force.

Pressure is defined as the force divided by the area perpendicular to the force over which the force is applied.

The SI unit for pressure is the pascal, where 1 Pa = 1 N/m^2.

Q\ **An astronaut is working outside the International Space Station where the atmospheric pressure is essentially zero. The pressure gauge on her air tank reads 6.90×10^6 Pa . What force does the air inside the tank exert on the flat end of the cylindrical tank, a disk 0.150 m in diameter?**

Solution:

We can find the force exerted from the definition of pressure given in

$$P = \frac{F}{A}$$

provided we can find the area A acted upon.

By rearranging the definition of pressure to solve for force, we see that F = PA. Here, the pressure P is given, as is the area of the end of the cylinder A , given by A = πr^2 . Thus,

$$\begin{aligned} F &= \left(6.90\times10^6 \text{ N/m}^2\right)(3.14)(0.0750 \text{ m})^2 \\ &= 1.22\times10^5 \text{ N.} \end{aligned}$$

Variation of Pressure with Depth in a Fluid

If your ears have ever popped on a plane flight or ached during a deep dive in a swimming pool, you have experienced the effect of depth on pressure in a fluid. At the Earth's surface, the air pressure exerted on you is a result of the weight of air above you. This pressure is reduced as you climb up in altitude and the weight of air above you decreases. Under water, the pressure exerted on you increases with increasing depth. In this case, the pressure being exerted upon you is a result of both the weight of water above you and that of the atmosphere above you.

That pressure is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container):

$$P = \frac{mg}{A}.$$

We can find the mass of the fluid from its volume and density:

$$m = \rho V.$$

The volume of the fluid V is related to the dimensions of the container. It is

$$V = Ah,$$

where A is the cross-sectional area and h is the depth. Combining the last two equations gives

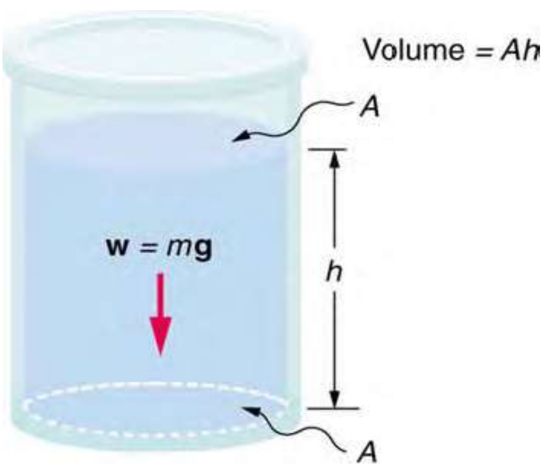
$$m = \rho Ah.$$

If we enter this into the expression for pressure, we obtain

$$P = \frac{(\rho Ah)g}{A}.$$

The area cancels, and rearranging the variables yields

$$P = h\rho g.$$



Q\ In Fig below, we calculated the mass of water in a large reservoir. We will now consider the pressure and force acting on the dam retaining water. The dam

is 500 m wide, and the water is 80.0 m deep at the dam. (a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam and compare it with the weight of water in the dam (previously found to be 1.96×10^{13} N).

Solution:

The average pressure $\bar{P}$due to the weight of the water is the pressure at the average depth $\bar{h}$of 40.0 m, since pressure increases linearly with depth.

Solution for (a)

The average pressure due to the weight of a fluid is

$$\bar{P} = \bar{h}\rho g.$$

$$\begin{aligned}\bar{P} &= (40.0\text{ m})\left(10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.80\ \frac{\text{m}}{\text{s}^2}\right)\\ &= 3.92\times10^5\ \frac{\text{N}}{\text{m}^2} = 392\text{ kPa.}\end{aligned}$$

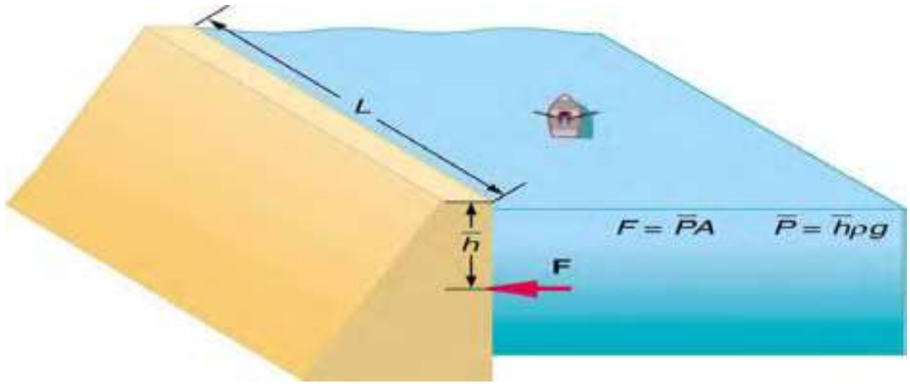
for (b)

The force exerted on the dam by the water is the average pressure times the area of contact:

$$F = \bar{P}A.$$

We have already found the value for $\bar{P}$. The area of the dam is A= $80.0\text{m}\times500\text{m}=4.00\times10^4\text{m}^2$, so that

$F= (3.92\times10^5\text{N/m}^2)(4.00\times10^4\text{m}^2)= 1.57\times10^{10}$N.

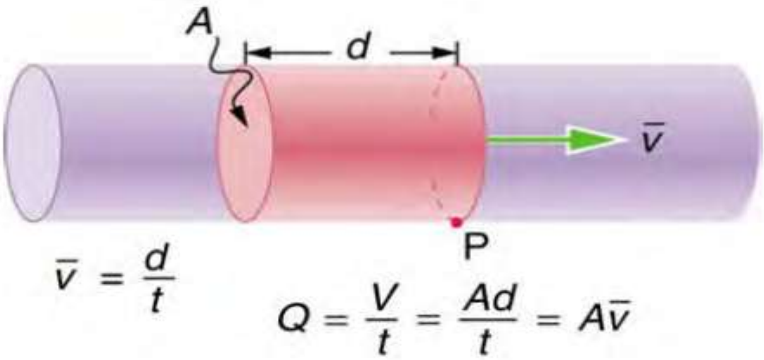


Flow Rate and Its Relation to Velocity

Flow rate Q is defined to be the volume of fluid passing by some location through an area during a period of time. In symbols, this can be written as:

$$Q = \frac{V}{t},$$

where **V** is the volume and **t** is the elapsed time. The SI unit for flow rate is m^3 /s.



Flow rate is the volume of fluid per unit time flowing past a point through the area A. Here the shaded cylinder of fluid flows past point P in a uniform pipe in time t. The volume of the cylinder is Ad and the average velocity is $\bar{v} = d/t$ so that the flow rate is $Q = Ad/t = A\bar{v}$.

Q\ How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min?

Solution:
Q=V/t for volume gives

V=Qt

$$V = \left(\frac{5.00\ \mathrm{L}}{1\ \mathrm{min}}\right)(75\ \mathrm{y})\left(\frac{1\ \mathrm{m}^3}{10^3\ \mathrm{L}}\right)\left(5.26\times10^5\frac{\mathrm{min}}{\mathrm{y}}\right)$$
$$= 2.0\times10^5\ \mathrm{m}^3.$$

<u>Ideal Fluids in Motion</u>

1. Steady flow In steady (or laminar) flow, the velocity of the moving fluid at any fixed point does not change with time. The gentle flow of water near the center of a quiet stream is steady; the flow in a chain of rapids is not. Nonlaminar or turbulent) flow for a rising stream of smoke. The speed of the smoke particles increases as they rise and, at a certain critical speed, the flow changes from steady to nonsteady.

2. Incompressible flow We assume, as for fluids at rest, that our ideal fluid is incompressible; that is, its density has a constant, uniform value.

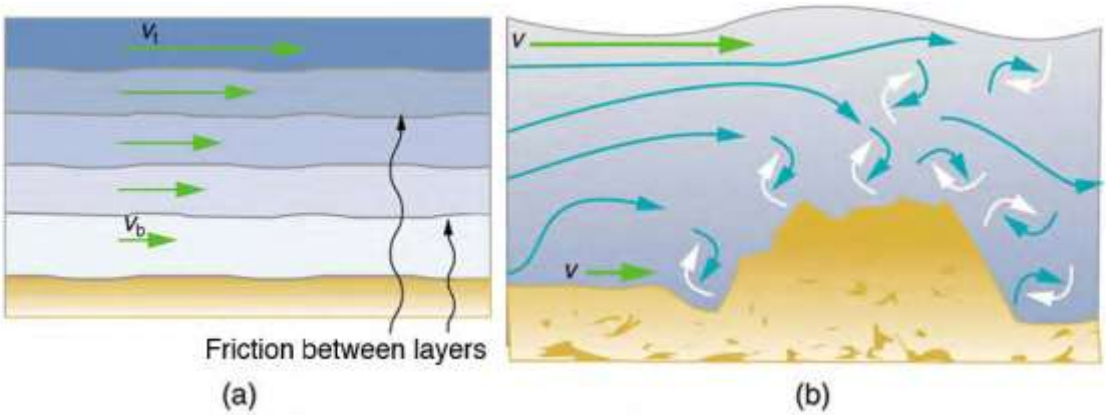
3. Nonviscous flow Roughly speaking, the viscosity of a fluid is a measure of how resistive the fluid is to flow. For example, thick honey is more resistive to flow than water, and so honey is said to be more viscous than water. Viscosity is the fluid analog of friction between solids; both are mechanisms by which the kinetic energy of moving objects can be transferred to thermal energy. In the absence of friction, a block could

glide at constant speed along a horizontal surface. In the same way, an object moving through a nonviscous fluid would experience no viscous drag force—that is, no resistive force due to viscosity; it could move at constant speed through the fluid.

Viscosity and Laminar and turbulence Flow:

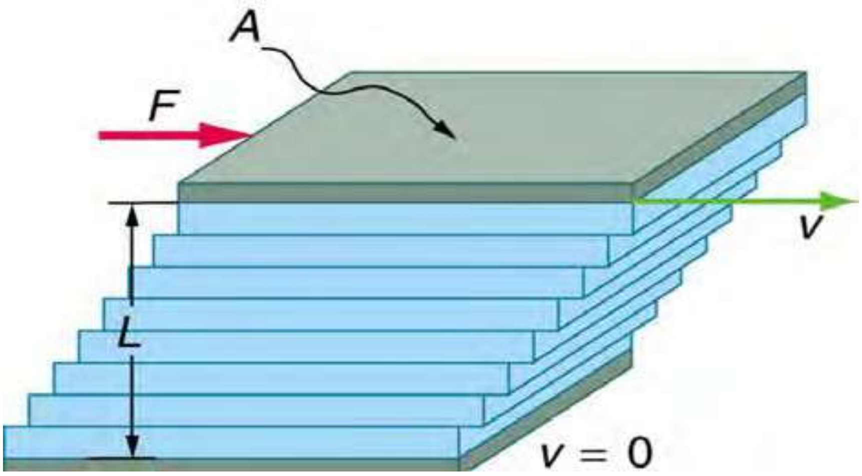
Laminar flow is characterized by the smooth flow of the fluid in layers that do not mix.

Turbulent flow, or turbulence, is characterized by eddies and swirls that mix layers of fluid together.



(a) Laminar flow occurs in layers without mixing.
(b) An obstruction in the vessel produces turbulence. Turbulent flow mixes the fluid.

Figure below shows how viscosity is measured for a fluid. Two parallel plates have the specific fluid between them. The bottom plate is held fixed, while the top plate is moved to the right, dragging fluid with it. The layer (or lamina) of fluid in contact with either plate does not move relative to the plate, and so the top layer moves at v while the bottom layer remains at rest. Each successive layer from the top down exerts a force on the one below it, trying to drag it along, producing a continuous variation in speed from v to 0 as shown. Care is taken to insure that the flow is laminar; that is, the layers do not mix. The motion in Figure is like a continuous shearing motion. Fluids have zero shear strength, but the rate at which they are sheared is related to the same geometrical factors A and L as is shear deformation for solids.



A force F is required to keep the top plate in Figure moving at a constant velocity v , and experiments have shown that this force depends on four factors. First, F is directly proportional to v (until the speed is so high that turbulence occurs–then a much larger force is needed, and it has a more complicated dependence on v). Second, F is proportional to the area A of the plate. This relationship seems reasonable, since A is directly proportional to the amount of fluid being moved. Third, F is inversely proportional to the distance between the plates L . This relationship is also reasonable; L is like a lever arm, and the greater the lever arm, the less force that is needed. Fourth, F is directly proportional to the coefficient of viscosity, η . The greater the viscosity, the greater the force required. These dependencies are combined into the equation:

$$F = \eta \frac{vA}{L},$$

which gives us a working definition of fluid viscosity η . Solving for η gives:

$$\eta = \frac{FL}{vA},$$

which defines viscosity in terms of how it is measured. The SI unit of viscosity is N.m /[(m/s)m^2] = (N/m^2)s or Pa.s .

The Onset of Turbulence

Sometimes we can predict if flow will be laminar or turbulent. We know that flow in a very smooth tube or around a smooth, streamlined object will be laminar at low velocity. We also know that at high velocity, even flow in a smooth tube or around a smooth object will experience turbulence. In between, it is more difficult to predict. In fact, at intermediate velocities, flow may oscillate back and forth indefinitely between laminar and turbulent.

An occlusion, or narrowing, of an artery, such as shown in Figure, is likely to cause turbulence because of the irregularity of the blockage, as well as the complexity of blood as a fluid. Turbulence in the circulatory system is noisy and can sometimes be detected with a stethoscope, such as when measuring diastolic pressure in the upper arm’s partially collapsed brachial artery. These turbulent sounds, at the onset of blood flow when the cuff pressure becomes sufficiently small, are called Korotkoff sounds. Aneurysms, or ballooning of arteries, create significant turbulence and can sometimes be detected with a stethoscope. Heart murmurs, consistent with their name, are sounds produced by turbulent flow around damaged and insufficiently closed heart valves. Ultrasound can also be used to detect turbulence as a medical indicator in a process analogous to Doppler-shift radar used to detect storms.

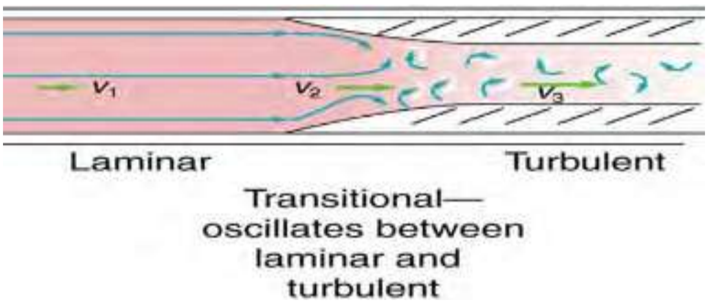


Figure show: Flow is laminar in the large part of this blood vessel and turbulent in the part narrowed by plaque, where velocity is high. In the transition region, the flow can oscillate chaotically between laminar and turbulent flow.

Oscillation Motion

Our world is filled with oscillations in which objects move back and forth repeatedly. Many oscillations are merely amusing or annoying, but many others are dangerous or financially important. Here are a few examples: When a bat hits a baseball, the bat may oscillate enough to sting the batter's hands or even to break apart.

Simple Harmonic Motion

Figure (a) shows a sequence of "snapshots" of a simple oscillating system, a particle moving repeatedly back and forth about the origin of an *x* axis. In this section we simply describe the motion. Later, we shall discuss how to attain such motion.

One important property of oscillatory motion is its **frequency,** or number of oscillations that are completed each second. The symbol for frequency is *f*, and its SI unit is the **hertz** (abbreviated Hz), where

1 hertz =1 Hz = 1 oscillation per second = 1 s^{-1}.

Related to the frequency is the **period** *T* of the motion, which is the time for one complete oscillation (or **cycle**); that is,

$$T = \frac{1}{f}.$$

Any motion that repeats itself at regular intervals is called periodic motion or harmonic motion. We are interested here in motion that repeats itself in a particular way—namely, like that in Fig. (a). For such motion the displacement x of the particle from the origin is given as a function of time by

$$x(t) = x_m \cos(\omega t + \phi) \qquad \text{(displacement),}$$

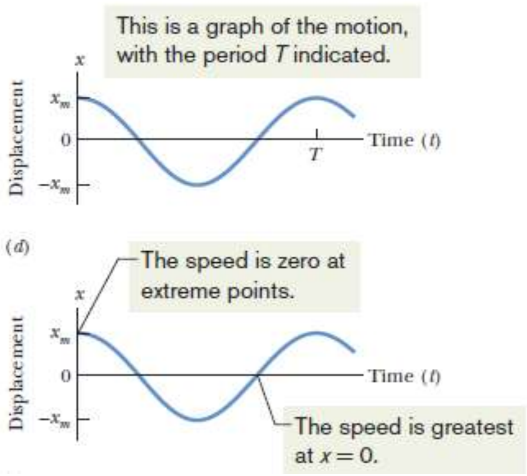


Figure shows: (SHM), a term that means the periodic motion is a sinusoidal function of time.

with their names. We now shall define those quantities.

The quantity x_m, called the **amplitude** of the motion, is a positive constant whose value depends on how the motion was started. The subscript *m* stands for *maximum* because the amplitude is the magnitude of the maximum displacement of the particle in either direction. The cosine function in Eq. varies between the limits ±1; so the displacement $x(t)$ varies between the limits $\pm x_m$.

The time-varying quantity ($\omega t + \Phi$) in Eq. is called the **phase** of the motion, and the constant Φ is called the **phase constant** (or **phase angle**). The value of Φ depends on the displacement and velocity of the particle at time $t = 0$. For the $x(t)$ plots of Fig. *a*, the phase constant Φ is zero.

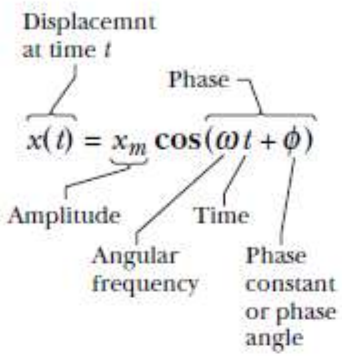
To interpret the constant ω, called the **angular frequency** of the motion, we first note that the displacement $x(t)$ must return to its initial value after one period T of the motion; that is, $x(t)$ must equal $x(t+T)$ for all t. To simplify this analysis, let us put $\Phi = 0$in Eq.. From that equation we then can write

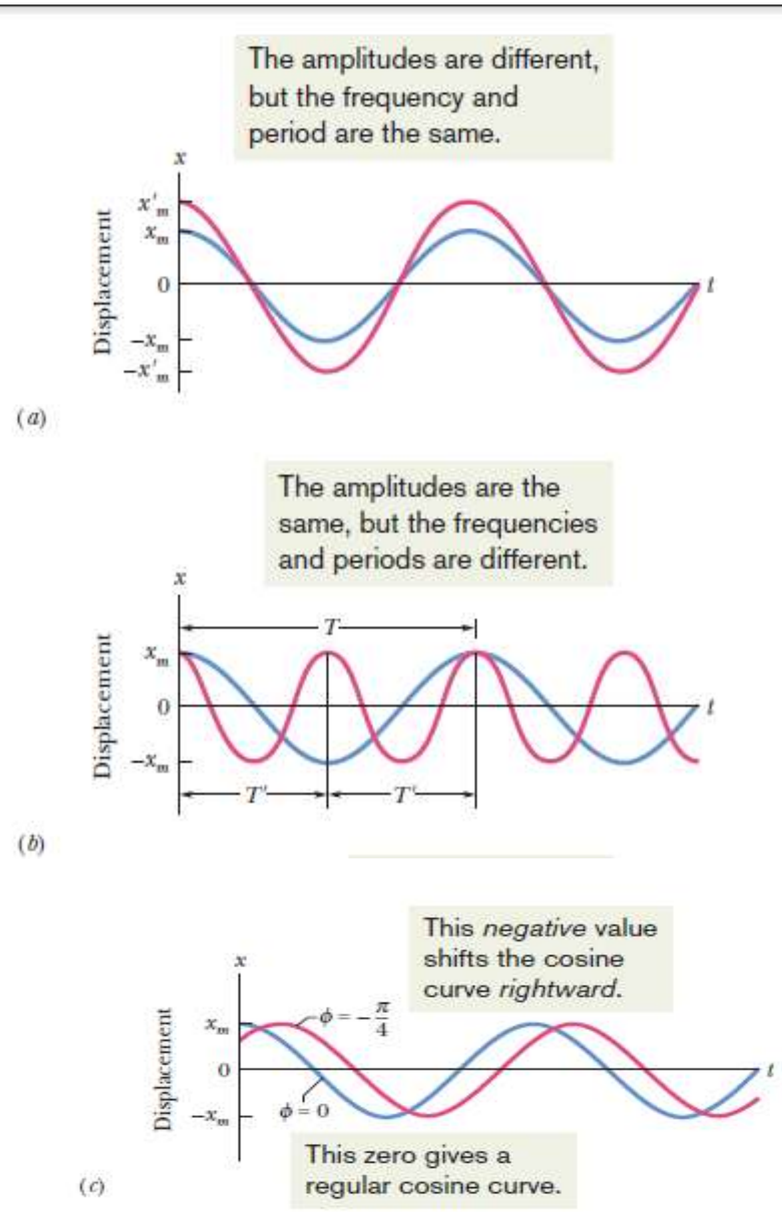
$$x_m \cos \omega t = x_m \cos \omega (t + T).$$

The cosine function first repeats itself when its argument (the phase) has increased by 2π rad; so Eq. gives us

$$\omega (t + T) = t + 2\pi$$

$$\text{or } \omega T = 2\pi$$





Thus, from Eq. 15-2 the angular frequency is

$$\omega = \frac{2\pi}{T} = 2\pi f.$$

The Velocity of SHM

We can find an expression for the velocity of a particle moving with simple harmonic motion; that is,

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}[x_m \cos(\omega t + \phi)]$$

$$v(t) = -\omega x_m \sin(\omega t + \phi) \qquad \text{(velocity).}$$

Figure *a* is a plot of Eq. with Φ= 0. Figure b shows Eq., also with Φ =0. Analogous to the amplitude x_m in Eq., the positive quantity ωx_m in Eq. is called the **velocity amplitude** v_m.As you can see in Fig. (b), the velocity of the oscillating particle varies between the limits $\pm v_m = \pm \omega x_m$. Note also in that figure that the curve of $v(t)$ is *shifted* (to the left) from the curve of $x(t)$ by one-quarter period; when the magnitude of the displacement is greatest (that is, $x(t)$ =(x_m), the magnitude of the velocity is least (that is, $v(t)$ = 0).When the magnitude of the displacement is least (that is, zero), the magnitude of the velocity is greatest (that is, $v_m = \omega x_m$).

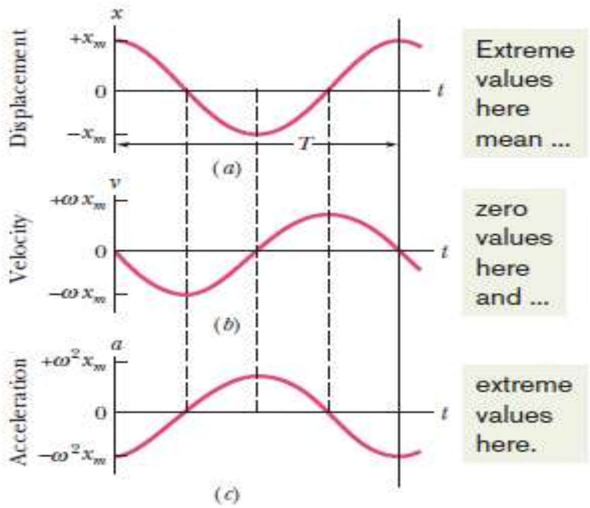


Fig. 15-4 (a) The displacement x(t) of a particle oscillating in SHM with phase angle Φ equal to zero. The period T marks one complete oscillation. (b) The velocity v(t) of the particle. (c) The acceleration a(t) of the particle.

The Acceleration of SHM

Knowing the velocity $v(t)$ for simple harmonic motion, we can find an expression for the acceleration of the oscillating particle by differentiating once more. Thus, we have, from Eq.

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}[-\omega x_m \sin(\omega t + \phi)]$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi) \quad \text{(acceleration).}$$

Figure (c) is a plot of Eq. for the case $\Phi = 0$.The positive quantity $\omega^2 x_m$ in Eq. is called the **acceleration amplitude** a_m; that is, the acceleration of the particle varies between the limits $\pm\, a_m = \pm\, \omega^2 x_m$, as Fig. (c) shows. Note also that the acceleration curve $a(t)$ is shifted (to the left) by $\frac{1}{4}T$ relative to the velocity curve $v(t)$.

We can combine

$$a(t) = -\omega^2 x(t),$$

Thus, as Fig. shows, when the displacement has its greatest positive value, the acceleration has its greatest negative value, and conversely. When the displacement is zero, the acceleration is also zero.

The Force Law for Simple Harmonic Motion

Once we know how the acceleration of a particle varies with time, we can use Newton's second law to learn what force must act on the particle to give it that acceleration. If we combine Newton's second law and Eq. we find, for simple harmonic motion,

$$F = ma = -(m\omega^2)x.$$

This result—a restoring force that is proportional to the displacement but opposite in sign—is familiar. It is Hooke's law,

$$F = -kx,$$

for a spring, the spring constant here being

$$k = m\omega^2.$$

The block– spring system of Fig. forms a linear simple harmonic oscillator (linear oscillator, for short), where "linear" indicates that F is proportional to x rather than to some other power of x. The angular frequency ω of the simple harmonic motion of the block is related to the spring constant k and the mass m of the block by Eq., which yields

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency).}$$

We can write, for the period of the linear oscillator

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period).}$$

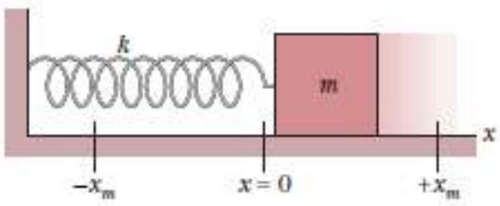


Fig. shows: A linear simple harmonic oscillator.

Q\ A block whose mass *m* is 680 g is fastened to a spring whose spring constant *k* is 65 N/m. The block is pulled a distance *x* = 11 cm from its equilibrium position at *x* = 0 on a frictionless surface and released from rest at *t* = 0.

(a) What are the angular frequency, the frequency, and the period of the resulting motion?

The block–spring system forms a linear simple harmonic oscillator, with the block undergoing SHM.

Calculations: The angular frequency is given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65\ \text{N/m}}{0.68\ \text{kg}}} = 9.78\ \text{rad/s}$$

$$\approx 9.8\ \text{rad/s.} \qquad \text{(Answer)}$$

The frequency follows

$$f = \frac{\omega}{2\pi} = \frac{9.78\ \text{rad/s}}{2\pi\ \text{rad}} = 1.56\ \text{Hz} \approx 1.6\ \text{Hz.} \qquad \text{(Answer)}$$

The period follows

$$T = \frac{1}{f} = \frac{1}{1.56\ \text{Hz}} = 0.64\ \text{s} = 640\ \text{ms.} \qquad \text{(Answer)}$$

(b) What is the amplitude of the oscillation?

With no friction involved, the mechanical energy of the spring–block system is conserved.

Reasoning: The block is released from rest 11 cm from its equilibrium position, with zero kinetic energy and the elastic potential energy of the system at a maximum. Thus, the block will have zero kinetic energy whenever it is again 11 cm from its equilibrium position, which means it will never be farther than 11 cm from that position. Its maximum displacement is 11 cm:

$$x_m = 11\ \text{cm.} \qquad \text{(Answer)}$$

(c) What is the maximum speed v_m of the oscillating block, and where is the block when it has this speed?

The maximum speed v_m is the velocity amplitude ωx_m

$$v_m = \omega x_m = (9.78 \text{ rad/s})(0.11 \text{ m})$$
$$= 1.1 \text{ m/s.} \qquad \text{(Answer)}$$

(d) What is the magnitude *am* of the maximum acceleration of the block?

$$a_m = \omega^2 x_m = (9.78 \text{ rad/s})^2(0.11 \text{ m})$$
$$= 11 \text{ m/s}^2. \qquad \text{(Answer)}$$

(e) What is the phase constant Φ for the motion?

gives the displacement of the block as a function of time. We know that at time $t = 0$, the block is located at $x = x_m$. Substituting these *initial conditions,* as they are called, into Eq. 15-3 and canceling x_m give us

$$1 = \cos \phi.$$

Taking the inverse cosine then yields

$$\phi = 0 \text{ rad.} \qquad \text{(Answer)}$$

(Any angle that is an integer multiple of 2π rad also satisfies

(f) What is the displacement function $x(t)$ for the spring–block system?

$$x(t) = x_m \cos(\omega t + \phi)$$
$$= (0.11 \text{ m}) \cos[(9.8 \text{ rad/s})t + 0]$$
$$= 0.11 \cos(9.8t), \qquad \text{(Answer)}$$

where x is in meters and t is in seconds.

Q\ We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let's try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of 0.400 µs. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

Solution a

1. Substitute 0.400 μs for T in

$$f = \frac{1}{T}.$$

$$f = \frac{1}{T} = \frac{1}{0.400 \times 10^{-6}\ \text{s}}.$$

$$f = 2.50 \times 10^{6}\ \text{Hz}.$$

Solution b

1. Identify the known values:

The time for one complete oscillation is the period T :

$$f = \frac{1}{T}.$$

2. Solve for T :

$$T = \frac{1}{f}.$$

3. Substitute the given value for the frequency into the resulting expression:

$$T = \frac{1}{f} = \frac{1}{264\ \text{Hz}} = \frac{1}{264\ \text{cycles/s}} = 3.79 \times 10^{-3}\ \text{s} = 3.79\ \text{ms}.$$

Energy in Simple Harmonic Motion

The energy of a linear oscillator transfers back and forth between kinetic energy and potential energy, while the sum of the two-the mechanical energy *E* of the oscillator—remains constant. We now consider this situation quantitatively.

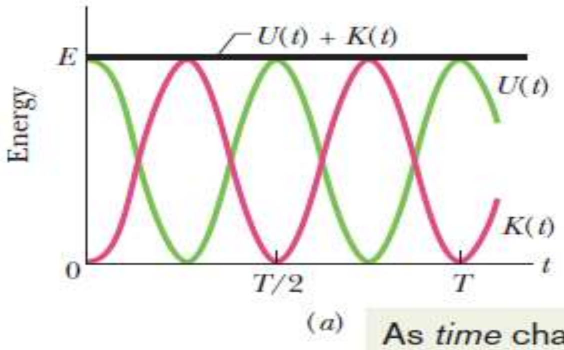
The potential energy of a linear oscillator like associated entirely with the spring. Its value depends on how much the spring is stretched or compressed—that is, on $x(t)$.We can use Eqs. to find

$$U(t) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_m^2\cos^2(\omega t + \phi).$$

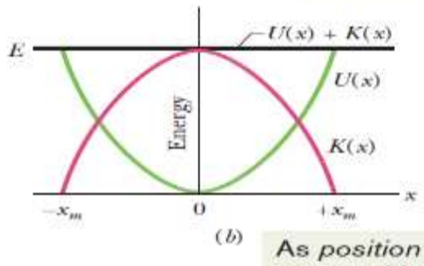
A function written in the form $\cos^2 A$ (as here) means $(\cos A)^2$ and is not the same as one written $\cos A^2$,which means $\cos(A^2)$.

The kinetic energy of the system is associated entirely with the block. Its value depends on how fast the block is moving—that is, on v(t).We can use to find

$$K(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2 x_m^2 \sin^2(\omega t + \phi).$$



As *time* changes, the energy shifts between the two types, but the total is constant.



As *position* changes, the energy shifts between the two types, but the total is constant.

$$K(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_m^2 \sin^2(\omega t + \phi).$$

The mechanical energy

$$E = U + K$$
$$= \tfrac{1}{2}kx_m^2 \cos^2(\omega t + \phi) + \tfrac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$
$$= \tfrac{1}{2}kx_m^2 [\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)].$$

For any angle α,

$$\cos^2 \alpha + \sin^2 \alpha = 1.$$

Thus, the quantity in the square brackets above is unity and we have

$$E = U + K = \tfrac{1}{2}kx_m^2.$$

An Angular Simple Harmonic Oscillator

Figure shows an angular version of a simple harmonic oscillator; the element of springiness or elasticity is associated with the twisting of a suspension wire rather than the extension and compression of a spring as we previously had. The device is called a torsion pendulum, with torsion referring to the twisting.

If we rotate the disk in Fig. by some angular displacement - from its rest position and release it, it will oscillate about that position in angular simple harmonic motion. Rotating the disk through an angle - in either direction introduces a restoring torque given by

$$\tau = -\kappa\theta.$$

Here / (Greek kappa) is a constant, called the torsion constant, that depends on the length, diameter, and material of the suspension wire.

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \qquad \text{(torsion pendulum).}$$

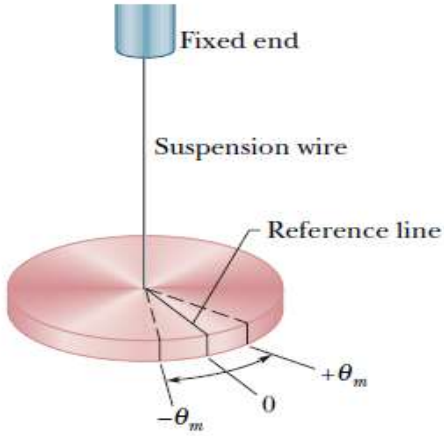


Figure shows: A torsion pendulum is an angular version of a linear simple harmonic Oscillator.

Pendulums

We turn now to a class of simple harmonic oscillators in which the springiness is associated with the gravitational force rather than with the elastic properties of a twisted wire or a compressed or stretched spring.

The Simple Pendulum

If an apple swings on a long thread, does it have simple harmonic motion? If so, what is the period *T*? T$_o$ answer, we consider a **simple pendulum,** which consists of a particle of mass *m* (called the *bob* of the pendulum) suspended from one end of an unstretchable, massless string of length *L* that is fixed at the other end, as in Fig. (a).The bob is free to swing back and forth in the plane of the page, to the left and right of a vertical line through the pendulum's pivot point. The forces acting on the bob are the force from the string and the gravitational force *g*, as shown in Fig. (b), where the string makes an angle θ with the vertical. We resolve *g* into a radial component F_g cos θ and a component F_g sin θ that is tangent to the path taken by the bob. This tangential component produces a restoring torque about the pendulum's pivot point because the component always acts opposite the displacement of the bob so as to bring the bob back toward its central location. That location is called the *equilibrium position*(θ=0)

because the pendulum would be at rest there were it not swinging.

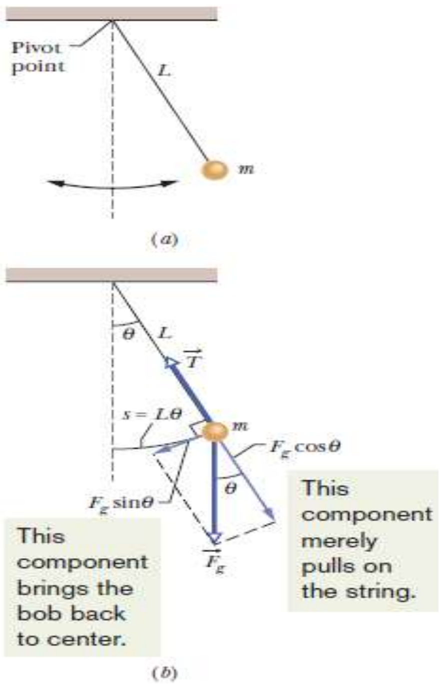


Figure shows:(a) A simple pendulum. (b) The forces acting on the bob are the gravitational force g and the force from the string. The tangential component F_g sin θof the gravitational force is a restoring force that tends to bring the pendulum back to its central position.

we can write this restoring torque as

$$\tau = -L(F_g \sin\theta),$$

where the minus sign indicates that the torque acts to reduce θ and L is the moment arm of the force component $F_g \sin\theta$ about the pivot point.

$$-L(mg\sin\theta) = I\alpha,$$

where I is the pendulum's rotational inertia about the pivot point and α is its angular acceleration about that point.

$$\alpha = -\frac{mgL}{I}\theta.$$

It tells us that the angular acceleration α of the pendulum is proportional to the angular displacement θ but opposite in sign. Thus, as the pendulum bob moves to the right, as in Fig. (a), its acceleration to the left increases until the bob stops and begins moving to the left. Then, when it is to the left of the equilibrium position, its acceleration to the right tends to return it to the right, and so on, as it swings back and forth in SHM. More precisely, the motion of a simple pendulum swinging through only small angles is approximately SHM. We can state this restriction to small angles another way: The angular amplitude θ_m of the motion (the maximum angle of swing) must be small.

the angular frequency of the pendulum is $\omega = \sqrt{\frac{mgL}{I}}$

Next, if we substitute this expression for ω into Eq. ($\omega = \frac{2\pi}{T}$), we see that the period of the pendulum may be written as

$$T = 2\pi\sqrt{\frac{I}{mgL}}.$$

All the mass of a simple pendulum is concentrated in the mass m of the particle like bob, which is at radius L from the pivot point. Thus, we can use ($I=mr^2$), to write $I=mL^2$ for the rotational inertia of the pendulum.

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{(simple pendulum, small amplitude).}$$

Q\ What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Solution

We are asked to find g given the period T and the length L of a pendulum. We can solve $T = 2\pi\frac{L}{g}$e

for g , assuming only that the angle of deflection is less than 15°.

1. Square $T = 2\pi\frac{L}{g}$ and solve for g :

$$g = 4\pi^2\frac{L}{T^2}.$$

2. Substitute known values into the new equation:

$$g = 4\pi^2\frac{0.75000\text{ m}}{(1.7357\text{ s})^2}.$$

3. Calculate to find g :

$$g = 9.8281\text{ m/s}^2.$$

Wave Motion

Types of Waves

Waves are of three main types:

1. ***Mechanical waves.*** These waves are most familiar because we encounter them almost constantly; common examples include water waves, sound waves, and seismic waves. All these waves have two central features: They are governed by Newton's laws, and they can exist only within a material medium, such as water, air, and rock.

2. ***Electromagnetic waves.*** These waves are less familiar, but you use them constantly; common examples include visible and ultraviolet light, radio and television waves, microwaves, x rays, and radar waves. These waves require no material medium to exist. Light waves from stars, for example, travel through the vacuum of space to reach us. All electromagnetic waves travel through a vacuum at the same speed c = 299 792 458 m/s.

3. ***Matter waves.*** Although these waves are commonly used in modern technology, they are probably very unfamiliar to you. These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. Because we commonly think of these particles as constituting matter, such waves are called matter waves.

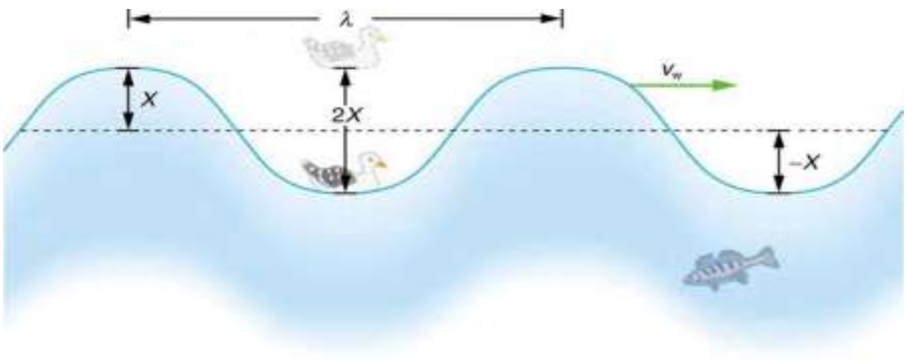


Figure show: *An idealized ocean wave passes under a sea gull that bobs up and down in simple harmonic motion. The wave has a wavelength λ , which is the distance between adjacent identical parts of the wave. The up and down disturbance of the surface propagates parallel to the surface at a speed* v_w *.*

The water wave in the figure also has a length associated with it, called its **wavelength** λ , the distance between adjacent identical parts of a wave. (λ is the distance parallel to the direction of propagation.) The speed of propagation v_w is the distance the wave travels in a given time, which is one wavelength in the time of one period. In equation form, that is

$$v_w = \frac{\lambda}{T}$$

$$v_w = f\lambda.$$

This fundamental relationship holds for all types of waves. For water waves, v_w is the speed of a surface wave; for sound, v_w is the speed of sound; and for visible light, v_w is the speed of light, for example.

Q\ Calculate the wave velocity of the ocean wave in Figure above ,if the distance between wave crests is 10.0 m and the time for a sea gull to bob up and down is 5.00 s.

Solution

1. Enter the known values into

$$v_w = \frac{\lambda}{T}$$

$$v_w = \frac{10.0\ \text{m}}{5.00\ \text{s}}.$$

2. Solve for v_w to find $v_w = 2.00$ m/s.

Transverse and Longitudinal Waves

A simple wave consists of a periodic disturbance that propagates from one place to another. The wave in Figure propagates in the horizontal direction while the surface is disturbed in the vertical direction. Such a wave is called a transverse wave or shear wave; in such a wave, the disturbance is perpendicular to the direction of propagation.

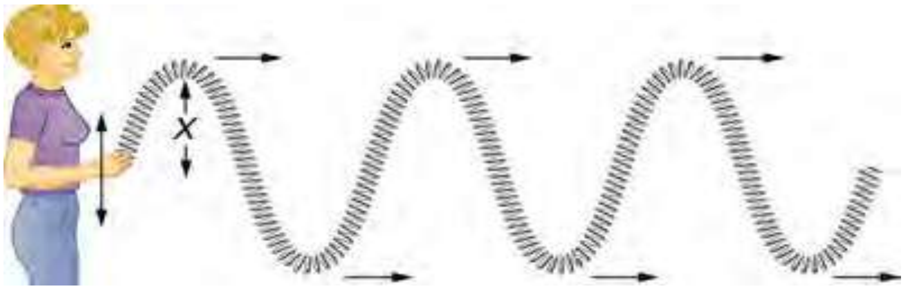


Figure : *In this example of a transverse wave, the wave propagates horizontally, and the disturbance in the cord is in the vertical direction.*

In contrast, in a longitudinal wave or compressional wave, the disturbance is parallel to the direction of propagation. Figure shows an example of a longitudinal wave. The size of the disturbance is its amplitude X and is completely independent of the speed of propagation v_w .

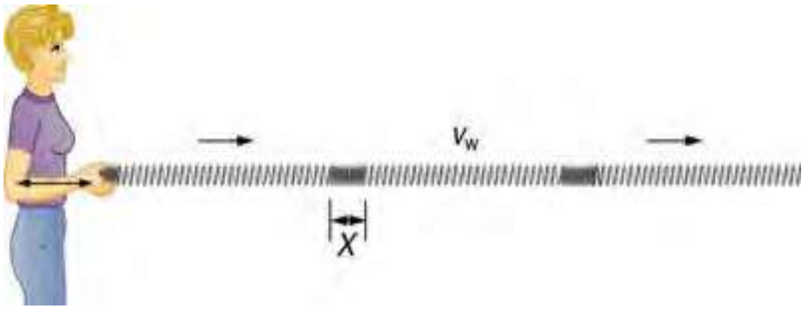


Figure shows: In this example of a longitudinal wave, the wave propagates horizontally, and the disturbance in the cord is also in the horizontal direction.

Wavelength and Frequency

To completely describe a wave on a string (and the motion of any element along its length), we need a function that gives the shape of the wave. This means that we need a relation in the form

$$y = h(x, t),$$

in which y is the transverse displacement of any string element as a function h of the time t and the position x of the element along the string. In general, a sinusoidal shape like the wave in Fig. (b) can be described with h being either a sine or cosine function; both give the same general shape for the wave. Imagine a sinusoidal wave like that of Fig. (b) traveling in the positive direction of an x axis. As the wave sweeps through succeeding elements (that is, very short sections) of the string, the elements oscillate parallel to the y axis. At time t, the displacement y of the element located at position x is given by

$$y(x, t) = y_m \sin(kx - \omega t).$$

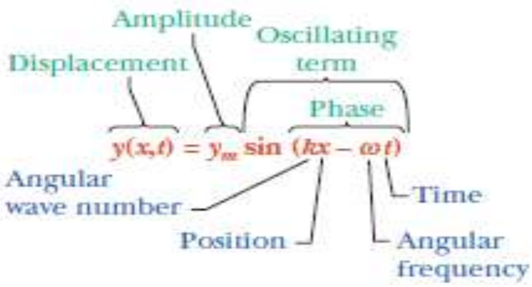


Figure shows: The names of the quantities in Eq., for a transverse sinusoidal wave.

Because this equation is written in terms of position x, it can be used to find the displacements of all the elements of the string as a function of time. Thus, it can tell us the shape of the wave at any given time and how that shape changes as the wave moves along the string.

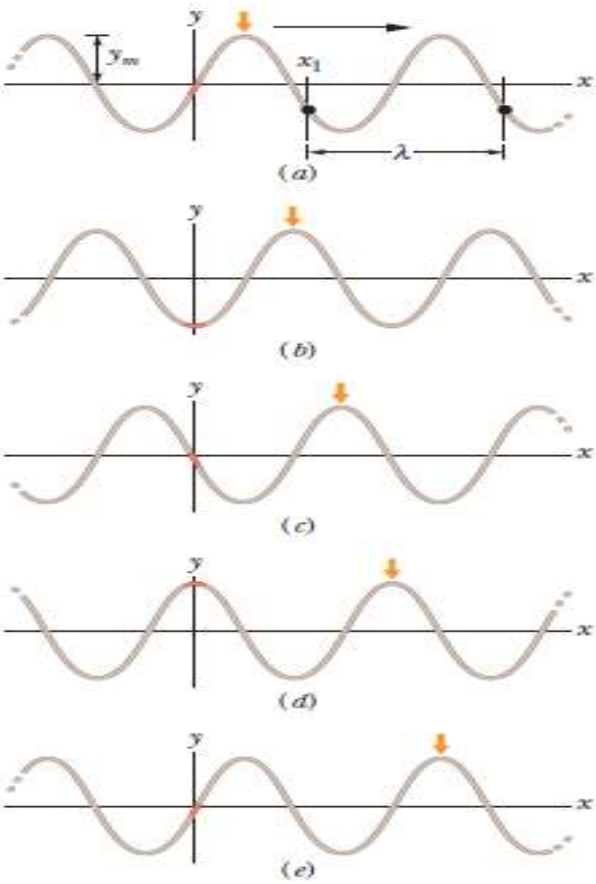


Figure shows: Five "snapshots" of a string wave traveling in the positive direction of *an x axis. The amplitude y_m is indicated. A typical wavelength l, measured from an arbitrary position x1, is also indicated.*

Amplitude and Phase

The **amplitude** y_m of a wave, is the magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them. (The subscript *m* stands for maximum.) Because y_m is a magnitude, it is always a positive quantity, even if it is measured downward instead of upward as drawn in Fig. (a).

The phase of the wave is the argument $kx - \omega t$ of the sine in Eq.. As the wave sweeps through a string element at a particular position x, the phase changes linearly with time t. This means that the sine also changes, oscillating between +1 and -1. Its extreme positive value (+1) corresponds to a peak of the wave moving through the element; at that instant the value of y at position x is y_m. Its extreme negative value (-1) corresponds to a valley of the wave moving through the element; at that instant the value of y at position x is $-y_m$. Thus, the sine function and the time-dependent phase of a wave correspond to the oscillation of a string element, and the amplitude of the wave determines the extremes of the element's displacement.

Wavelength and Angular Wave Number

The wavelength l of a wave is the distance (parallel to the direction of the wave's travel) between repetitions of the shape of the wave (or wave shape). A typical wavelength is marked in Fig. (a), which is a snapshot of the wave at time t = 0.

At that time, Eq. gives, for the description of the wave shape,

$$y(x,0) = y_m \sin kx.$$

By definition, the displacement y is the same at both ends of this wavelength- that is, at $x = x_1$ and $x = x_1\ \lambda$. Thus

$$y_m \sin kx_1 = y_m \sin k(x_1 + \lambda)$$
$$= y_m \sin(kx_1 + k\lambda).$$

A sine function begins to repeat itself when its angle (or argument) is increased by 2π rad. we must have $k\lambda=2\pi$, or

$$k = \frac{2\pi}{\lambda} \quad \text{(angular wave number).}$$

We call k the angular wave number of the wave; its SI unit is the radian per meter, or the inverse meter. (Note that the symbol k here does not represent a spring constant as previously.)

Period, Angular Frequency, and Frequency

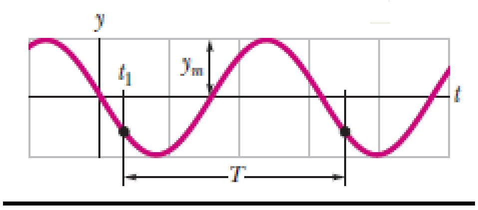


Figure shows a graph of the displacement y of Eq. versus time t at a certain position along the string, taken to be x = 0. If you were to monitor the string, you would see that the single element of the string at that position moves up and down in simple harmonic motion given by Eq. with x = 0:

$$y(0,t) = y_m \sin(-\omega t)$$
$$= -y_m \sin \omega t \qquad (x = 0).$$

Here we have made use of the fact that sin(-α)=-sinα, where αis any angle. Figure is a graph of this equation, with displacement plotted versus time; it does not show the shape of the wave.

We define the period of oscillation T of a wave to be the time any string element takes to move through one full oscillation

$$-y_m \sin \omega t_1 = -y_m \sin \omega(t_1 + T)$$
$$= -y_m \sin(\omega t_1 + \omega T).$$

This can be true only if ωT=2π, or if

$$\omega = \frac{2\pi}{T} \qquad \text{(angular frequency).}$$

The frequency f of a wave is defined as 1/T and is related to the angular frequency ω by

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \qquad \text{(frequency).}$$

Q\ A wave traveling along a string is described by y(x, t) = 0.00327 sin(72.1x - 2.72t), in which the numerical constants are in SI units (0.00327 m, 72.1 rad/m, and 2.72 rad/s).

(a) What is the amplitude of this wave?

$$y = y_m \sin(kx - \omega t),$$

so we have a sinusoidal wave. By comparing the two equations, we can find the amplitude.

Calculation: We see that

$$y_m = 0.00327 \text{ m} = 3.27 \text{ mm}. \qquad \text{(Answer)}$$

(b) What are the wavelength, period, and frequency of this wave?

$$k = 72.1 \text{ rad/m} \quad \text{and} \quad \omega = 2.72 \text{ rad/s.}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi \text{ rad}}{72.1 \text{ rad/m}}$$
$$= 0.0871 \text{ m} = 8.71 \text{ cm.} \qquad \text{(Answer)}$$

Next, we relate T to ω with .

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{2.72 \text{ rad/s}} = 2.31 \text{ s,} \qquad \text{(Answer)}$$

$$f = \frac{1}{T} = \frac{1}{2.31 \text{ s}} = 0.433 \text{ Hz.} \qquad \text{(Answer)}$$

(c) What is the velocity of this wave?

$$v = \frac{\omega}{k} = \frac{2.72 \text{ rad/s}}{72.1 \text{ rad/m}} = 0.0377 \text{ m/s}$$
$$= 3.77 \text{ cm/s.}$$

(d) What is the displacement y of the string at x = 22.5 cm and t = 18.9 s?

$$y = 0.00327 \sin(72.1 \times 0.225 - 2.72 \times 18.9)$$
$$= (0.00327 \text{ m}) \sin(-35.1855 \text{ rad})$$
$$= (0.00327 \text{ m})(0.588)$$
$$= 0.00192 \text{ m} = 1.92 \text{ mm.}$$

Superposition and Interference

When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called **superposition**. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple addition of the disturbances of the individual waves—that is, their amplitudes add.

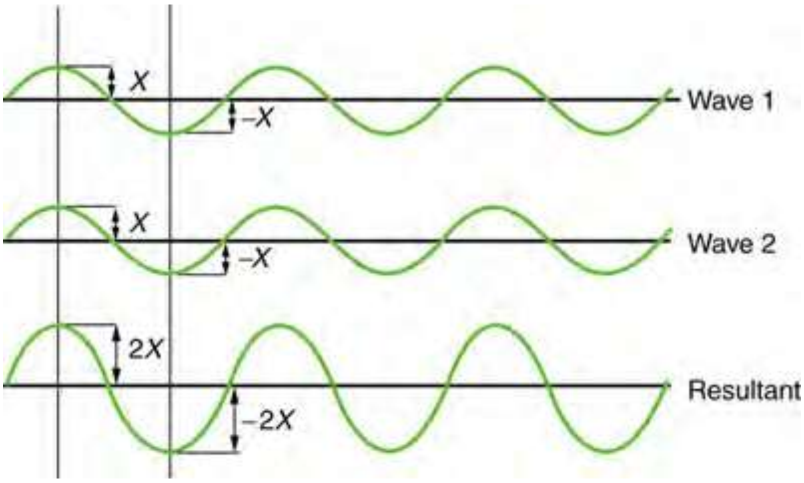


Figure shows: two identical waves that arrive at the same point exactly in phase. The crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure constructive interference. Because the disturbances add, pure constructive interference produces a wave that has twice the amplitude of the individual waves, but has the same wavelength.

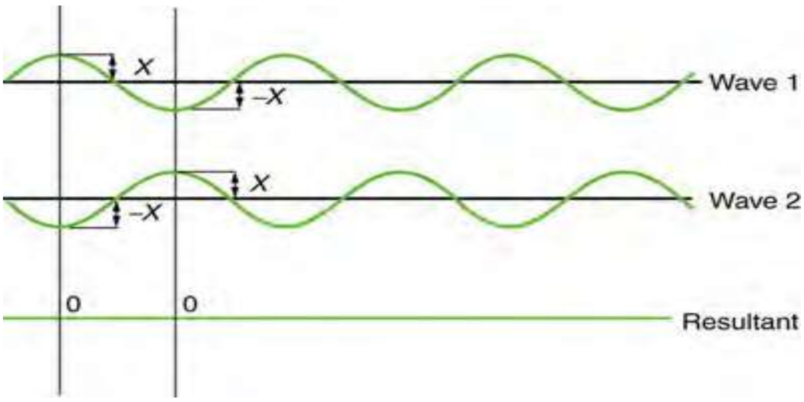


Figure shows :two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—producing pure destructive interference. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude is zero for pure destructive interference—the waves completely cancel.

Interference of Waves

Suppose we send two sinusoidal waves of the same wavelength and amplitude in the same direction along a stretched string. The superposition principle applies.

What resultant wave does it predict for the string?

The resultant wave depends on the extent to which the waves are in phase (in step) with respect to each other—that is, how much one wave form is shifted from the other wave form. If the waves are exactly in phase (so that the peaks and valleys of one are exactly aligned with those of the other), they combine to double the displacement of either wave acting alone. If they are exactly out of phase (the peaks of one are exactly aligned with the valleys of the other), they combine to cancel everywhere, and the string remains straight. We call this phenomenon of combining waves interference, and the waves are said to interfere. (These terms refer only to the wave displacements; the travel of the waves is unaffected.)
Let one wave traveling along a stretched string be given by

$$y_1(x,t) = y_m \sin(kx - \omega t)$$

and another, shifted from the first, by

$$y_2(x,t) = y_m \sin(kx - \omega t + \phi).$$

These waves have the same angular frequency ω (and thus the same frequency f), the same angular wave number k (and thus the same wavelength λ), and the same amplitude y_m. They both travel in the positive direction of the x axis, with the same speed, given by Eq.They differ only by a constant angle Φ, the phase constant. These waves are said to be out of phase by Φ or to have a phase difference of Φ, or one wave is said to be phase-shifted from the other by Φ.

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$
$$= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi).$$

In Appendix E we see that we can write the sum of the sines of two angles α and β as

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta).$$

Applying this relation

$$y'(x,t) = [2y_m \cos\tfrac{1}{2}\phi]\sin(kx - \omega t + \tfrac{1}{2}\phi).$$

$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{[2y_m \cos\tfrac{1}{2}\phi]}_{\text{Magnitude gives amplitude}}\ \underbrace{\sin(kx - \omega t + \tfrac{1}{2}\phi)}_{\text{Oscillating term}}$$

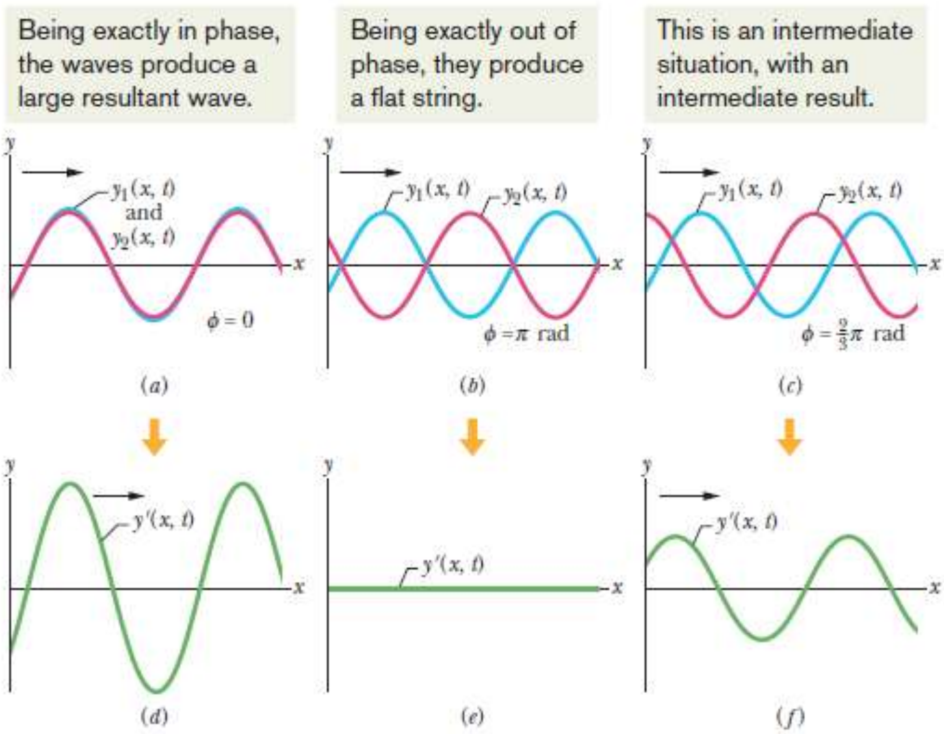


Figure shows: Two identical sinusoidal waves, y1(x, t) and y2(x, t), travel along a string in the positive direction of an x axis.They interfere to give a resultant wave y/(x, t).

Standing Waves

Sometimes waves do not seem to move; rather, they just vibrate in place. Unmoving waves can be seen on the surface of a glass of milk in a refrigerator, for example. Vibrations from the refrigerator motor create waves on the milk that oscillate up and down but do not seem to move across the surface. These waves are formed by the superposition of two or more moving waves, such as illustrated in **Figure** for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength, then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a standing wave. Waves on the glass of milk are one example of standing waves. There are other standing waves, such as on guitar strings and in organ pipes. With the glass of milk, the two waves that produce standing waves may come from reflections from the side of the glass.

A closer look at earthquakes provides evidence for conditions appropriate for resonance, standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.

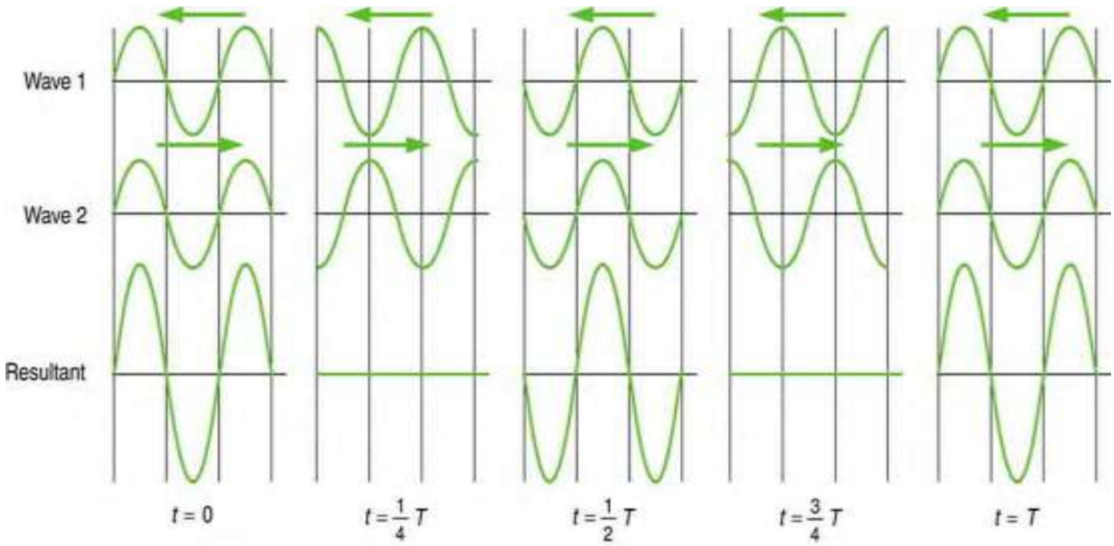


Figure shows: *Standing wave created by the superposition of two identical waves moving in opposite directions. The oscillations are at fixed locations in space and result from alternately constructive and destructive interference.*

Standing waves are also found on the strings of musical instruments and are due to reflections of waves from the ends of the string. Nodes are the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word antinode is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed v_w of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

The lowest frequency, called the fundamental frequency, is thus for the longest wavelength, which is seen to be $\lambda_1 = 2L$. Therefore, the fundamental frequency is $f_1 = v_w / \lambda_1 = v_w / 2L$. In this case, the overtones or harmonics are multiples of the fundamental frequency. As seen in Figure, the first harmonic can easily be calculated since $\lambda_2 = L$. Thus, $f_2 = v_w / \lambda_2 = v_w / 2L = 2 f_1$. Similarly, $f_3 = 3 f_1$, and so on. All of these frequencies can be changed by adjusting the tension in the string. The greater the tension, the greater v_w is and the higher the frequencies. This observation is familiar to anyone who has ever observed a string instrument being tuned. We will see in later chapters that standing waves are crucial to many resonance phenomena, such as in sounding boxes on string instruments.

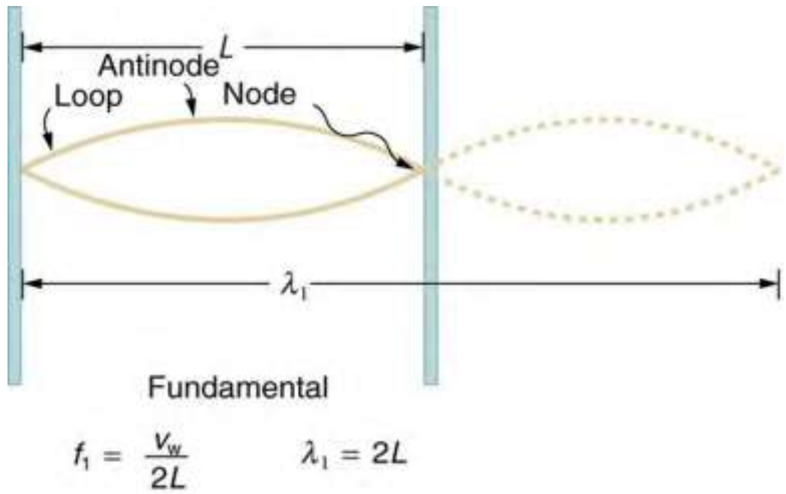


Figure shows: *The figure shows a string oscillating at its fundamental frequency.*

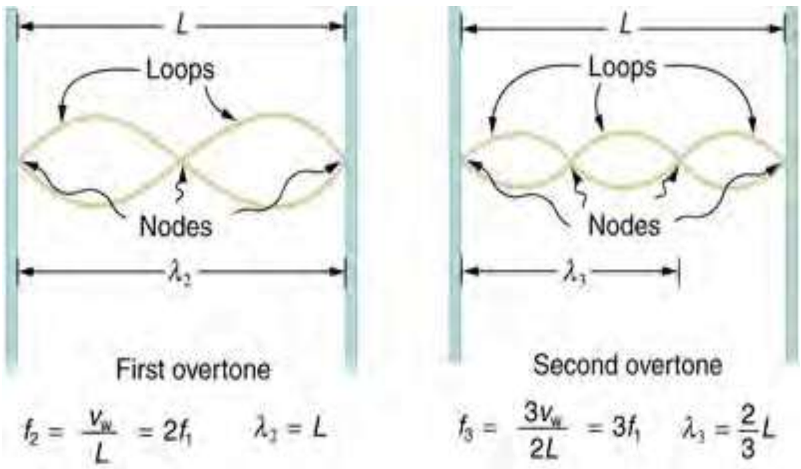


Figure shows: *First and second harmonic frequencies are shown.*

Speed of Light

The speed of light (c) not only affects refraction, it is one of the central concepts of Einstein's theory of relativity. As the accuracy of the measurements of the speed of light were improved, *c* was found not to depend on the velocity of the source or the observer. However, the speed of light does vary in a precise manner with the material it traverses. These facts have far-reaching implications, as we will see in **Special Relativity**. It makes connections between space and time and alters our expectations that all observers measure the same time for the same event, for example. The speed of light is so important that its value in a vacuum is one of the most fundamental constants in nature as well as being one of the four fundamental SI units.

The speed of light is now known to great precision. In fact, the speed of light in a vacuum c is so important that it is accepted as one of the basic physical quantities and has the fixed value $c = 2.9972458 \times 10^8$ m/s ≈ 3.00×10^8 m/s,

where the approximate value of 3.00×10^8 m/s is used whenever three-digit accuracy is sufficient. The speed of light through matter is less than it is in a vacuum, because light interacts with atoms in a material. The speed of light depends strongly on the type of material, since its interaction with different atoms, crystal lattices, and other substructures varies. We define the index of refraction n of a material to be

$$n = \frac{c}{v},$$

where v is the observed speed of light in the material. Since the speed of light is always less than c in matter and equals c only in a vacuum, the index of refraction is always greater than or equal to one.

Q\ Calculate the speed of light in zircon, a material used in jewelry to imitate diamond.

Solution

The equation for index of refraction states that n = c / v . Rearranging this to determine v gives

$$v = \frac{c}{n}.$$

The index of refraction for zircon is given as 1.923, and c is given in the equation for speed of light. Entering these values in the last expression gives

$$\begin{aligned} v &= \frac{3.00\times10^{8}\ \text{m/s}}{1.923} \\ &= 1.56\times10^{8}\ \text{m/s}. \end{aligned}$$

The Law of Reflection

The law of reflection is illustrated in Figure (a), which also shows how the angles are measured relative to the perpendicular to the surface at the point where the light ray strikes. We expect to see reflections from smooth surfaces, but Figure (b) illustrates how a rough surface reflects light. Since the light strikes different parts of the surface at different angles, it is reflected in many different directions, or diffused.

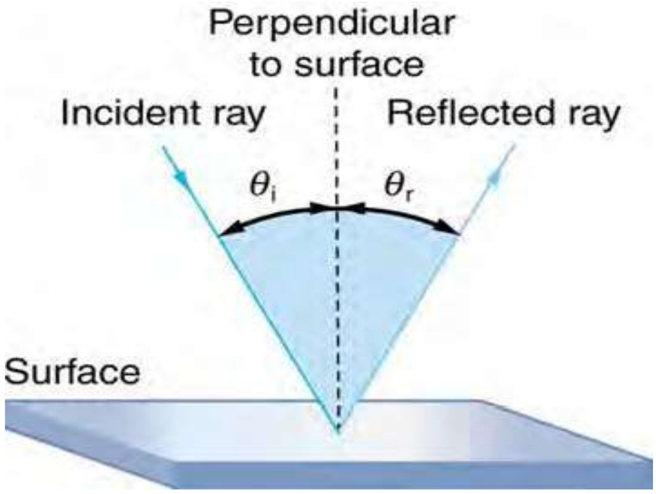


Figure (a): The law of reflection states that the angle of reflection equals the angle of incidence— θr = θi . The angles are measured relative to the perpendicular to the surface at the point where the ray strikes the surface.

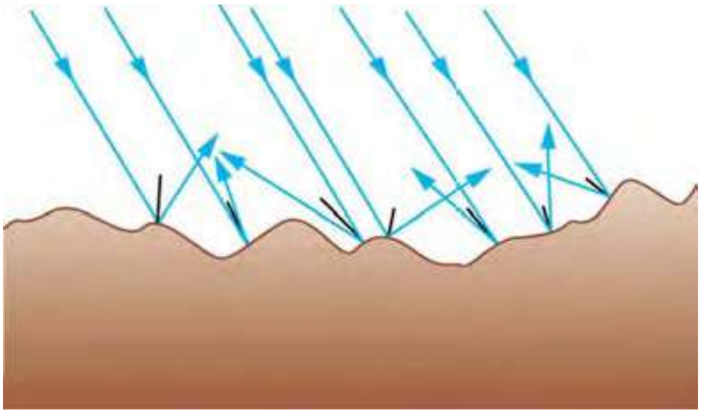


Figure (b): Light is diffused when it reflects from a rough surface. Here many parallel rays are incident, but they are reflected at many different angles since the surface is rough.

Law of Refraction

Figure shows how a ray of light changes direction when it passes from one medium to another. As before, the angles are measured relative to a perpendicular to the surface at the point where the light ray crosses it. (Some of the incident light will be reflected from the surface, but for now we will concentrate on the light that is transmitted.) The change in direction of the light ray depends on how the speed of light changes. The change in the speed of light is related to the indices of refraction of the media involved. In the situations shown in Figure, medium 2 has a greater index of refraction than medium 1. This means that the speed of light is less in medium 2 than in medium 1. Note that as shown in Figure (a), the direction of the ray moves closer to the perpendicular when it slows down. Conversely, as shown in Figure (b), the direction of the ray moves away from the perpendicular when it speeds up.

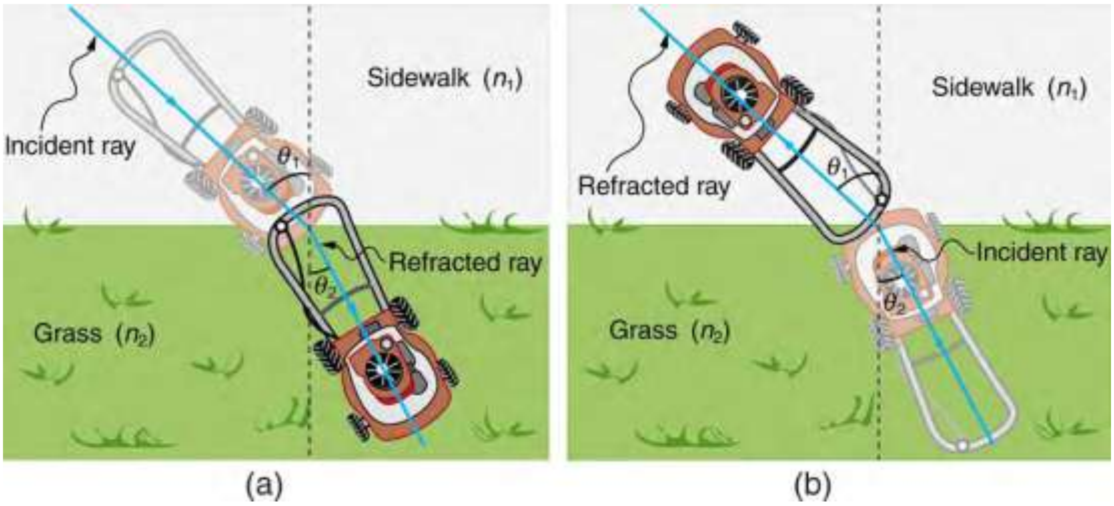


Figure shows: *The change in direction of a light ray depends on how the speed of light changes when it crosses from one medium to another.*

The amount that a light ray changes its direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in angle. The exact mathematical relationship is the law of refraction, or "Snell's Law," which is stated in equation form as

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

Here n_1 and n_2 are the indices of refraction for medium 1 and 2, and θ_1 and θ_2 are the angles between the rays and the perpendicular in medium 1 and 2.

Q\ Find the index of refraction for medium 2 in Figure above(a), assuming medium 1 is air and given the incident angle is 30.0º and the angle of refraction is 22.0º

Solution

The index of refraction for air is taken to be 1 in most cases (and up to four significant figures, it is 1.000). Thus $n_1 = 1.00$ here. From the given information, $\theta_1 = 30.0^\circ$ and $\theta_2 = 22.0^\circ$. With this information, the only unknown in Snell's law is n_2 , so that it can be used to find this unknown.

Snell's law is

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

Rearranging to isolate n_2 gives.

$$n_2 = n_1 \frac{\sin \theta_1}{\sin \theta_2}.$$

Entering known values,

$$\begin{aligned} n_2 &= 1.00 \frac{\sin 30.0^\circ}{\sin 22.0^\circ} = \frac{0.500}{0.375} \\ &= 1.33. \end{aligned}$$

Q\ Suppose that in a situation like that in Example above, light goes from air to diamond and that the incident angle is 30.0º . Calculate the angle of refraction θ_2 in the diamond.

Solution:

Again the index of refraction for air is taken to be $n_1 = 1.00$, and we are given $\theta_1 = 30.0^{\circ}$. The index of refraction for diamond ,finding $n_2 = 2.419$. The only unknown in is θ_2 , which we wish to determine.

Solving Snell's law for sin θ_2 yields

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1.$$

Entering known values,

$$\sin \theta_2 = \frac{1.00}{2.419} \sin 30.0^{\circ} = \left(0.413\right)(0.500) = 0.207.$$

The angle is thus

$$\theta_2 = \sin^{-1} 0.207 = 11.9^{\circ}.$$

<u>Total Internal Reflection</u>

A good-quality mirror may reflect more than 90% of the light that falls on it, absorbing the rest. But it would be useful to have a mirror that reflects all of the light that falls on it. Interestingly, we can produce total reflection using an aspect of refraction.

Consider what happens when a ray of light strikes the surface between two materials, such as is shown in Figure (a). Part of the light crosses the boundary and is refracted; the rest is reflected. If, as shown in the figure, the index of refraction for the second medium is less than for the first, the ray bends away from the perpendicular. (Since $n_1 > n_2$, the angle of refraction is greater than the angle of incidence–that is, $\theta_1 > \theta_2$.) Now imagine what happens as the incident angle is increased. This causes θ_2 to increase also. The largest the angle of refraction θ_2 can be is 90° , as shown in Figure (b).The critical angle θ_c for a combination of materials is defined to be the incident angle θ_1 that produces an angle of refraction of 90° . That is, θ_c is the incident angle for which $\theta_2 = 90^\circ$. If the incident angle θ_1 is greater than the critical angle, as shown in Figure (c), then all of the light is reflected back into medium 1, a condition called total internal reflection.

Critical Angle

The incident angle θ_1 that produces an angle of refraction of 90° is called the critical angle, θ_c .

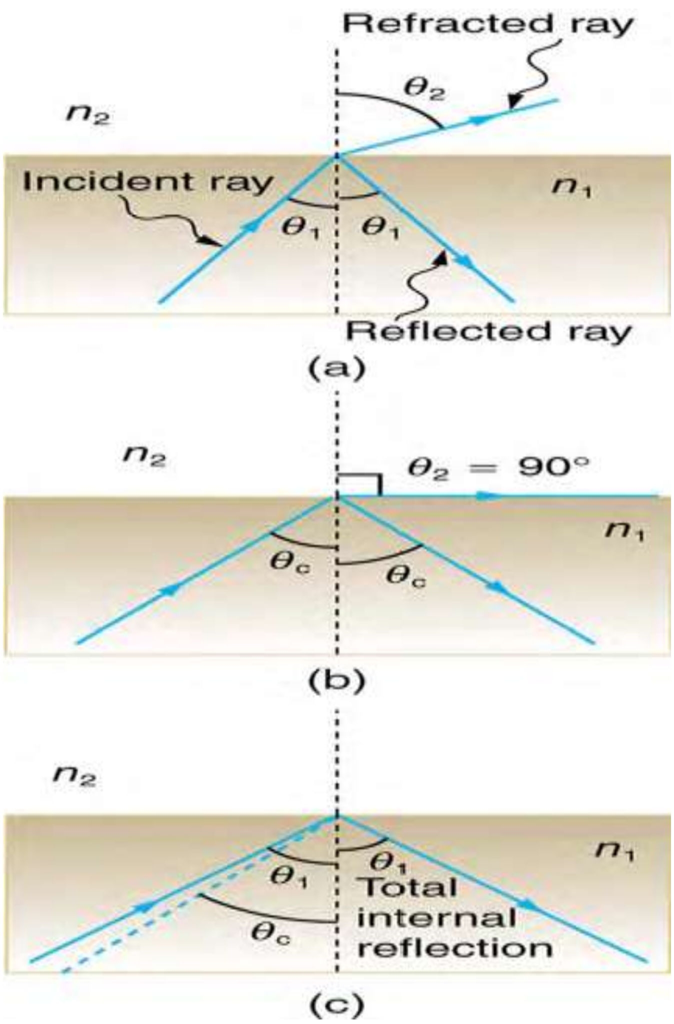


Figure shows: (a) A ray of light crosses a boundary where the speed of light increases and the index of refraction decreases. That is, $n_2 < n_1$. The ray bends away from the perpendicular. (b) The critical angle θ_c is the one for which the angle of refraction is . (c) Total internal reflection occurs when the incident angle is greater than the critical angle.

Snell's law states the relationship between angles and indices of refraction. It is given by

$$\mathbf{n_1 \sin \theta_1 = n_2 \sin \theta_2.}$$

When the incident angle equals the critical angle ($\theta_1 = \theta_c$), the angle of refraction is 90° ($\theta_2 = 90^\circ$). Noting that $\sin 90^\circ = 1$, Snell's law in this case becomes

$$n_1 \sin \theta_1 = n_2.$$

The critical angle θ_c for a given combination of materials is thus

$$\theta_c = \sin^{-1}(n_2 / n_1) \text{ for } n_1 > n_2.$$

for $n_1 > n_2$.

Total internal reflection occurs for any incident angle greater than the critical angle θ_c , and it can only occur when the second medium has an index of refraction less than the first. Note the above equation is written for a light ray that travels in medium 1 and reflects from medium 2, as shown in the figure.

Q\ What is the critical angle for light traveling in a polystyrene (a type of plastic) pipe surrounded by air?

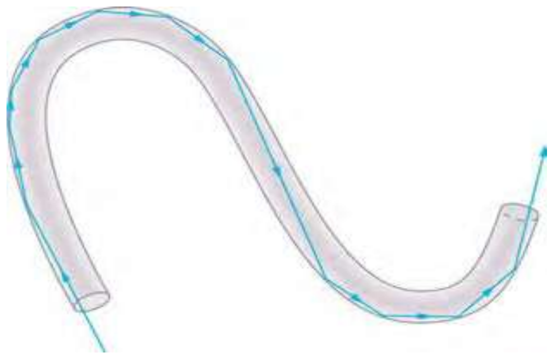


Figure shows: *Light entering a thin fiber may strike the inside surface at large or grazing angles and is completely reflected if these angles exceed the critical angle. Such rays continue down the fiber, even following it around corners, since the angles of reflection and incidence remain large.*

Solution :

The index of refraction for polystyrene is found to be 1.49 in Figure, and the index of refraction of air can be taken to be 1.00, as before. Thus, the condition that the second medium (air) has an index of refraction less than the first (plastic) is satisfied, and the equation

$$\theta_c = \sin^{-1}(n_2 / n_1)$$

can be used to find the critical angle θ_c . Here, then, n_2 = 1.00 and n_1 = 1.49.

The critical angle is given by

$$\theta_c = \sin^{-1}(n_2 / n_1).$$

Substituting the identified values gives

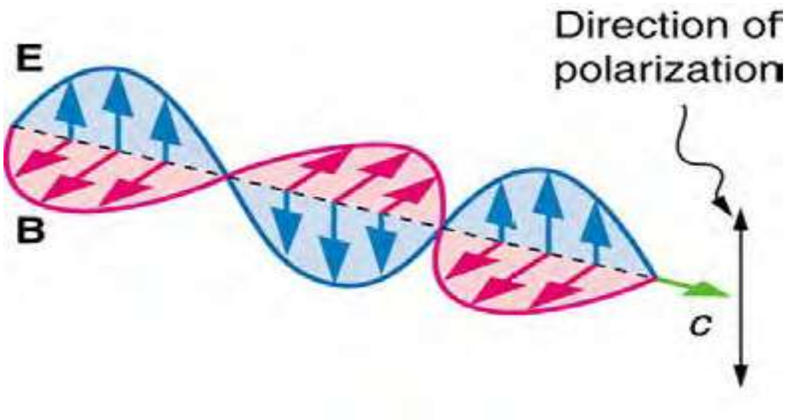
$$\theta_c = \sin^{-1}(1.00 / 1.49) = \sin^{-1}(0.671)$$
$$42.2^{\circ}.$$

Polarization

Light is one type of electromagnetic (EM) wave. As noted earlier, EM waves are *transverse waves* consisting of varying electric and magnetic fields that oscillate perpendicular to the direction of propagation (Figure 1). There are specific directions for the oscillations of the electric and magnetic fields. **Polarization** is the attribute that a wave's oscillations have a definite direction relative to the direction of propagation of the wave.

(This is not the same type of polarization as that discussed for the separation of charges.) Waves having such a direction are said to be **polarized**.

For an EM wave, we define the **direction of polarization** to be the direction parallel to the electric field. Thus we can think of the electric field arrows as showing the direction of polarization, as in **Figure**.



Figure(1): An EM wave, such as light, is a transverse wave. The electric and magnetic fields are perpendicular to the direction of propagation.

Polarized Light

The electromagnetic waves emitted by a television station all have the same polarization, but the electromagnetic waves emitted by any common source of light (such as the Sun or a bulb) are polarized randomly, or unpolarized (the two terms mean the same thing).That is, the electric field at any given point is always perpendicular to the direction of travel of the waves but changes directions randomly. Thus, if we try to represent a head-on view of the oscillations over some time period, we do not have a simple drawing with a single double arrow like that of Fig. (2b); instead we have a mess of double arrows like that in Fig. (2a).

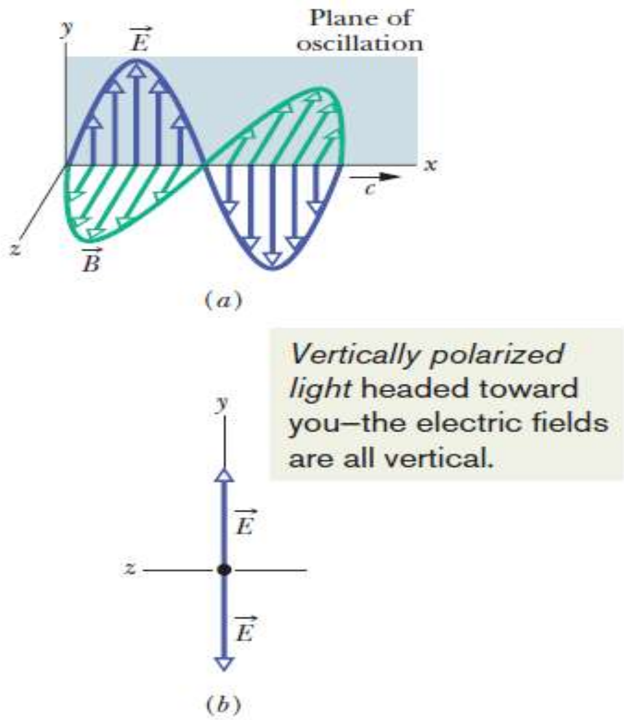


Figure (2): (a) The plane of oscillation of a polarized electromagnetic wave. (b) To represent the polarization, we view the plane of oscillation head-on and indicate the directions of the oscillating electric field with a double arrow.

In principle, we can simplify the mess by resolving each electric field of Fig. (3a) into y and z components. Then as the wave travels past us, the net y component oscillates parallel to the y axis and the net z component oscillates parallel to the z axis. We can then represent the unpolarized light with a pair of double arrows as shown in Fig. (3b).The double arrow along the y axis represents the oscillations of the net y component of the electric field. The double arrow along the z axis represents the oscillations of the net z component of the electric field. In doing all this, we effectively change unpolarized light into the superposition of two polarized waves whose planes of oscillation are perpendicular to each other—one plane contains the y axis and the other contains the z axis. One reason to make this change is that drawing Fig. 33-10b is a lot easier than drawing Fig. (3a). We can draw similar figures to represent light that is partially polarized (its field oscillations are not completely random as in Fig. (3a), nor are they parallel to a single axis as in Fig. (2b). For this situation, we draw one of the double arrows in a perpendicular pair of double arrows longer than the other one.

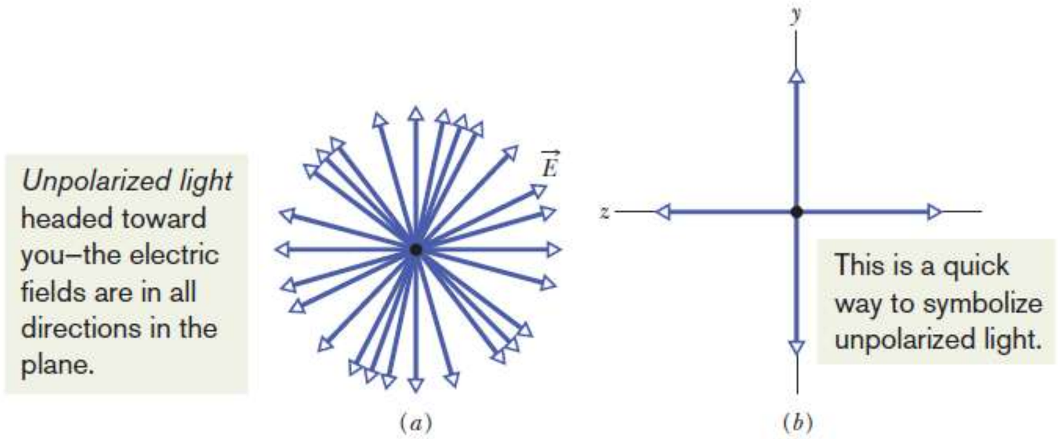


Figure (3): (a) Unpolarized light consists of waves with randomly directed electric fields. Here the waves are all traveling along the same axis, directly out of the page, and all have the same amplitude E. (b) A second way of representing unpolarized light—the light is the superposition of two polarized waves whose planes of oscillation are perpendicular to each other.

We can transform unpolarized visible light into polarized light by sending it through a polarizing sheet, as is shown in Fig.. Such sheets, commercially known as Polaroids or Polaroid filters, were invented in 1932 by Edwin Land while he was an undergraduate student. A polarizing sheet consists of certain long molecules embedded in plastic.

When the sheet is manufactured, it is stretched to align the molecules in parallel rows, like rows in a plowed field. When light is then sent through the sheet, electric field components along one direction pass through the sheet, while components perpendicular to that direction are absorbed by the molecules and disappear.

Thus, the electric field of the light emerging from the sheet consists of only the components that are parallel to the polarizing direction of the sheet; hence the light is polarized in that direction. In Fig (4), the vertical electric field components are transmitted by the sheet; the horizontal components are absorbed. The transmitted waves are then vertically polarized.

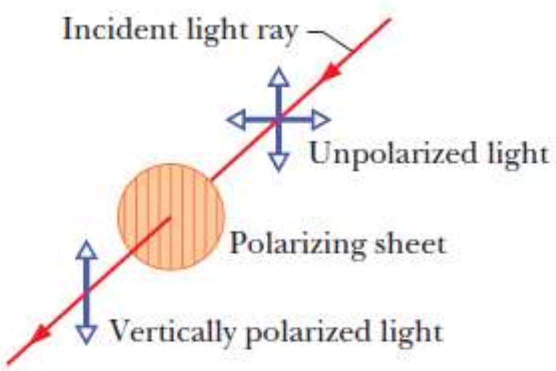


Figure (4): Unpolarized light becomes polarized when it is sent through a polarizing sheet. Its direction of polarization is then parallel to the polarizing direction of the sheet, which is represented here by the vertical lines drawn in the sheet.

Intensity of Transmitted Polarized Light

We now consider the intensity of light transmitted by a polarizing sheet. We start with unpolarized light, whose electric field oscillations we can resolve into y and z components as represented in Fig. (3b). Further, we can arrange for the y axis to be parallel to the polarizing direction of the sheet. Then only the y components of the light's electric field are passed by the sheet; the z components are absorbed. As suggested by Fig. (3b), if the original waves are randomly oriented, the sum of the y components and the sum of the z components are equal. When the z components are absorbed, half the intensity I_0 of the original light is lost. The intensity I

of the emerging polarized light is then

$$I = \frac{1}{2} I_0.$$

Let us call this the one-half rule; we can use it only when the light reaching a polarizing sheet is unpolarized. Suppose now that the light reaching a polarizing sheet is already polarized. Figure (5) shows a polarizing sheet in the plane of the page and the electric field $\vec{E}$of such a polarized light wave traveling toward the sheet (and thus prior to any absorption).We can resolve $\vec{E}$ into two components relative to the polarizing direction of the sheet: parallel component E_y is transmitted by the sheet, and perpendicular component E_z is absorbed. Since θ is the angle between $\vec{E}$ and the polarizing direction of the sheet, the transmitted parallel component is

$$E_y = E \cos \theta.$$

In our present case then, the intensity I of the emerging wave is proportional to and the intensity I0 of the original wave is proportional to E_y^2. Hence, we can write I/I_0 " $\cos^2 \theta$, or

$$I = I_0 \cos^2 \theta.$$

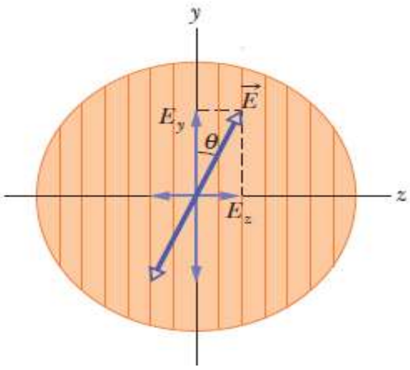


Figure (5): Polarized light approaching a polarizing sheet.

The travel of light through a surface (or interface) that separates two media is called refraction, and the light is said to be refracted. Unless an incident beam of light is perpendicular to the surface, refraction changes the light's direction of travel. For this reason, the beam is said to be "bent" by the refraction. Note in Fig. (6a) that the bending occurs only at the surface; within the water, the light travels in a straight line.

In Figure (6b), the beams of light in the photograph are represented with an incident ray, a reflected ray, and a refracted ray (and wavefronts). Each ray is oriented with respect to a line, called the normal, that is perpendicular to the surface at the point of reflection and refraction. In Fig. (6b), the angle of incidence is θ_1, the angle of reflection is , and the angle of refraction isθ_2, all measured relative to the normal. The plane containing the incident ray and the normal is the plane of incidence, which is in the plane of the page in Fig (6b).

Experiment shows that reflection and refraction are governed by two laws: Law of reflection: A reflected ray lies in the plane of incidence and has an angle of reflection equal to the angle of incidence (both relative to the normal). In Fig. (6b), this means that

$$\theta_1' = \theta_1 \qquad \text{(reflection).}$$

(We shall now usually drop the prime on the angle of reflection.)

Law of refraction: A refracted ray lies in the plane of incidence and has an angle of refraction θ_2 that is related to the angle of incidence θ_1 by

$$n_2 \sin\theta_2 = n_1 \sin\theta_1 \qquad \text{(refraction).}$$

Here each of the symbols n_1 and n_2 is a dimensionless constant, called the index of refraction, that is associated with a medium involved in the refraction. We derive this equation, called Snell's law

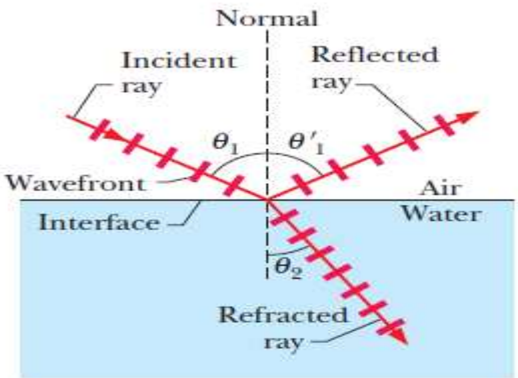


Figure (6):(Continued) (b) A ray representation of (a).The angles of incidence (θ1), reflection , and refraction (θ2) are marked.

Q\ What angle is needed between the direction of polarized light and the axis of a polarizing filter to reduce its intensity by 90.0% ?

Solution

When the intensity is reduced by 90.0% , it is 10.0% or 0.100 times its original value. That is, $I = 0.100I_0$. Using this information, the equation $I = I_0 \cos_2 \theta$ can be used to solve for the needed angle.

Solving the equation $I = I_0 \cos_2 \theta$ for $\cos\theta$ and substituting with the relationship between I and I_0 gives

$$\cos\theta = \sqrt{\frac{I}{I_0}} = \sqrt{\frac{0.100 I_0}{I_0}} = 0.3162.$$

$$\theta = \cos^{-1} 0.3162 = 71.6°.$$

Polarization by Reflection

The part of the light that is not reflected is refracted, the amount of polarization depends on the indices of refraction of the media involved. It can be shown that reflected light is completely polarized at a angle of reflection θ_b , given by

$$\tan\theta_b = \frac{n_2}{n_1},$$

where n_1 is the medium in which the incident and reflected light travel and n_2 is the index of refraction of the medium that forms the interface that reflects the light. This equation is known as Brewster's law, and θ_b is known as Brewster's angle, named after the 19th-century Scottish physicist who discovered them.

Q\ **At what angle will light traveling in air be completely polarized horizontally when reflected from water? (b) From glass?**

Solution (a):

All we need to solve these problems are indices of refraction. Air has n_1=1.00, water has n_2=1.333, and crow glass has $\acute{n}_{2=1.520}$. This equation $tan\theta_{b=\frac{n_2}{n_1}}$ can be directly applied to find θ_b in each case.

$$\tan\theta_{b=\frac{n_2}{n_1}}$$

$$\tan\theta_{b=\frac{n_2}{n_1}} = \frac{1.333}{1.00} = 1.333$$

$$\theta_{b=\tan^{-1}1.333=53.1}$$

Solution (b):

Similarly for crow glass and air,

$$\tan\theta'_{b=\frac{n'_2}{n_1}} = \frac{1.520}{1.00} = 1.520$$

$$\theta'_{b=\tan^{-1}1.52=56.7^0}$$

Microscopy Enhanced by the Wave Characteristics of Light

Microscopy use to observe small details is limited by the wave nature of light. Ultraviolet (UV) microscopes have been constructed with special lenses that transmit UV rays and utilize photographic or electronic techniques to record images. The shorter UV wavelengths allow somewhat greater detail to be observed, but drawbacks, such as the hazard of UV to living tissue and the need for special detection devices and lenses (which tend to be dispersive in the UV), severely limit the use of UV microscopes. Elsewhere, we will explore practical uses of very short wavelength EM waves, such as x rays, and other short-wavelength probes, such as electrons in electron microscopes, to detect small details.

Another difficulty in microscopy is the fact that many microscopic objects do not absorb much of the light passing through them. The lack of contrast makes image interpretation very difficult. Contrast is the difference in intensity between objects and the background on which they are observed.

Stains (such as dyes, fluorophores, etc.) are commonly employed to enhance contrast, but these tend to be application specific. More general wave interference techniques can be used to produce contrast.

Interference microscopes

Enhance contrast between objects and background by superimposing a reference beam of light upon the light emerging from the sample. Since light from the background and objects differ in phase, there will be different amounts of constructive and destructive interference, producing the desired contrast in final intensity. Figure shows schematically how this is done. Parallel rays of light from a source are split into two beams by a half-silvered mirror. These beams are called the object and reference beams. Each beam passes through identical optical elements, except that the object beam passes through the object we wish to observe microscopically. The light beams are recombined by another half-silvered mirror and interfere. Since the light rays passing through different parts of the object have different phases, interference will be significantly different and, hence, have greater contrast between them.

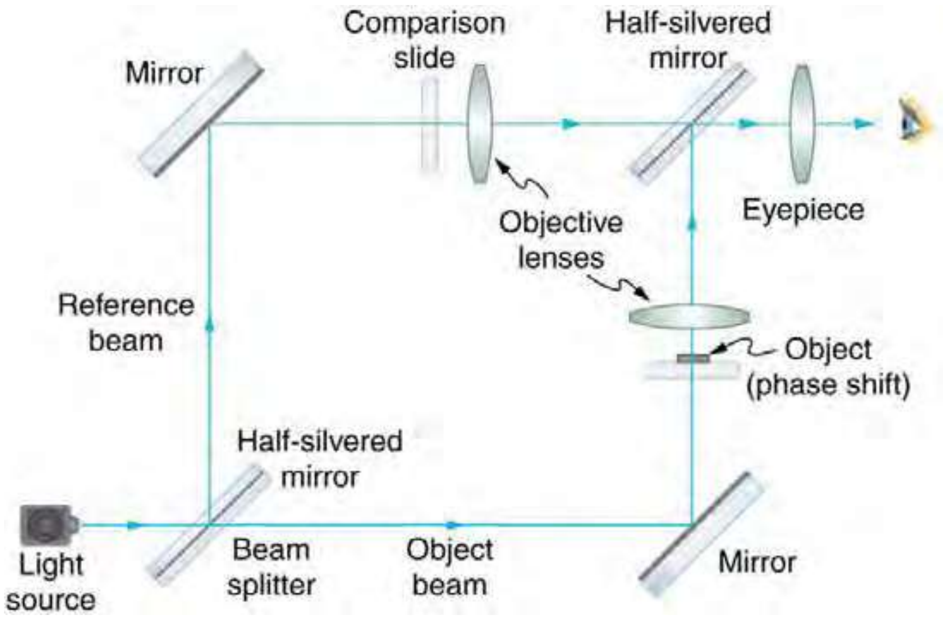
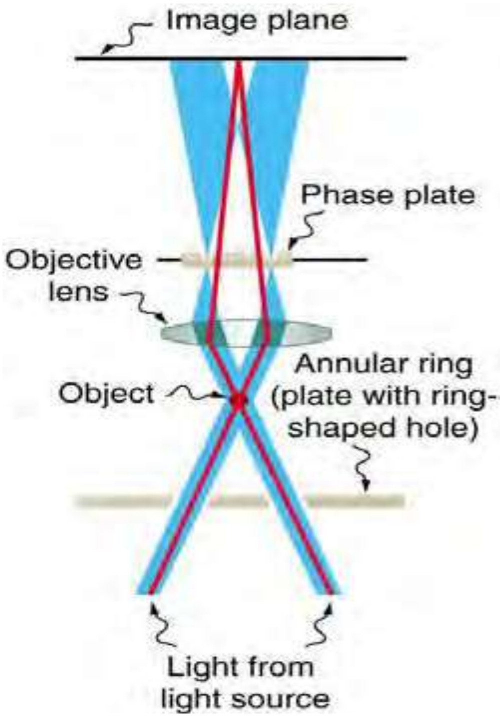


Figure shows: An interference microscope utilizes interference between the reference and object beam to enhance contrast. The two beams are split by a half-silvered mirror; the object beam is sent through the object, and the reference beam is sent through otherwise identical optical elements. The beams are recombined by another half-silvered mirror, and the interference depends on the various phases emerging from different parts of the object, enhancing contrast.

Another type of microscope utilizing wave interference and differences in phases to enhance contrast is called the phase-contrast microscope.

While its principle is the same as the interference microscope, the phase-contrast microscope is simpler to use and construct. Its impact (and the principle upon which it is based) was so important that its developer, the Dutch physicist Frits Zernike (1888–1966), was awarded the Nobel Prize in 1953. Figure shows the basic construction of a phase-contrast microscope. Phase differences between light passing through the object and background are produced by passing the rays through different parts of a phase plate (so called because it shifts the phase of the light passing through it). These two light rays are superimposed in the image plane, producing contrast due to their interference.



Simplified construction of a phase-contrast microscope. Phase differences between light passing through the object and background are produced by passing the rays through different parts of a phase plate. The light rays are superimposed in the image plane, producing contrast due to their interference.

polarization microscope

Also enhances contrast by utilizing a wave characteristic of light. Polarization microscopes are useful for objects that are optically active or birefringent, particularly if those characteristics vary from place to place in the object. Polarized light is sent through the object and then observed through a polarizing filter that is perpendicular to the original polarization direction. Nearly transparent objects can then appear with strong color and in high contrast. Many polarization effects are wavelength dependent, producing color in the processed image. Contrast results from the action of the polarizing filter in passing only components parallel to its axis.

Confocal microscopes

which use the extended focal region to obtain three-dimensional images rather than two-dimensional images. Here, only a single plane or region of focus is identified; out-of-focus regions above and below this plane are subtracted out by a computer so the image quality is much better. This type of microscope makes use of fluorescence, where a laser provides the excitation light. Laser light passing through a tiny aperture called a pinhole forms an extended focal region within the specimen. The reflected light passes through the objective lens to a second pinhole and the photomultiplier detector, see Figure. The second pinhole is the key here and serves to block much of the light from points that are not at the focal point of the objective lens. The pinhole is conjugate (coupled) to the focal point of the lens. The second pinhole and detector are scanned, allowing reflected light from a small region or section of the extended focal region to be imaged at any one time. The out-of-focus light is excluded. Each image is stored in a computer, and a full scanned image is generated in a short time. Live cell processes can also be imaged at adequate scanning speeds allowing the imaging of three-dimensional microscopic movement. Confocal microscopy enhances images over conventional optical microscopy, especially for thicker specimens, and so has become quite popular.

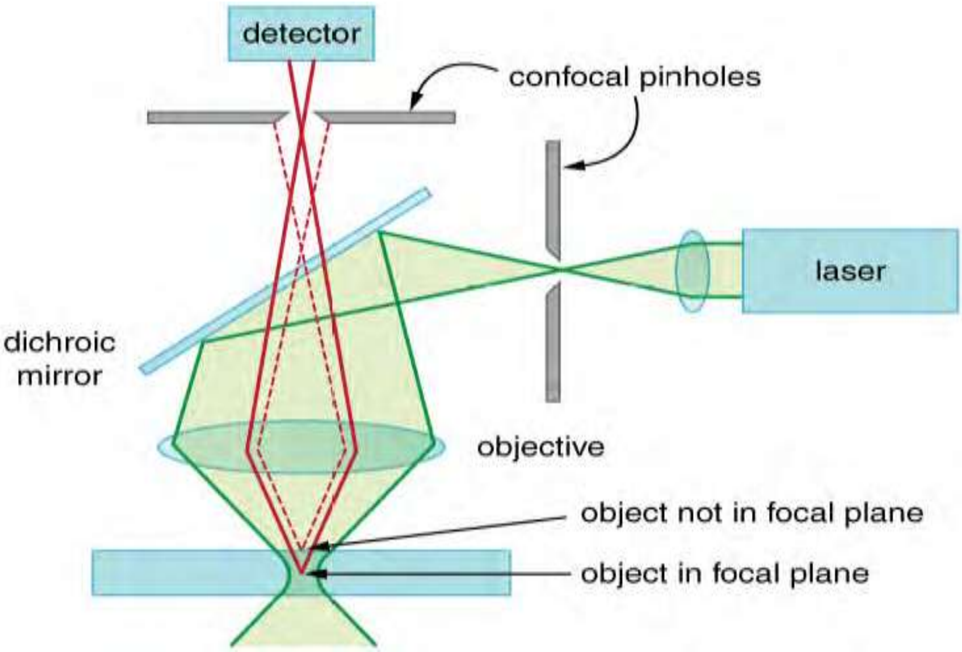


Figure shows: A confocal microscope provides three-dimensional images using pinholes and the extended depth of focus as described by wave optics.

Electric Current

Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, electric current I is defined to be

$$I = \frac{\Delta Q}{\Delta t},$$

where ΔQ is the amount of charge passing through a given area in time Δt. (As in previous chapters, initial time is often taken to be zero, in which case Δt = t .) (See Figure). The SI unit for current is the ampere (A), named for the French physicist André-Marie Ampère (1775–1836). Since I = ΔQ / Δt , we see that an ampere is one coulomb per second: 1 A = 1 C/s Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.

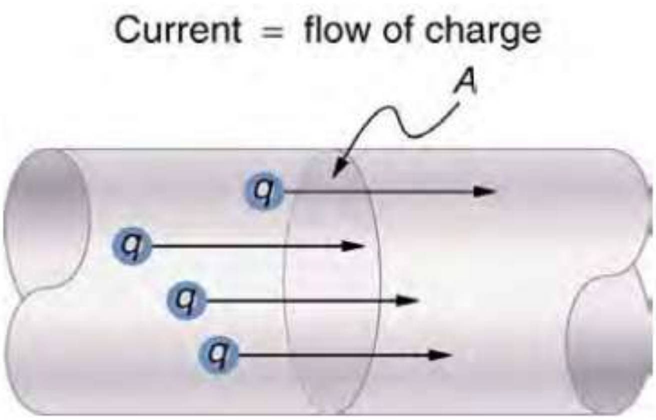


Figure shows: The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.

Q\ (a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

Solution for (a)

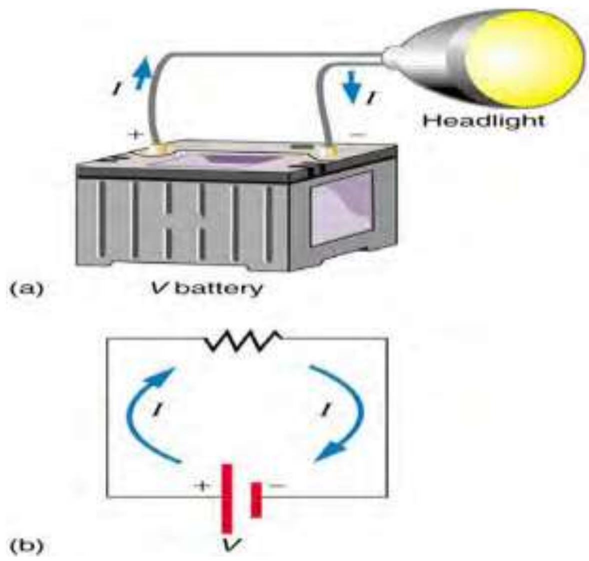
Entering the given values for charge and time into the definition of current gives

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} = \frac{720\ \text{C}}{4.00\ \text{s}} = 180\ \text{C/s} \\ &= 180\ \text{A.} \end{aligned}$$

Solution for (b)

Solving the relationship I = ΔQ / Δt for time Δt , and entering the known values for charge and current gives

$$\begin{aligned} \Delta t &= \frac{\Delta Q}{I} = \frac{1.00\ \text{C}}{0.300\times 10^{-3}\ \text{C/s}} \\ &= 3.33\times 10^{3}\ \text{s.} \end{aligned}$$



Note that the direction of current flow in Figure above is from positive to negative. The direction of conventional current is the direction that positive charge would flow. Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons.

Q\ If the 0.300-mA current through the calculator mentioned in the Example above example is carried by electrons, how many electrons per second pass through it?

Solution:

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite, $I_{electrons} = -0.300 \times 10^{-3}$ C/s .Since each electron (e–) has a charge of -1.60×10^{-19} C , we can convert the current in coulombs per second to electrons per second.

$$I_{\text{electrons}} = \frac{\Delta Q_{\text{electrons}}}{\Delta t} = \frac{-0.300\times10^{-3}\ \text{C}}{\text{s}}.$$

We divide this by the charge per electron, so that

$$\frac{e^-}{\text{s}} = \frac{-0.300\times10^{-3}\ \text{C}}{\text{s}} \times \frac{1\ e^-}{-1.60\times10^{-19}\ \text{C}}$$

$$= 1.88\times10^{15}\ \frac{e^-}{\text{s}}.$$

Ohm's Law

The current that flows through most substances is directly proportional to the voltage *V* applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

$$I \propto V.$$

This important relationship is known as **Ohm's law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect.

This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn't always occur.

Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called **resistance** *R* . Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

$$I \propto \frac{1}{R}.$$

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

$$I = \frac{V}{R}.$$

This relationship is also called Ohm's law. Ohm's law in this form really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid. The many substances for which Ohm's law holds are called ohmic. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I . An object that has simple resistance is called a resistor, even if its resistance is small. The unit for resistance is an ohm and is given the symbol Ω (upper case Greek omega). Rearranging I = V/R gives R = V/I , and so the units of resistance are 1 ohm = 1 volt per ampere:

$$1\ \Omega = 1\frac{V}{A}.$$

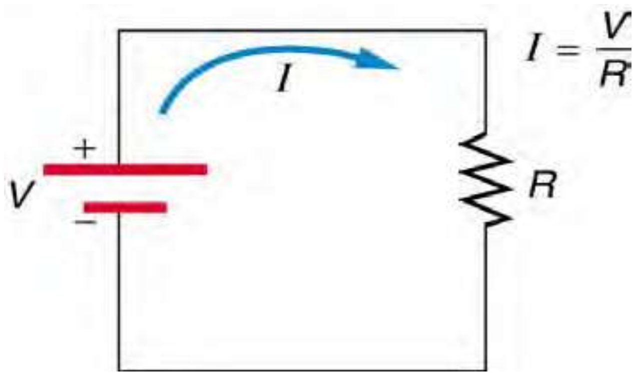


Figure shows the schematic for a simple circuit. A simple circuit has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R .

Q\ shows the schematic for a simple circuit. A simple circuit has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R .

solution:

Rearranging I = V/R and substituting known values gives

$$R = \frac{V}{I} = \frac{12.0\ \text{V}}{2.50\ \text{A}} = 4.80\ \Omega.$$

Alternating Current versus Direct Current

Alternating Current

Most of the examples dealt with so far, and particularly those utilizing batteries, have constant voltage sources. Once the current is established, it is thus also a constant. Direct current (DC) is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit. Most well-known applications, however, use a time-varying voltage source. Alternating current (AC) is the flow of electric charge that periodically reverses direction. If the source varies periodically, particularly sinusoidally, the circuit is known as an alternating current circuit. Examples include the commercial and residential power that serves so many of our needs. Figure shows graphs of voltage and current versus time for typical DC and AC power. The AC voltages and frequencies commonly used in homes and businesses vary around the world.

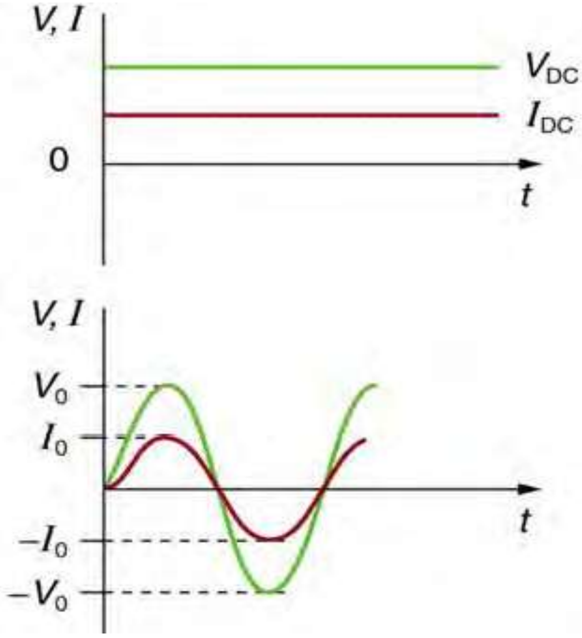


Figure shows:(a) DC voltage and current are constant in time, once the current is stablished. (b) A graph of voltage and current versus time for 60-Hz AC power. The voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of AC sources differ greatly.

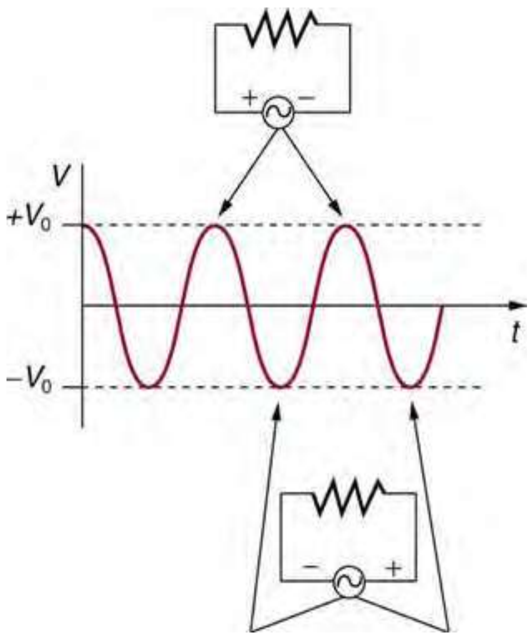


Figure shows: The potential difference V between the terminals of an AC voltage source fluctuates as shown. The mathematical expression for V is given by $V = V_0 \sin 2\pi ft$.

Figure shows a schematic of a simple circuit with an AC voltage source. The voltage between the terminals fluctuates as shown, with the AC voltage given by

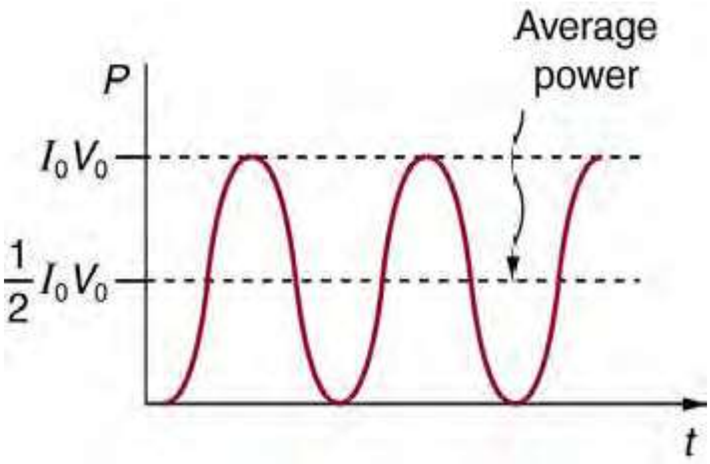
$$V = V_0 \sin 2\pi ft,$$

where V is the voltage at time t , V0 is the peak voltage, and f is the frequency in hertz. For this simple resistance circuit, I = V/R , and so the AC current is

$$I = I_0 \sin 2\pi ft,$$

where I is the current at time t , and $I_0 = V_0/R$ is the peak current. For this example, the voltage and current are said to be in phase, as seen in figure. Current in the resistor alternates back and forth just like the driving voltage, since I = V/R . If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you have your hand back and forth between your face and a fluorescent light, you will see a stroboscopic effect evidencing AC. The fact that the light output fluctuates means that the power is fluctuating. The power supplied is P = IV . Using the expressions for I and V above, we see that the time dependence of power is

$P = I_0V_0 \sin^2 2\pi ft$, as shown in Figure



AC power as a function of time. Since the voltage and current are in phase here, their product is non-negative and fluctuates between zero and I_0 V_0 . Average power is (1 / 2)I_0 V_0

We are most often concerned with average power rather than its fluctuations–that 60-W light bulb in your desk lamp has an average power consumption of 60 W, for example. As illustrated in Figure, the average power Pave is

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0.$$

This is evident from the graph, since the areas above and below the (1 / 2)I_0 V_0 line are equal, but it can also be proven using trigonometric identities. Similarly, we define an average or rms current I_{rms} and average or rms voltage V_{rms} to be, respectively,

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$$

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}.$$

where rms stands for root mean square, a particular kind of average. In general, to obtain a root mean square, the particular quantity is squared, its mean (or average) is found, and the square root is taken. This is useful for AC, since the average value is zero. Now,

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}},$$

$$P_{\text{ave}} = \frac{I_0}{\sqrt{2}} \cdot \frac{V_0}{\sqrt{2}} = \frac{1}{2} I_0 V_0,$$

as stated above. It is standard practice to quote I_{rms} , V_{rms} , and Pave rather than the peak values. For example, most household electricity is 120 V AC, which means that V_{rms} is 120 V. The common 10-A circuit breaker will interrupt a sustained I_{rms} greater than 10 A. Your 1.0-kW microwave oven consumes Pave = 1.0 kW , and so on. You can think of these rms and average values as the equivalent DC values for a simple resistive circuit.

To summarize, when dealing with AC, Ohm's law and the equations for power are completely analogous to those for DC, but rms and average values are used for AC. Thus, for AC, Ohm's law is written

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}.$$

The various expressions for AC power P_{ave} are

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}},$$

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R},$$

$$P_{\text{ave}} = I_{\text{rms}}^2 R.$$

Q\ What is the value of the peak voltage for 120-V AC power? (b) What is the peak power consumption rate of a 60.0-W AC light bulb?

Solution for (a)

Solving the equation $V_{rms}=\frac{V_0}{\sqrt{2}}$ for the peak voltage V_0 and substituting the known value for V_{rms} gives

$$V_0 = \sqrt{2}V_{\text{rms}} = 1.414(120\text{ V}) = 170\text{ V}.$$

Solution for (b)

Peak power is peak current times peak voltage. Thus,

$$P_0 = I_0V_0 = 2\left(\frac{1}{2}I_0V_0\right) = 2P_{\text{ave}}.$$

$$P_0 = 2(60.0\text{ W}) = 120\text{ W}.$$

Q\ What current is needed to transmit 100 MW of power at 200 kV? (b) What is the power dissipated by the transmission lines if they have a resistance of 1.00 Ω ? (c) What percentage of the power is lost in the transmission lines?

Solution (a):

We are given Pave = 100 MW , V_{rms} = 200 kV , and the resistance of the lines is R = 1.00 Ω . Using these givens, we can find the current flowing (from $P = IV$) and then the power dissipated in the lines ($P = I^2R$), and we take the ratio to the total power transmitted.

To find the current, we rearrange the relationship Pave = $I_{\text{rms}}V_{\text{rms}}$ and substitute known values. This gives

$$I_{\text{rms}} = \frac{P_{\text{ave}}}{V_{\text{rms}}} = \frac{100\times10^6\text{ W}}{200\times10^3\text{ V}} = 500\text{ A}.$$

Solution (b):

Knowing the current and given the resistance of the lines, the power dissipated in them is found from Pave $= I_{rms}^2 R$. Substituting the known values gives

$$P_{\text{ave}} = I_{\text{rms}}^2 R = (500\ \text{A})^2 (1.00\ \ \Omega\ ) = 250\ \text{kW}.$$

Solution (c):

The percent loss is the ratio of this lost power to the total or input power, multiplied by 100:

$$\%\ \text{loss} = \frac{250\ \text{kW}}{100\ \text{MW}} \times 100 = 0.250\ \%.$$

Introduction to Radioactivity and Nuclear Physics

Nuclear Radioactivity

In 1896, the French physicist Antoine Henri Becquerel (1852–1908) accidentally found that a uranium-rich mineral called pitchblende emits invisible, penetrating rays that can darken a photographic plate enclosed in an opaque envelope. The rays therefore carry energy; but amazingly, the pitchblende emits them continuously without any energy input. This is an apparent violation of the law of conservation of energy, one that we now understand is due to the conversion of a small amount of mass into energy, as related in Einstein's famous equation $E = mc^2$. It was soon evident that Becquerel's rays originate in the nuclei of the atoms and have other unique characteristics. The emission of these rays is called **nuclear radioactivity** or simply **radioactivity**. The rays themselves are called **nuclear radiation**. A nucleus that spontaneously destroys part of its mass to emit radiation is said to **decay** (a term also used to describe the emission of radiation by atoms in excited states). A substance or object that emits nuclear radiation is said to be **radioactive**.

<u>Alpha, Beta, and Gamma</u>

There are three types were distinguished and named alpha (α) , beta(β) , and gamma (γ) , because, like x-rays, their identities were initially unknown. Figure shows what happens if the rays are passed through a magnetic field. The γ s are unaffected, while the γ s and β s are deflected in opposite directions, indicating the α s are positive, the β s negative, and the γ s uncharged. Rutherford used both magnetic and electric fields to show that α s have a positive charge twice the magnitude of an electron, or +2 | qe | . In the process, he found the α s charge to mass ratio to be several thousand times smaller than the electron's. Later on, Rutherford collected α s from a radioactive source and passed an electric discharge through them, obtaining the spectrum of recently discovered helium gas. Among many important discoveries made by Rutherford and his collaborators was the proof that α radiation is the emission of a helium nucleus. Rutherford won the Nobel Prize in chemistry in 1908 for his early work. He continued to make important contributions until his death in 1934.

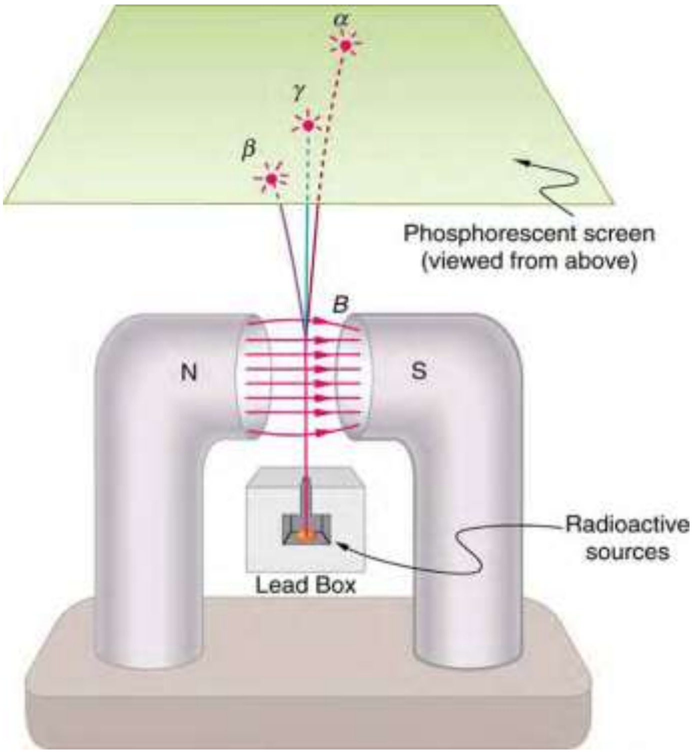


Figure shows: Alpha, beta, and gamma rays are passed through a magnetic field on the way to a phosphorescent screen.

Ionization and Range

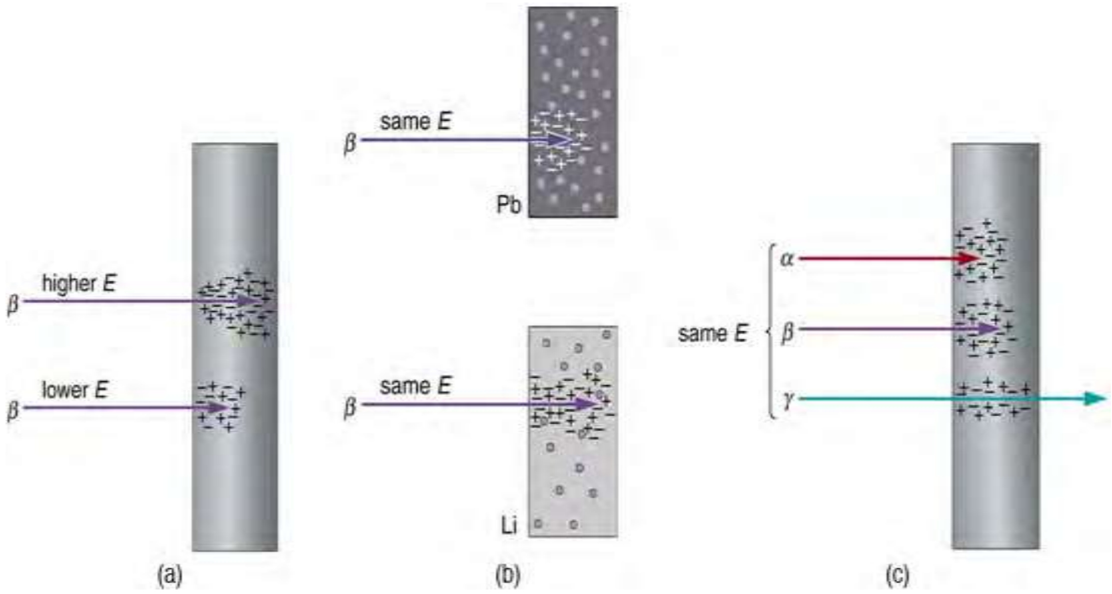
Two of the most important characteristics of α , β , and γ rays were recognized very early. All three types of nuclear radiation produce ionization in materials, but they penetrate different distances in materials−

that is, they have different ranges. Let us examine why they have these characteristics and what are some of the consequences.

Like x rays, nuclear radiation in the form of α s, β s, and γ s has enough energy per event to ionize atoms and molecules in any material. The energy emitted in various nuclear decays ranges from a few keV to more than 10 MeV , while only a few eV are needed to produce ionization.

The effects of x rays and nuclear radiation on biological tissues and other materials, such as solid state electronics, are directly related to the ionization they produce. All of them, for example, can damage electronics or kill cancer cells. In addition, methods for detecting x rays and nuclear radiation are based on ionization, directly or indirectly. All of them can ionize the air between the plates of a capacitor, for example, causing it to discharge. This is the basis of inexpensive personal radiation monitors, such as pictured in Figure. Apart from α , β , and γ , there are other forms of nuclear radiation as well, and these also produce ionization with similar effects. We define ionizing radiation as any form of radiation that produces ionization whether nuclear in origin or not, since the effects and detection of the radiation are related to ionization.

Figure shows: These dosimeters (literally, dose meters) are personal radiation monitors that detect the amount of radiation by the discharge of a rechargeable internal capacitor. The *amount of discharge is related to the amount of ionizing radiation encountered, a measurement of dose. One dosimeter is shown in the charger. Its scale is read through an eyepiece on the top.*



The range of radiation is defined to be the distance it can travel through a material. Range is related to several factors, including the energy of the radiation, the material encountered, and the type of radiation (see Figure). The higher the energy, the greater the range, all other factors being the same. This makes good sense, since radiation loses its energy in materials primarily by producing ionization in them, and each ionization of an atom or a molecule requires energy that is removed from the radiation. The amount of ionization is, thus, directly proportional to the energy of the particle of radiation, as is its range.

Substructure of the Nucleus

What is inside the nucleus? Why are some nuclei stable while others decay?

We have already identified protons as the particles that carry positive charge in the nuclei. However, there are actually two types of particles in the nuclei–the proton and the neutron, referred to collectively as nucleons, the constituents of nuclei. As its name implies, the neutron is a neutral particle (q = 0) that has nearly the same mass and intrinsic spin as the proton.

Note how close the proton and neutron masses are, but the neutron is slightly more massive once you look past the third digit. Both nucleons are much more massive than an electron. In fact, $m_p = 1836m_e$ (as noted in Medical Applications of Nuclear Physics and $m_n = 1839m_e$

Table also gives masses in terms of mass units that are more convenient than kilograms on the atomic and nuclear scale. The first of these is the unified atomic mass unit (u), defined as

$$1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg.}$$

This unit is defined so that a neutral carbon ^{12}C atom has a mass of exactly 12 u. Masses are also expressed in units of MeV/c^2 . These units are very convenient when considering the conversion of mass into energy (and vice versa), as is so prominent in nuclear processes. Using $E = mc^2$ and units of m in MeV/c^2 , we find that c^2 cancels and E comes out conveniently in MeV. For example, if the rest mass of a proton is converted entirely into energy, then

$$E = mc^2 = (938.27 \text{ MeV}/c^2)c^2 = 938.27 \text{ MeV.}$$

It is useful to note that 1 u of mass converted to energy produces 931.5 MeV, or $1 \text{ u} = 931.5 \text{ MeV}/c^2$

All properties of a nucleus are determined by the number of protons and neutrons it has. A specific combination of protons and neutrons is called a nuclide and is a unique nucleus. The following notation is used to represent a particular nuclide:

$$^{A}_{Z}X_{N},$$

where the symbols A , X , Z , and N are defined as follows: The number of protons in a nucleus is the atomic number Z , as defined in Medical Applications of Nuclear Physics. X is the symbol for the element, such as Ca for calcium. However, once Z is known, the element is known; hence, Z and X are redundant. For example, Z = 20 is always calcium, and calcium always has Z = 20 . N is the number of neutrons in a nucleus. In the notation for a nuclide, the subscript N is usually omitted. The symbol A is defined as the number of nucleons or the total number of protons and neutrons,

$$\mathbf{A = N + Z,}$$

where A is also called the mass number. This name for A is logical; the mass of an atom is nearly equal to the mass of its nucleus, since electrons have so little mass. The mass of the nucleus turns out to be nearly equal to the sum of the masses of the protons and neutrons in it, which is proportional to A . In this context, it is particularly convenient to express masses in units of u. Both protons and neutrons have masses close to 1 u, and so the mass of an atom is close to A u. For example, in an oxygen nucleus with eight protons and eight neutrons, A = 16 , and its mass is 16 u. As noticed, the unified atomic mass unit is defined so that a neutral carbon atom (actually a ^{12}C atom) has a mass of exactly 12 u . Carbon was chosen as the standard, partly because of its importance in organic chemistry .

Masses of the Proton, Neutron, and Electron

Particle	Symbol	kg	u	MeVc^2
Proton	*p*	1.67262×10^{-27}	1.007276	938.27
Neutron	*n*	1.67493×10^{-27}	1.008665	939.57
Electron	*e*	9.1094×10^{-31}	0.00054858	0.511

Since you are given that there are no neutrons, the mass number A is also 1. Suppose you are told that the helium nucleus or α particle has two protons and two neutrons. You can then see that it is written $^{4}_{2}He2$ There is a scarce form of hydrogen found in nature called deuterium; its nucleus has one proton and one neutron and, hence, twice the mass of common hydrogen. The symbol for deuterium is, thus, $^{2}_{1}H1$ (sometimes D is used, as for deuterated water D_2O). An even rarer—and radioactive—form of hydrogen is called tritium, since it has a single proton and two neutrons, and it is written $^{3}_{1}H_2$. These three varieties of hydrogen have nearly identical chemistries, but the nuclei differ greatly in mass, stability, and other characteristics. Nuclei (such as those of hydrogen) having the same Z and different N s are defined to be isotopes of the same element.

There is some redundancy in the symbols A , X , Z , and N . If the element X is known, then Z can be found in a periodic table and is always the same for a given element. If both A and X are known, then N can also be determined (first find Z ; then, N = A − Z). Thus the simpler notation for nuclides is

$$^{A}X,$$

which is sufficient and is most commonly used. For example, in this simpler notation, the three isotopes of hydrogen are ^{1}H, ^{2}H, and ^{3}H, while the α particle is ^{4}He

We read this backward, saying helium-4 for ^{4}He , or uranium-238 for ^{238}U . So for ^{238}U , should we need to know, we can determine that Z = 92 for uranium from the periodic table, and, thus, N = 238 − 92 = 146.

A variety of experiments indicate that a nucleus behaves something like a tightly packed ball of nucleons, as illustrated in Figure. These nucleons have large kinetic energies and, thus, move rapidly in very close contact. Nucleons can be separated by a large force, such as in a collision with another nucleus, but resist strongly being pushed closer together. The most compelling evidence that nucleons are closely packed in a nucleus is that the radius of a nucleus, r , is found to be given approximately by

$$r = r_0 A^{1/3},$$

where r_0 = 1.2 fm and A is the mass number of the nucleus. Note that $r^3 \propto A$. Since many nuclei are spherical, and the volume of a sphere is $V = (4/3)\pi r^3$, we see that $V \propto A$ − that is, the volume of a nucleus is proportional to the number of nucleons in it. This is what would happen if you pack nucleons so closely that there is no empty space between them

Figure shows: A model of the nucleus.

Nucleons are held together by nuclear forces and resist both being pulled apart and pushed inside one another. The volume of the nucleus is the sum of the volumes of the nucleons in it, here shown in different colors to represent protons and neutrons.

Example:

(a) Find the radius of an iron-56 nucleus. (b) Find its approximate density in kg / m^3 , approximating the mass of ^{56} Fe to be 56 u.

Solution:

(a) Finding the radius of 56 Fe is a straightforward application of $r = r_0A^{1/3}$, given A = 56 . (b) To find the approximate density, we assume the nucleus is spherical (this one actually is), calculate its volume using the radius found in part (a), and then find its density from ρ = m/V .

Finally, we will need to convert density from units of u / fm^3 to kg / m^3 .

Solution

(a) The radius of a nucleus is given by

$$r = r_0 A^{1/3},$$

Substituting the values for r_0 and A yields

$r = (1.2\ fm)(56)^{1/3} = (1.2\ fm)(3.83) = 4.6\ fm.$

(b) Density is defined to be ρ = m/V , which for a sphere of radius r is

$$\rho = \frac{m}{V} = \frac{m}{(4/3)\pi r^3}.$$

$$\rho = \frac{56\ \text{u}}{(1.33)(3.14)(4.6\ \text{fm})^3} = 0.138\ \text{u/fm}^3.$$

Converting to units of kg / m^3 , we find

$$\rho = (0.138\ \text{u/fm}^3)(1.66\times10^{-27}\ \text{kg/u})\left(\frac{1\ \text{fm}}{10^{-15}\ \text{m}}\right) = 2.3\times10^{17}\ \text{kg/m}^3.$$

Half-Life

The time in which half of the original number of nuclei decay is defined as the half-life, $t_{1/2}$. Half of the remaining nuclei decay in the next half-life. Further, half of that amount decays in the following half-life. Therefore, the number of radioactive nuclei decreases from N to N / 2 in one half-life, then to N / 4 in the next, and to N / 8 in the next, and so on. If N is a large number, then many half lives (not just two) pass before all of the nuclei decay.

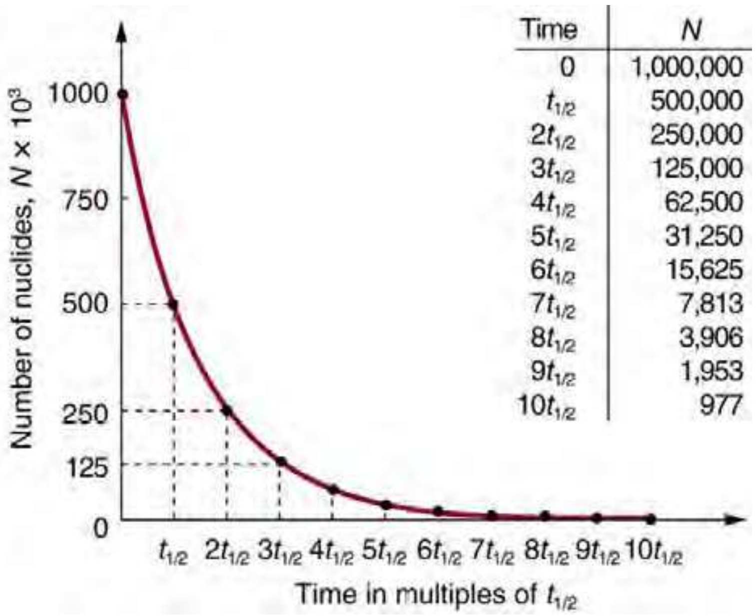


Figure shows: Radioactive decay reduces the number of radioactive nuclei over time. In one half-life$t_{1/2}$, the number decreases to half of its original value. Half of what remains decay in the next half-life, and half of those in the next, and so on. This is an exponential decay, as seen in the graph of the number of nuclei present as a function of time.

The following equation gives the quantitative relationship between the original number of nuclei present at time zero (N_0) and the number (N) at a later time t :

$$N = N_0 e^{-\lambda t},$$

where e = 2.71828... is the base of the natural logarithm, and λ is the decay constant for the nuclide. The shorter the half-life, the larger is the value of λ , and the faster the exponential $e^{-\lambda t}$ decreases with time. The relationship between the decay constant λ and the half-life $t_{1/2}$is

$$\lambda = \frac{\ln(2)}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}.$$

To see how the number of nuclei declines to half its original value in one half-life, let t = $t_{1/2}$ in the exponential in the equation $N = N_0 e^{-\lambda t}$. This gives $N = N_0\, e^{-\lambda t} = N_0\, e^{-0.693} = 0.500 N_0$.

Q\ Calculate the age of the Shroud of Turin given that the amount of ^{14}C found in it is 92% of that in living tissue.

Solution:

Knowing that 92% of the ^{14}C remains means that $N / N_0 = 0.92$. Therefore, the equation $N = N_0 e^{-\lambda t}$ can be used to find λt . We also know that the half-life of ^{14}C is 5730 y, and so once λt is known, we can use the equation $\lambda = \frac{0.693}{t_{1/2}}$ to find λ and then find t as requested. Here, we postulate that the decrease in ^{14}C is solely due to nuclear decay.

Solving the equation $N = N_0 e^{-\lambda t}$ for N / N_0 gives

$$\frac{N}{N_0} = e^{-\lambda t}.$$

$$0.92 = e^{-\lambda t}.$$

Taking the natural logarithm of both sides of the equation yields

$$\ln 0.92 = -\lambda t$$

so that

$$-0.0834 = -\lambda t.$$

Rearranging to isolate t gives

$$t = \frac{0.0834}{\lambda}.$$

Now, the equation $\lambda = \frac{0.693}{t_{1/2}}$ can be used to find λ for ^{14}C. Solving for λ and substituting the known half-life gives

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{5730\text{ y}}.$$

We enter this value into the previous equation to find t:

$$t = \frac{0.0834}{\frac{0.693}{5730\text{ y}}} = 690\text{ y}.$$

Biology department

Biophysics

Dr. Samar Imran Essa

<u>Syllabus</u>

Week	Subject Name
1	Rotational Motion
2	Heat
3	Kinetic Theory of Gases
4	Fluid Statics, Fluid Dynamics and its Biological and Medical Applications
5	Oscillation Motion
6	Wave Motion
7	The Ray Aspect of Light
8	Polarization
9	Microscopy Enhanced by the Wave Characteristics of Light
10	Electric Current
11	Introduction to Radioactivity and Nuclear Physics