حسبان التفاضل والتكامل
Calculus (1)

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Calculus

تعريف Calculus

Calculus is the branch of mathematics that includes differential calculus and integral calculus. The study of calculus started from the 17 th century by well-known authors like Newton. It is used in various areas of mathematics, as well as in physics and in engineering fields and others.

Sets and Numbers:

1- Natural Numbers

$N = \{1, 2, 3, \ldots\}$ is the set of all natural numbers.

$N_0 = \{0, 1, 2, \ldots\}$ is the set of all natural numbers that includes zero.

2- Integers Numbers

$Z = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ is the set of all integers numbers

$Z_+ = \{1, 2, 3, \ldots\}$ is the set of all positive

$Z_- = \{-1, -2, -3, \ldots\}$ " " negative

Remark:

1- n is said to be an even number if $n = 2k$, $k \in Z$.

2- n is said to be an odd number if $n = 2k+1$, $k \in Z$.

3- Rational Numbers

The rational numbers are the numbers that can be expressed in the form of a fraction a/b, where a and b are integers and $b \neq 0$. The set of all rational numbers denoted by $\mathbb{Q}$.

i.e $\mathbb{Q} = \{ \frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0 \}$ is the set of all rational numbers.

Remark:

The rational numbers have decimal expansions that are either

(1) Terminating, for example, $\frac{1}{4} = 0.24$

(2) Repeating, for example, $\frac{1}{3} = 0.333\ldots = 0.\overline{3}$

$\frac{23}{11} = 2.0909\ldots = 2.\overline{09}$

4- Irrational Numbers

An irrational number is a number that cannot be expressed as a fraction a/b for any integers a and b. Irrational number have decimal expansions that neither terminate nor repeating. The set of all irrational numbers denoted by $\overline{\mathbb{Q}}$. For instance, $\pi = 3.14159\ldots$

$\sqrt{2} = 1.414213\ldots$

Remark: Many square roots, cube roots, etc are also irrational numbers. Example

$\sqrt{3} = 1.732050807568877292935$

$\sqrt{99} = 9.949874371066199547$

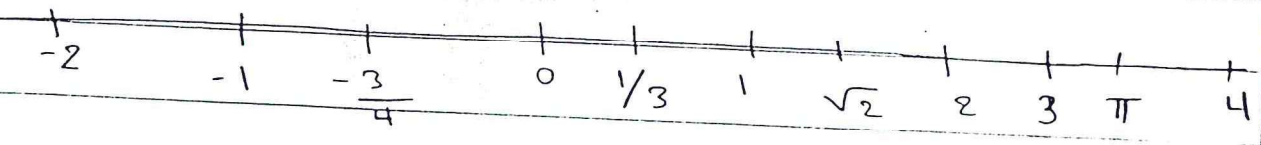
But $\sqrt{4} = 2$ (rational), and $\sqrt{9} = 3$ (rational)

So not all roots are irrational.

5 Real Numbers

$\mathbb{R} = \mathbb{Q} \cup \overline{\mathbb{Q}}$ is the set of all real numbers.

The real numbers can be represented geometrically as points on a number line called the real line.



Remark:

1- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subset \mathbb{R}$

2- $\overline{\mathbb{Q}} \subseteq \mathbb{R}$

Properties of Real Numbers:

Let a, b and c are real numbers:

① Commutative Property:

$$a+b = b+a$$

$$a \cdot b = b \cdot a$$

② Associative Property:

$$(a+b)+c = a+(b+c)$$

$$(a \cdot b) \cdot c = a \cdot (bc)$$

③ Distributive Property:

$$(a+b)c = ac+bc$$

$$a(b+c) = ab+ac$$

④ Identity Property: $a+0 = 0+a = a$

(0 is the identity element under addition)

$$a \cdot 1 = 1 \cdot a = a$$

(1 is the identity element under multiplication)

⑤ Inverse Property: $a+(-a) = (-a)+a = 0$

($-a$ is the additive inverse)

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1 \quad \text{where } a \neq 0$$

($\frac{1}{a}$ is the multiplicative inverse)

Inequality Rules: قانون المتباينة

If a, b and $c \in \mathbb{R}$, then

① If $a<b \Rightarrow a+c<b+c$

② If $a<b \Rightarrow a-c<b-c$

③ If $a<b$ and $c>0 \Rightarrow ac<bc$

④ If $a<b$ and $c<0 \Rightarrow bc<ac$

⑤ If $a>0 \Rightarrow \frac{1}{a}>0$

⑥ If $0<a<b$ or $a<b<0 \Rightarrow \frac{1}{b}<\frac{1}{a}$

Ex: $0<2<3 \Rightarrow \frac{1}{3}<\frac{1}{2}$

$-3<-2<0 \Rightarrow \frac{1}{-2}<-\frac{1}{3}$

Remark:

① $<$ (less than)

② $>$ (greater than)

③ $\leq$ (less than or equal to)

④ $\geq$ (greater than or equal to)

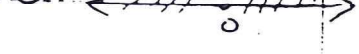
Intervals:

An interval is the set of all real numbers lying between two given numbers.

Ex:

① The set of all number x satisfying $0 \leq x \leq 2$ is an interval

② The set of all real numbers $\mathbb{R}$ is an interval.



Remark:

Type of brackets are used: $(\ )$, $[\ ]$, $[\ )$, $(\ ]$