~~[illegible]~~ Numerical Analysis التحليل العددي

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المصدر: كتاب التحليل العددي

علي سيفي

# Numerical Analysis
## التحليل العددي
## (Chapter one)

**Introduction:-**

In any numerical Analysis, Errors will arise during the calculations. so that, To able to deal with the issue of Errors, we need to

- Identify where the error is coming from and quantifying the Error. and minimize the Error as per our need.

يستخدم التحليل العددي في حل المعادلات الرياضية المعقدة والتي تحتاج وقت طويل للحل

- The solution of same equations may be obtained by **direct methods**

The direct method is produce an exact answer

$$x^2 + 2x + 1 = \text{zero}$$

$$(x+1)(x+1) = \text{zero}$$

$$x = -1$$ يمثل جذر المعادلة

لإثبات ذلك

let proofe

$$(-1)^2 + (2 \times -1) + 1 = \text{zero}$$

$$1 - 2 + 1 = \text{zero}$$

$$\text{zero} = \text{zero}$$

العلاقة هي التي تصفر المعادلة أي الدالة تساوي صفر

## What is true Error (absolute Error)

الخطأ المطلق

True Error denoted by $E_t$ is the difference between the true Value (also called the exact Value) and the approximate Value

القيمة الحقيقية

القيمة التقريبية

$$\boxed{E_t = \text{True Value} - \text{Approximate Value}}$$

**Example:-** The derivative of a function $f(x)$ at a particular Value of $x$ can be approximately calculated by

$$\bar{f}(x) = \frac{f(x+h) - f(x)}{h}$$

of $\bar{f}(2)$ for $f(x) = 7e^{0.5x}$ and $h = 0.3$

find (a) The approximate value of $\bar{f}(2)$

(b) The true value of $\bar{f}(2)$

(c) The true Error for part a

$$f(x) = \frac{f(x+h) - f(x)}{h}$$ for $x=2$, $h = 0.3$, $f(x) = 7e^{0.5x}$ (a)

$$\bar{f}(x) = \frac{f(2+0.3) - f(2)}{0.3}$$

$$= \frac{f(2.3) - f(2)}{0.3}$$

$$= \frac{7e^{0.5(x+h)} - 7e^{0.5x}}{h}$$

$$= \frac{7e^{0.5(2.3)} - 7e^{0.5\times 2}}{0.3}$$

approximate

$$= \frac{22.107 - 19.028}{0.3} = \boxed{10.265}$$

(b) the exact value of $\bar{f}(2)$ canbe calcul by using our knowledge of differential calculus

$f(x) = 7e^{0.5x}$ مشتقة

$\bar{f}(x) = 7 \times 0.5 \times e^{0.5x} = 3.5e^{0.5x} = 3.5e^{0.5}$

(c)

$E_t$ = Truevalue − Approximate value

$= 9.5140 - 10.265$

$= -075061$

## What is relative True Error (relative Error)
الخطأ النسبي

Relative true Error denoted by $\epsilon_r$ and defined as the ratio between the true error and the true Value

$$\epsilon_r = \left| \frac{E_t}{x_0} \right| = \left| \frac{x_0 - x}{x_0} \right| = \left| \frac{x_0}{x_0} - \frac{x}{x_0} \right|$$

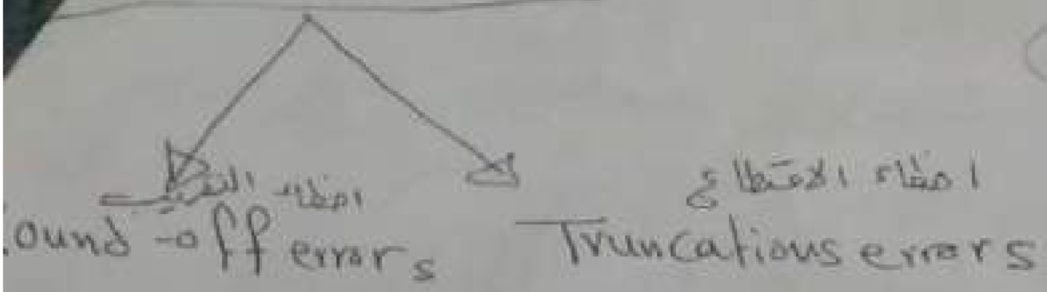
$$= \left| 1 - \frac{x}{x_0} \right|$$

where $x_0$ = exact number

$x$ = approximation number

خلي أرقام فرع المثال السابق وهو حساب relative error ؟

# Types of Error




## ① Round-off Errors

signification figure, means the numbers of digit in a figure

**exp :-**

64903 have 5 significant figure

0.014 have 2 significant figure

**Nots :-**

1- Find the position of the last digits.
- look at the next digit or the right - the decider
- if the decider is 5 or more, round up the last di
- if the decider is 4 or less, leave the last digi as it is
- fill in any gaps befor the decimal point with ze

ظة في الرقم الذي يلي الرقم المطلوب اذا كان اكثر من ٥ نضيف واحد للرقم المطلوب , اذا اقل لا يتغير ويبقى نفس الرقم.

Ex

50 have 1 S.F

500 have 1 S.F

5000.0 have 4 S.F

0.004 have 1 S.F

0.00435 have 3 S.F

0.00708 have 3 S.F

0.05060 have 4 S.F

Exp Rounding using significant figure Decimals

0.054076

a- 1 S.F 0.05

b- 2 S.F 0.054

c 3 S.F 0.0541

d- 4 S.F 0.05408

اذا كان الرقم خمسة فما فوق يعطيا واحد للرقم المطلوب كما في فرع d واذا كان اقل من خمسة لا يعطيا ويبقى الرقم نفسه كما في فرع b

H.w

* Rounding Using significant figure, Decimals

a- 0.4965 for 3 S.F

ans. 0.497

b- 1.0355 for 3S.F

ans. 1.04

c. 12.904 for 2 S.F

ans. 13

* Rounding

7462.5996 to 1S.F 7000.00

to 2S.F 7500.00

to 3SF 7460.00

to 4 S.F 7463.00

H.w// Rounding 0.39052 to

1- 1 S.F => 0.4

- 2 S F => 0.39

- 3 S.F => 0.391

- 4 S.F => 0.3905

Note_ (1) If we have zero at the start or end the number it does not count as sig. digits

لو كان الصفر في البداية او الاخر لا تحسب كرقم

(2) The zero between 2 digits does count

اما اذا كان الصفر بين رقمين تحسب

## (2) Truncation Errors :- أخطاء الاقتطاع

Truncation Error is defined as the Error caused by truncation a mathematical procedure

Exp. // The Maclaurin series for $e^x$ is given as

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

This series has an infinite number of terms if we take Three terms to calculated $e^x$

اقتطاع ٣ حدود

$$\therefore e^x = 1 + x + \frac{x^2}{2!}$$

The truncation Error for such as approximation

الخطأ - القيمة

$$\text{Truncation error} = e^x - \left\{1 - x + \frac{x^2}{2!}\right\} \text{ ---- } (1)$$

$$\because e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots \quad (2)$$

في (2) في (1)

$$\text{Truncat error} = \cancel{1 + x + \frac{x^2}{2!}} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} - \left\{\cancel{1 + x + \frac{x^2}{2!}}\right\} = \frac{x^3}{3!} + \dots$$

Maclaurin

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\log(x+1) = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

truncat error four terms

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$$\therefore \text{truncat error} = \cos x - \left\{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right\}$$

$$= \cancel{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}} + \frac{x^8}{8!} - \frac{x^{10}}{10!} - \left\{\cancel{1 - \frac{x^2}{2!}}\right.$$

$$\left. \cancel{+ \frac{x^4}{4!} - \frac{x^6}{6!}}\right\}$$

$$= \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots$$

# Numerical Analysis

## Chapter two

## Solving a Non linear Equation

حل المعادلات غير الخطية

## ① Bisection Method طريقة التنصيف المتتالي

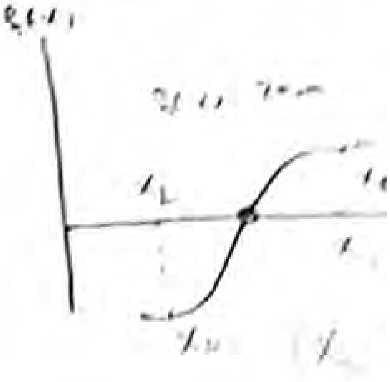
to find the root of $f(x) = zero$

**Theorem**

إيجاد جذر المعادلة بطريقة التنصيف المتتالي

1- if $f(x) = zero$, $f(x)$ is a real continuous function has at least one root between $X_L$ and $X_R$ so that $f(X_L)\, f(X_R) < 0$ (−)

2- if $f(X_L)\, f(X_R) > 0$ (+)
There are not be any root between $X_L$, $X_R$



3- if $f(X_L)\, f(X_R) < 0$ There are more root between $X_L$, $X_R$

جذر المعادلة عندما تكون الدالة وتعويض القيمة في الدالة وتساوي = صفر

# Algorithm for the Bisection method

1- choose $X_L$ . $X_R$ as two guesses values

if $f(X_L) \cdot f(X_R) < 0$
(a) (b)

2- Estimate the root, $X_m$ of equation

$f(x) = $ zero as the mid-point between $X_L, X_R$

$$\boxed{X_m = \frac{X_L + X_R}{2}}$$

3- Now check the following

(a) if $f(X_L)\, f(X_m) < 0$

then root lies between $X_L$ and $X_m$ become so

$X_L = X_L$ , $X_R = X_m$

a = a   b = w

(b) if $f(X_L)\, f(X_m) > 0$

the root lies between $X_R$ and $X_m$ so

a = w   b = b

$X_L = X_m$ and $X_R = X_R$

(c) if $f(X_L) \times f(X_m) =$ zero

the root is $X_m$

Stope the algorithm if this is true

ايقاف العمليات الحسابية وهنا يكون هو جذر المعادلة

Find the new estimate of the root

$$x_m = \frac{x_L + x_R}{2}$$

Find the absolut relative approximate error

$$|\epsilon_r| = \left| \frac{x_m^{new} - x_m^{old}}{x_m^{new}} \right| \times 100\%$$

where $x_m^{new}$ = estimated root from present itera

$x_m^{old}$ = estimated root from previous iteration

5- Compare the absolute relative approxim error $|\epsilon_r|$ with the pre-specified relative error tolerance $\epsilon_s$

if $|\epsilon_a| > \epsilon_s$ then go to step 3

else stop the algorithm.

Exp/ $f(x) = x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$

a $x_L = 0$ , $x_R = 0.11$ (E)

n=0 (1) check if the function changes sign between $x_L$ and $x_R$

$f(x_L) = f(0) = 0^3 - 0.165(0)^2 + 3.993\times10^{-4}$

$= (+)\ 3.993\times10^{-4}$

$f(x_R) = f(0.11) = (0.11)^3 - 0.165(0.11)^2 + 3.993\times10^{-4}$

$= (-)\ 2.662\times10^{-4}$

$f(x_L)\,f(x_R) = f(0) * f(0.11)$

$= 3.993\times10^{-4} * (-2.662\times10^{-4})$

$= - < 0$ سالب

$n_1 = 1$

$x_m = \frac{x_L + x_R}{2} = \frac{0 + 0.11}{2} = 0.055$

$f(x_m) = f(0.055) = (0.055)^3 - 0.165(0.055)^2 - 3.99$

$= +6.65$

$f(x_L)\,f(x_m) = f(0) * f(0.055) = 3.99\times10^{-4} \times 6.655\times10^{-5}$

$= + > 0$ موجب

$x_L = x_m$ , $x_R = x_R$

$x_L = \boxed{0.055}$ $x_R = 0.11$

$n = 2$

$x_m = \frac{x_L + x_R}{2} = \frac{0.055 + 0.11}{2} = 0.0825$

$$f(X_m) = f(0.0825) = (0.0825)^3 - 0.165(0.0825)^2 + 3.99\times10^{-4} = -1.622\times10$$

$$f(X_L)\,f(X_m) = f(0.055) * f(0.0825) = (6.655\times10^{-5})(-1.62\ldots) = - < 0$$

$X_L = X_L$ , $X_R = X_m$ الجديدة

$X_L = 0.055 \quad X_R = 0.0825$

$$|\epsilon_a| = \frac{X_m^{new} - X_m^{old}}{X_m^{new}} = \left|\frac{0.0825 - 0.055}{0.0825}\right| \times 100\% = 33.33\%$$

$\boxed{n=3}$

$X_L = 0.055$ ; $X_R = 0.0825$

$$X_m = \frac{X_L + X_R}{2} = \frac{0.055 + 0.0825}{2} = 0.06875$$

$$f(X_m) = f(0.06875) = -5.563\times10^{-5}$$

$$f(X_L)\,f(X_m) = f(0.055) * f(0.06875) = - < 0$$

$X_L = X_L$ ; $X_R = X_m$

$X_L = 0.055 \quad X_R = 0.06875$

$$|\epsilon_a| = \left|\frac{X_m^{new} - X_m^{old}}{X_m^{old}}\right| \times 100\% = \left|\frac{0.06875 - 0.0825}{0.06875}\right|$$

$$= 20\%$$

تم ايقاف العملية عند الوصول الى نسبة الخطأ المطلوبة
و ان قيم الدالة اقرب الى الصفر $f(x) = -1.0804\times10^{-5}$
كما طلب السؤال $h = ?$

Exp/ $X^2 + X - 0.75 = \text{zero}$ , $X_L = 0.4$ , $X_R = 0.$

$f(X_L) = f(0.4) = (0.4)^2 + 0.4 - 0.75$

$= -\,0.19$

$f(X_R) = f(0.6) = (0.6)^2 + 0.6 - 0.75$

$= 0.36 + 0.6 - 0.75$

$= +\,0.21$

$f(X_L)\,f(X_R) = f(0.4) * f(0.6)$

$= -0.19 * +0.21$

$= - \quad < 0$

$$X_m = \frac{X_L + X_R}{2} = \frac{0.4 + 0.6}{2} = \frac{1.0}{2} = 0.5$$

$f(X_m) = f(0.5) = (0.5)^2 + 0.5 - 0.75$

$= 0.25 + 0.5 - 0.75$

$= 0.75 - 0.75 = \text{zero}$

so that it the solution $\boxed{X = 0.5}$

كانت الدالة = صفر يعني $X_m$ هو الجذر المطلوب وهو الحل للدالة.

H.W// solve $X^3 - 9X + 1 = 0$

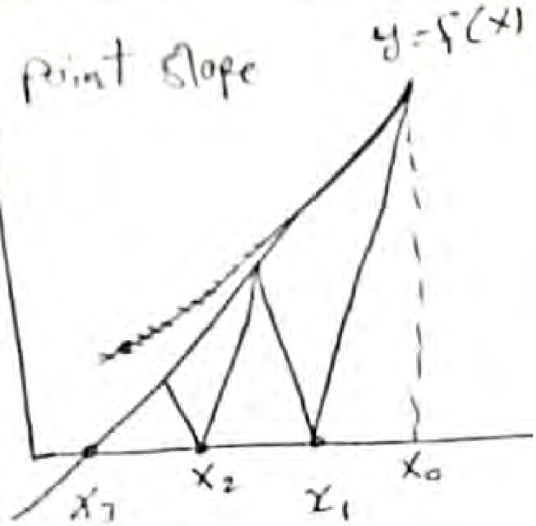
$X_L = 2$ , $X_R = 4$

To use Bisection method

s.l

## (2) Newton-Raphson method

الطريقة الثانية طريقة نيوتن - رافسون




At that point $(x_0, y_0)$, using the point slope from

$$(y - y_0) = (x - x_0)\, f'(x_1)$$

the points $x, x_1, x_2, x_3$, is the value that make the $y = \text{zero}$

substituting

$y_0 = f(x_0)$ $\quad \therefore y - y_0 = (x - x_0)\, f'(x_0)$

$$f'(x) = \frac{y - y_0}{x - x_0}$$

$$0 - F(x_0) = (x_1 - x_0)\, f'(x_0)$$

Note. $\text{slope} = \frac{\Delta y}{\Delta}$

$$-\frac{F(x_0)}{f'(x_0)} = x_1 - x_0$$

$f'(x) = \text{slope} = \frac{d}{d}$

$$x_1 = x_0 - \frac{F(x_0)}{f'(x_0)}$$

In General

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

حفظ

This is called Newton-Raphson for

$\sqrt{A} = x$

$x^2 - A = 0$

$x^2 = A$

s.t. $x^2 - A = \text{zero}$

let $f(x) = x^2 - A \quad \dots (1)$

$f'(x) = 2x \quad \dots (2)$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ [illegible]

$$x_{n+1} = x_n - \frac{x_n^2 - A}{2x_n}$$

[illegible]

$$= \frac{2x_n^2 - x_n^2 + A}{2x_n} = \frac{x_n^2 + A}{2x_n}$$

$$= \frac{x_n^2}{2x_n} + \frac{A}{2x_n}$$ [illegible]

$$= \frac{1}{2}\left[\frac{x_n}{1} + \frac{A}{x_n}\right]$$

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{A}{x_n}\right]$$

find $\sqrt{10}, \sqrt{33}, \sqrt{72}$

find $\sqrt{10}$

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{A}{x_n}\right]$$

let $x_0 = 3$

$n = 0$

$$x_1 = \frac{1}{2}\left[3 + \frac{10}{3}\right] = 0.5\left[3 + 3.333\right] = 3.1666$$

$n = 1$

$$x_2 = \frac{1}{2}\left[3.1666 + \frac{10}{3.1666}\right] = 0.5\left[3.1666 + \dots\right] = 3.162280$$

$n = 2$

$$x_3 = \frac{1}{2}\left[3.162280 + \frac{10}{3.162280}\right]$$
$$= 0.5\left[3.162280 + \frac{10}{3.162280}\right]$$
$$= 3.1622788301$$

$n = 3$

$$x_4 = \frac{1}{2}\left[3.1622788 + \frac{10}{3.1622788}\right]$$
$$= 3.16227776602$$

من السؤال n معينه نستمر بايجاد قيم $x$ الى الحد

المطلوب اما $x_n$ اي لدينا n معينه

Using Newton - Raphson method
Find the general formula

$\frac{1}{\sqrt{a}}$ , $\sqrt[N]{a}$ , $\sqrt{a}$

let

$$x = \left(\frac{1}{\sqrt{a}}\right)^2$$

تم التربيع للتخلص من الجذر
لسهولة الحل

$$\therefore x^2 = \frac{1}{a}$$

$$f(x) = x^2 - \frac{1}{a}$$

$$\bar{f}(x) = 2x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{\bar{f}(x_n)}$$

$$= x_n - \frac{x_n^2 - \frac{1}{a}}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 - \frac{1}{a}}{2x_n}$$

$$= \frac{x_n^2 - \frac{1}{a}}{2x_n} = \frac{x_n^2}{2x_n} - \frac{\frac{1}{a}}{2x_n}$$

$$= \frac{x_n}{2} - \frac{1}{2ax_n} = \frac{1}{2}\left[x_n - \frac{1}{2ax_n}\right]$$

$$= \frac{1}{2}\left[x_n - \frac{1}{2ax_n}\right]$$

د. هدى م

using Newton-Raphson method for $f(x) = x^3 - x - 1$ take $x_0 = 1.3$ find $n = 3$

② Using Newton-Raphson method to solve $f(x) = \log_{10} x - 0.5x$ stope at $n = 3$

Note ⟸ دوال ومشتقاتها

① $f(x) = e^x$

$\bar{f}(x) = e^x$

② $f(x) = \ln x$

$\bar{f}(x) = \dfrac{1}{x}$

③ $f(x) = \log_a x$

$\bar{f}(x) = \dfrac{1}{x \ln a}$

④ $f(x) = \sqrt[3]{x^4}$

$= x^{4/3}$

$\bar{f}(x) = \dfrac{4}{3} x^{\frac{1}{3}} = \dfrac{4}{3} \sqrt[3]{x}$

(3) Secant method

الطريقة الثالثة طريقة القاطع

The secant method may be regarded as a modification of the Newton-Raphson method.

$$x_{n+1} = x_n - \frac{f(x_n)}{\bar{f}(x_n)} \quad \dots\dots (1)$$

$$\bar{f}(x) = m = \frac{y_n - y_{n-1}}{x_n - x_{n-1}} = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} \quad \dots\dots (2)$$

نعوض (2) في (1)

$$x_{n+1} = x_n - \frac{f(x_n)}{\dfrac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}}$$

$$= x_n - \frac{f(x_n)\,(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$= \frac{x_n\big(f(x_n) - f(x_{n-1})\big) - f(x_n)\,(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$= \frac{\cancel{x_n f(x_n)} - x_n f(x_{n-1}) - \cancel{x_n f(x_n)} + x_{n-1} f(x_n)}{f(x_n) - f(x_{n-1})}$$

$$\boxed{x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}}$$

الصيغة العامة مهمة

$^{2}-9x+1$ by the ~~Newton Raphson~~ secant method

$x_1=2,\quad x_2=4$

$f(x_1)=f(2)=(2)^3-9(2)+1=-9$

$f(x_2)=f(4)=(4)^3-9(4)+1=29$

$f(2)\,f(4)=-9\times 29=-$

there is a root between $x_1, x_2$

$$x_{n+1}=\frac{x_{n-1}\,f(x_n)-x_n\,f(x_{n-1})}{f(x_n)-f(x_{n-1})}$$

$x_1=2$ $\quad n=2$

$x_2=4$

$n=2$

$$x_{i+1}=\frac{x_1\,f(x_2)-x_2\,f(x_1)}{f(x_2)-f(x_1)}$$

$$x_3=\frac{2(29)-4(-9)}{29-(-9)}$$

$$=\frac{38+36}{38}=2.47368$$

$x_2=4$

$x_3=2.47368$ $\qquad \frac{x_3-x_2}{x_3}$

$$|\epsilon_r|=\left|\frac{2.47368-4}{2.47368}\right|\times 100\%$$

Exp/ solve by secant method

(4)

$x^3 = 20$

$x_0 = 4.0 \quad , \quad x_1 = 5.5$

Find the estimate after 2 iterations

$x^3 = 20$

$f(x) = x^3 - 20 = 0$

$$x_{n+1} = \frac{x_{n-1}\, f(x_n) - x_n\, f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

n=1

$$x_2 = \frac{x_0\, f(x_1) - x_1\, f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{4.0\left((5.5)^3 - 20\right) - 5.5\left((4)^3 - 20\right)}{\left((5.5)^3 - 20\right) - \left((4)^3 - 20\right)}$$

$= 3.353$

$$|\epsilon_r| = \left|\frac{x_2 - x_1}{x_2}\right| \times 100\%$$

$$= \left|\frac{3.353 - 5.5}{3.353}\right| \times 100\% = 63.92$$

$n = 2$

تتمة

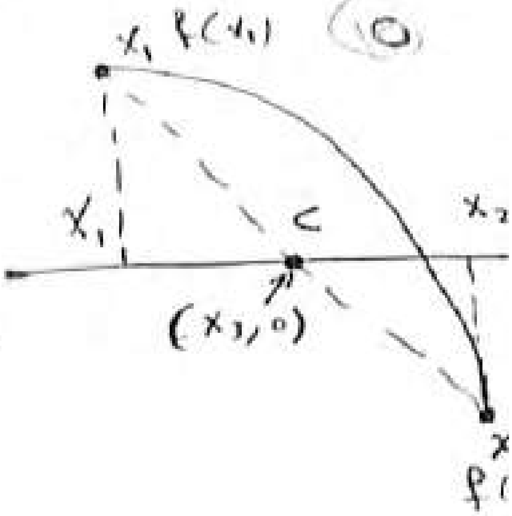
الثاني

## false position method

طريقة الموضع الكاذب

if $f(x_1)$ and $f(x_2)$ have opposite signs the bisection method used the midpoint of the interval $x_1, x_2$ as the next iterate

$$m_1 = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$




at point c

$$m_2 = \frac{0 - f(x_2)}{x_3 - x_2}$$

$$m_1 = m_2$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{0 - f(x_2)}{x_3 - x_2}$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

So that

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

الصيغة العامة

## false position method

طريقة الموضع الكاذب

if $f(x_1)$ and $f(x_2)$ have opposite signs the bisection method used the midpoint of the interval $x_1, x_2$ as the next iterate

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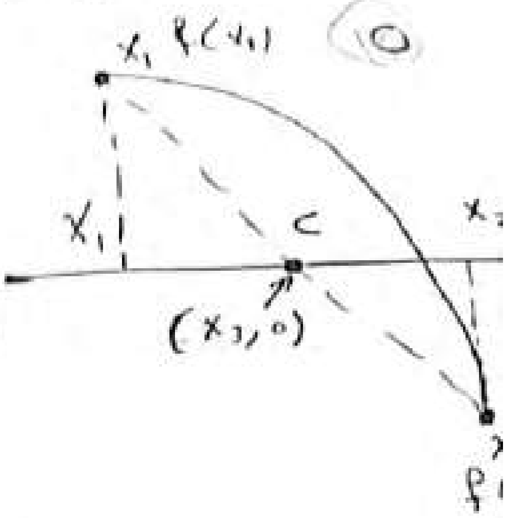



at point c

$$m_2 = \frac{0 - f(x_2)}{x_3 - x_2}$$

$$m_1 = m_2$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{0 - f(x_2)}{x_3 - x_2}$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

So that

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

المصمعادلة

Exp//

$x\sin(x)-1 = \text{zero}$ $\quad x_1 = 0$, $x_2 = 2$

1

$f(x) = x\sin(x) - 1$

$f(0) = 0\sin 0 - 1 = -1$

$f(2) = 2\sin 2 - 1 = \cancel{0.8185948}\ -0.930$

$$x_3 = x_2 - \frac{f(x_2)\,(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$= 2 - \frac{-0.\cancel{8185948}*\,(2-0)}{-0.\cancel{8185948} - (-1)}$$

$$= 2 - \frac{-1.86}{0.07}$$

$\cancel{-12.6}$

$$= 2 - \frac{-0.930\,(2-0)}{-0.930 - (-1)}$$

$$= 2 - \frac{-1.86}{0.07}$$

$$= 2 + 26.57 = 28.57$$

Numerical Analysis

①

Chapter Three

Principle of least squares (Linear Fitting)

Let the curve

$y = a + bx + cx^2 + \cdots + x^{m-1}$

be fitted to the set of n data points $(x_1, y_1)$.

$(x_2, y_2)$ . $(x_3, x_3)$ ⋯ $(x_n, y_n)$

At $(x = x_i)$ the observed (or experimental) value of the ordinate is $y_i = P_i m_i$ and the correspon value on the fitting curve (i) is

$$a + bx_i + cx_i^2 + \cdots kx_i^m = L_i m_i$$

which is the expected or calculated value.

The diffenece of the observed and the expected v is $P_i m_i - L_i m_i = e_i$

This diffenence is called error at $x = x_i$

clearly some of the error $e_1, e_2, e_3 \cdots e_n$ will be positive and other negative.

to make all errors positive we square each of the errors i.e

$$S = e_1^2 + e_2^2 + e_3^2 + \cdots + e_n^2$$

The curve of best fit is that for which e. are small as possible i.e (S)

The sum of the square of the errors is a minimum. This is known as the principle of least squares.

the theoretical value for $x_1, x_2, x_3 \dots, x_n$ may be

$$y_{x_1}, y_{x_2}, y_{x_n}$$

let a straight line

$$y = a + bx \quad \dots \quad (1)$$

which is fitted to the given data points $(x_1, y_1)$ $(x_2, y_2)$, $(x_3, y_3) \dots (x_n, y_n)$

let $y_{x_1}$ be the theoretical value for $x_1$

Then $e_1 = y_1 - y_{x_1}$

$$e_1 = y_1 - (a + bx_1)$$

$$e_1^2 = (y_1 - a - bx_1)^2$$

$$S = e_1^2 + e_2^2 + e_3^2 \dots + e_n^2$$

$$S = \sum_{i=1}^{n} (y_i - a - bx_i)^2$$

By the principle of least squares, the value of $S$ is minimum. Therefore

$$\frac{\partial S}{\partial a} = 0 \quad \cdots (2)$$

$$\frac{\partial S}{\partial b} = 0 \quad \cdots (3)$$

on solving equat (2) and (3) we have

$$\sum y = na + b\sum x \quad \cdots 4$$

$$\sum xy = a\sum x + b\sum x^2 \quad \cdots 5$$

on solving eq. 4 and 5 we get the Value a and b

putting the value of a and b in equa (1) we get the equat of line of best fit.

Exp// Find the best-fit values of a and b so they $y = a + bx$ fits the data given in the table

| | | | | | |
|---|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 |
| y | 1 | 1.8 | 3.3 | 4.5 | 6.3 |

| x | y | xy | $x^2$ |
|---|---|---|---|
| 0 | 1 | 0 | 0 |
| 1 | 1.8 | 1.8 | 1 |
| 2 | 3.3 | 6.6 | 4 |
| 3 | 4.5 | 13.5 | 9 |
| [illegible] | [illegible] | 25.2 | [illegible] |

$\sum x = 10$

$\sum y = 16.9$

$\sum xy = 47.1$

$\sum x^2 = 30$

Normal equation are

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

$n = 5$, $\sum x = 10$, $\sum y = 16.9$, $\sum xy = 47.1$, $\sum x^2 = 30$

$$16.9 = 5a + 10b$$

$$47.1 = 10a + 30b$$

حل المعادلات آنياً

$a = 0.72$

$b = 1.33$

$y = 0.72 + 1.33x$

H.W// Fit straight line to the given data regarding $x$ as the independent variable

| | | | | | | |
|---|---|---|---|---|---|---|
| $x =$ | 1 | 2 | 3 | 4 | 5 | 6 |
| $y =$ | 1200 | 900 | 600 | 200 | 110 | 50 |

ذكر، Ans. $y = 1361.97 - 243.42x$

| $x$ | $y$ | $xy$ | $x^2$ | $x^2y$ | $x^3$ | $x^4$ |
|---|---|---|---|---|---|---|
| 0 | -4 | 0 | 0 | 0 | 0 | 0 |
| 1 | -1 | -1 | 1 | -1 | 1 | 1 |
| 2 | 4 | 8 | 4 | 16 | 8 | 16 |
| 3 | 11 | 33 | 9 | 99 | 27 | 81 |
| 4 | 20 | 80 | 16 | 320 | 64 | 256 |

$\Sigma x = 10 \quad \Sigma y = 30 \quad \Sigma xy = 120 \quad \Sigma x^2 = 30 \quad \Sigma x^2 y = 434$

$\Sigma x^3 = 100, \quad \Sigma x^4 = 354$

$n = 5$

$$30 = 5a + 10b + 30c \quad \text{—} \quad (1)$$

$$120 = 10a + 30b + 100c \quad \text{—} \quad (2)$$

$$434 = 30a + 100b + 354c \quad \text{—} \quad (3)$$

حل المعادلات ① و ② آنية لإيجاد قيم a، b، c

$$30 = 5a + 10b + 30c \quad \text{—} \quad (1) \quad *2$$
$$120 = 10a + 30b + 100c \quad \text{—} \quad (2)$$

$$\mp 60 = \mp 10a \mp 20b \mp 60c$$
$$120 = 10a + 30b \mp 100c$$

$$60 = 10b + 40c \quad \div 10$$

$$6 = b + 4c \implies \boxed{b = 6 - 64c} \quad \text{—} \quad *$$

(2)

$120 = 10a + 30b + 100c \quad \cdots (1) \quad \times 3$

$434 = 30a + 100b + 354c \quad \cdots (2)$

---

$360 = 30a + 90b + 300c$

$434 = 30a + 100b + 354c$

---

$74 = 10b + 54c$

$74 = 10(6-4)c + 54c$

$74 = 60 - 40c + 54c$

$74 - 60 = 54c - 40c$

$\boxed{\therefore c = \frac{14}{14} = 1}$

نعوض في معادلة * لايجاد قيمة b

$b = 6 - 4c$

$= 6 - 4 \times 1$

$\boxed{b = 2}$

نعوض قيم c, b في معادلة (1) لايجاد a

$30 = 5a + 10 \times 2 + 30 \times 1$

$\boxed{a = -4}$

∴ المعادلة هي

$y = -4 + 2x + x^2$

Numerical Analysis

Chapter Four

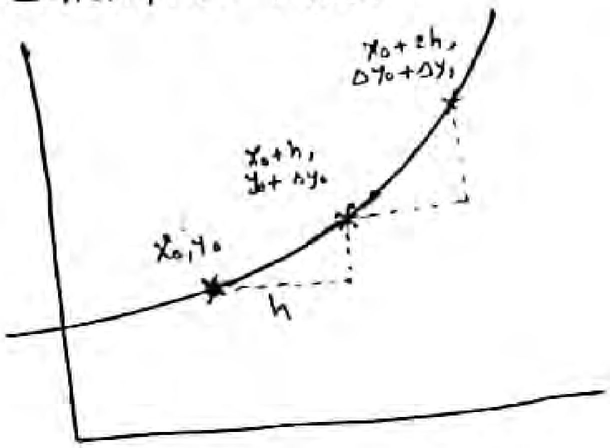
1

# Solution of differential equation

حل المعادلات التفاضلية

if $\bar{y} = f(x,y)$ with initial condition that

$x = x_0$ ; $y = y_0$




## (1) The Euler method

The solution at the equation

$$\bar{y} = f(x,y)$$

with initial condition that $y = y_0$, when $x =$

We can find a new point $(x_{n+1}, y_{n+1})$ by making use of the definition of the derivative

$$\bar{y} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

by approximate

$$\bar{y} = \frac{\Delta y}{\Delta x}$$

if $\Delta y = y_{n+1} - y_n$

$$\Delta x = x_{n+1} - x_n = h$$

so that $\bar{y} = \dfrac{y_{n+1} - y_n}{h}$

$$y_{n+1} = y_n + h\bar{y}$$

تعويض المعادلة

$$\boxed{\text{Euler eq.}:\ y_{n+1} = y_n + h f(x_n, y_n)}$$

Exp// solve

$$\bar{y} = xy \qquad \left[h = \frac{b-a}{m}\right]$$

the initial condition $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$\boxed{n=0}$ $\boxed{y_{n+1} = y_n + h f(x_n, y_n)}$

$$y_1 = y_0 + 0.1\, x_0 y_0$$

$$= 1 + 0.1(0)(1)$$

$$\boxed{y_1 = 1}$$

$n = 1$

$$y_2 = \underline{y_1} + 0.1 f(x_1, y_1)$$

$$= 1 + 0.1\, x_1 y_1$$

$$= 1 + 0.1\underset{x_1}{(0.1)}\underset{y_1}{(1)}$$

$$= 1 + 0.01$$

$$y_2 = 1.01$$

$\because \Delta x = h$

$\therefore x_{n+1} - x_n = h$

$$x_{0+1} - x_0 = h$$

$$\boxed{x_1 = x_0 + h}$$

$$= 0 + 0.1$$

$$\boxed{x_1 = 0.1}$$

$$x_2 = x_1 + h$$

$$x_3 = x_2 + h$$

Umerical Analysis

$y' = x^2 + y$  $y(0) = 1$, $h = 0.1$  find $y_3$

$y_{n+1} = y_n + h f(x_n, y_n)$

$n = 0$

$y_1 = y_0 + h f(x_0, y_0)$

$y_1 = 1 + 0.1 (x_0^2 + y_0)$

$= 1 + 0.1 (0^2 + 1) = 1.1$

$n = 1$

$y_2 = y_1 + h f(x_1, y_1)$

$= 1.1 + 0.1 (x_1^2 + y_1)$

$x_1 = x_0 + h = 0 + 0.1 = 0.1$

$= 1.1 + 0.1 ((0.1)^2 + (1.1))$

$= 1.1 + 0.1 (1.11)$

$= 1.211$

$n = 2$

$y_3 = y_2 + h f(x_2, y_2)$

$= 1.211 + 0.1 (x_2^2 + y_2)$

$x_2 = x_1 + h = 0.1 + 0.1 = 0.2$

$= 1.211 + 0.1 ((0.2)^2 + 1.211$

$= 1.211 + 0.1 (1.2511)$

$= 1.33611$

Ex P/11

$$Y' = \frac{Y - X}{X + Y} \qquad x_0 = 0,\quad y_0 = 1,\quad h = 0.1$$

(11)

$$x_{n+1} = x_n + h$$

$$Y'_n = f(x_n, y_n)$$

$$y_{n+1} = y_n + h f(x_n, y_n)$$

ادبلر

$n = 0$

$$Y_1 = Y_0 + h\, f(x_0, y_0)$$

$x_0 = 0$

$y_0 = 1$

$h = 0.1$

$$f(x_0, y_0) = \frac{y_0 - x_0}{x_0 + y_0} = \frac{1 - 0}{0 + 1} = 1$$

$$\therefore Y_1 = Y_0 + h f(x_0, y_0)$$

$$Y_1 = 1 + 0.1(1) = 1 + 0.1 = 1.1$$

$n = 1$

$$Y_2 = Y_1 + h\, f(x_1, y_1)$$

$y_1 = 1.1$

$x_1 = x_0 + h = 0 + 0.1 = 0.1$

$h = 0.1$

$$f(x_1, y_1) = \frac{y_1 - x_1}{x_1 + y_1} = \frac{1.1 - 0.1}{0.1 + 1.1} = \frac{1}{1.2}$$

$$Y_2 = 1.1 + 0.1\left(\frac{1}{1.2}\right) = 1.1 + \frac{0.1}{1.2}$$

$=$

Numerical Analysis

① $y' = x^2 + y$ $y(0) = 1$ $h = 0.1$ find $y_3$

$y_{n+1} = y_n + h f(x_n, y_n)$

$n = 0$

$y_1 = y_0 + h f(x_0, y_0)$ ②

$y_1 = 1 + 0.1\ (x_0^2 + y_0)$

$= 1 + 0.1\ (0^2 + 1) = 1.1$

$n = 1$

$y_2 = y_1 + h f(x_1, y_1)$

$= 1.1 + 0.1\ (x_1^2 + y_1)$

$x_1 = x_0 + h$
$= 0 + 0.1$
$= 0.1$

$= 1.1 + 0.1\ ((0.1)^2 + (1.1))$

$= 1.1 + 0.1\ (1.11)$

$= 1.211$

$n = 2$

$y_3 = y_2 + h f(x_2, y_2)$

$= 1.211 + 0.1\ (x_2^2 + y_2)$

$x_2 = x_1 + h$
$= 0.1 + 0.1$
$= 0.2$

$= 1.211 + 0.1\ ((0.2)^2 + 1.211$

$= 1.211 + 0.1\ (1.2511)$

$= 1.33611$

1 (2) Modified Euler method

if $\bar{y} = f(x, y)$ [illegible] ايلر المطورة

1 with initial - value $y = y_0$
$x = x_0$

we will truncate the Taylor series after The second derivative term.

$$y = y_0 + \bar{y}_0 (x - x_0) + \frac{\bar{\bar{y}}_0 (x - x_0)^2}{2!} + \dots \quad (1)$$

and $h = x_1 - x_0$

So

$$y_1 = y_0 + h\bar{y}_0 + \frac{h^2}{2} \boxed{\bar{\bar{y}}_0} \dots \quad (2)$$

Recalling forward differences

$$\Delta \bar{y}_0 = \bar{y}_1 - \bar{y}_0$$

and an approximation for

$$\boxed{\bar{\bar{y}}_0} = \frac{\bar{y}_1 - \bar{y}_0}{h}$$

Replacing the second derivative in (2) w

$$y_1 = y_0 + h\bar{y}_0 + \frac{h^2}{2} \left[ \frac{\bar{y}_1 - \bar{y}_0}{h} \right]$$

$$y_1 = y_0 + \frac{h}{2} \left[ \bar{y}_0 + \bar{y}_1 \right] \dots \quad (3)$$

(1)

we must
which requ
which

we must have a value for $\bar{y}_1$
which requires that we know $y_1$
which of course we can approximate by

$$y_1 = y_0 + h\bar{y}_0 \quad \text{Euler's method}$$

Now we find

$$\bar{y}_1 = f(x_1, y_1)$$

and substitute in eq (3)

The general form is

$$\boxed{y_{n+1} = y_n + \frac{h}{2}\left[\bar{y}_n + \dot{y}_{n+1}\right]}$$

اويلر
المطورة
[illegible]

where $\bar{y}_{n+1} = f(x_{n+1}, \bar{y}_{n+1})$

$$\bar{y}_{n+1} = y_n + h\bar{y}_n$$

Q/ solve $y' = xy$ by the modified Euler
the initial condition $x_0 = 0$
$y_0 = 1$
$h = 0.1$

$$y_{n+1} = y_n + \frac{h}{2}\left[ y'_n + \bar{y}'_{n+1} \right]$$

$$\bar{y}'_{n+1} = f(x_{n+1}, \bar{y}_{n+1})$$

$$\bar{y}_{n+1} = y_n + h\, y'_n$$

$$y_1 = y_0 + \frac{h}{2}\left[ y'_0 + \bar{y}'_1 \right]$$

$$\bar{y}_{n+1} = y_n + h\, y'_n$$

$$\bar{y}_1 = y_0 + h\, y'_0 = 1 + 0.1 \; f(x_n, y_n)$$
$$= 1 + 0.1 \; (x_0, y_0)$$
$$= 1 + 0.1 \; (0 \times 1)$$
$$= 1$$

$y' = xy$

$$x_{n+1} = x_n + h = 0 + 0.1 = 0.1$$

$$\bar{y}'_{n+1} = f(x_{n+1}, \bar{y}_{n+1})$$

$$\bar{y}'_1 = (x_1 \times \bar{y}_1) = 0.1 \times 1 = 0.1$$

$$= y_0 + \frac{h}{2}\left[ y'_0 + \bar{y}'_1 \right]$$ $\bar{y}'_0 = x_0 y_0 = 0 \times 1 = 0$

$$= 1 + \frac{0.1}{2}\left[ 0 + 0.1 \right]$$

$$= 1 + \frac{0.1}{2}\left[ 0.1 \ 0.1 \right]$$

$$= 1 + \frac{0.1 \times 0.1}{2} = 1 + 0.01 \times 0.5$$

$y_1 = 1.005$ , $x_1 = 0.1$ , $h = 0.1$

$n = 1$

$$y_2 = y_1 + \frac{h}{2}\left[ y'_1 + \bar{y}'_{1+1} \right]$$

$$y'_1 = f(x_1, y_1) = x_1 * y_1 = 0.1 * 1.005 = 0.1005$$

$$\bar{y}'_2 = f(x_2, \bar{y}_2) = x_2 * y_2 \ldots$$

$$x_2 = x_1 + h = 0.1 + 0.1 = 0.2$$

$$\bar{y}_2 = y_1 + h\, f(x_1, y_1)$$

$$= y_1 + h\,( x_1 * y_1)$$

$$= 1.005 + 0.1(0.1 * 1.005)$$

$$= 1.005 + 0.01005$$

$$\bar{y}_2 = 1.01505$$

$$\bar{y}'_2 = x_2 * \bar{y}_2 = 0.2 * 1.01505$$

$$= 0.20301$$

$$\therefore y_2 = y_1 + \frac{h}{2}\left[ y'_1 + \bar{y}'_2 \right] = 1.005 + \frac{0.1}{2}\{0.1005 + 0.20\ldots$$

$$= 1.020175$$

CamScanner الممسوحة ضوئيا بـ

[illegible]

[illegible] $h = 0.1$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] $= 1$

[illegible] $= 0.1$

[illegible] $= 1 + 0.1(1+1) = 1.2$

[illegible] $= 1.1 + 1.2 = 2.3$

[illegible] $[\,\ldots + 2.3\,]$

$= 1 + 0.05\,[\ldots\,1.3\,] = 1.055$

[illegible]

[illegible] $= 0.1,\ h = 0.1$

5

## Runge-Kutta method

The R-K method uses terms through the fourth Derivative
the equations are given below

$$\boxed{y_{n+1} = y_n + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]}$$

$$k_1 = h f(x_n, y_n)$$

$$k_2 = h f(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_1)$$

$$k_3 = h f(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_1)$$

$$k_4 = h f(x_n + h, y_n + k_3)$$

**Ex** $y' = \boxed{-y + x + 1}$ uses Runge-Kutta

$0 \leq x \leq 1$

$y(0) = 1$

$h = 0.1$

$\boxed{k_1 = -y_n + x_n + 1}$

$k_2 = f(x_n + \frac{h}{2}, y_n + \frac{1}{2}k_1)$

$n = 0$

$k_1 = -y_0 + x_0 + 1$

$= -1 + 0 + 1 = \boxed{0}$

$k_2 = 0.1(0 + \frac{1}{2} \times 0.1 - 1 + \frac{1}{2} \times 0) + 1$

$= 0.005$

$$k_3 = hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_2)$$
$$= 0.1(0 + \tfrac{1}{2} \times 0.1 - 1 + \tfrac{1}{2} \times 0.005 + 1)$$
$$= 0.47$$

$$k_4 = hf(x_n + h, y_n + k_3)$$
$$= 0.1(0 \times 0.1 - 1 \times 0.47 + 1)$$
$$= -0.037$$

$y_{n+1} = y_n$

$$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$
$$= 1 + \frac{1}{6}\{0 + 2 \times 0.005 + 2 \times 0.47 - 0.037]$$
$$= 1.152166.$$

**Exp.** solve $y' = x - y$ for initial condition $x_0 = 0,\ y_0 = 2,\ h = 0.1$
use R-K method

$$k_1 = hf(x_n, y_n) \quad ; k_1 = h(x-y)$$
$$= 0.1(0-2) = -0.2$$

$$k_2 = hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_1)$$

$x_0 = 0$
$0 + \frac{1}{2} \times 0.1 = 0.05$

$$= 0.1(0.05 - [2 + \tfrac{1}{2}(-0.2)]) = -0.185$$

$$k_3 = hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_2)$$
$$= 0.1(0.05 - [2 + \tfrac{1}{2}(-0.185)]) = -0.18575$$

$$k_4 = hf(x_n + h, y_n + k_3)$$
$$= 0.1\{0.1 - (2 - 0.18575)\} = 0.171425$$

$$y_{n+1} = y_n + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$y_1 = 2 + \frac{1}{6}\left[-0.2 + 2(-0.185) + 2(-0.18575) - 0.171425\right]$$

$$= 1.814525$$

Find $X_3 = ?$

$Y_3 = ?$

H.W Solve by Runge-Kutta

$$f(x,y) = 3x + \frac{y}{2}$$

$y_0 = 1$, $h = 0.1$ $\quad 0 \le X \le 1$

$y_1 ?$

H.W $y' = \frac{x-y}{2}$ using Runge-Ku

$y_0 = 1$, $x_0 = 0$, $h = 0.1$

$y_1 = ?$

// Solve by S.R [illegible]

(5)

$$\int_0^3 \frac{1}{5+x^5} \qquad n=6$$

$$\frac{b-a}{n} = \Delta x = \frac{3-0}{6} = \frac{1}{2}$$

| $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ |
|---|---|---|---|---|---|---|
| 0 | $\frac{1}{2}$ | 1 | $3/2$ | 2 | $5/2$ | 3 |

$$I = \frac{\Delta x}{3}\Big[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) + f(x_6)$$

$$= \frac{\frac{1}{2}}{3}\Big[f(0) + 4f(\tfrac{1}{2}) + 2f(1) + 4f(\tfrac{3}{2}) + 2f(2) + 4f(\tfrac{5}{2}) + f(3)$$

$$= \frac{1}{6}\Big[\frac{1}{5} + 4 * \frac{32}{161} + 2 * \frac{1}{6} + 4 * \frac{32}{403} + 2 * \frac{1}{37} + 4 * \frac{32}{3285} + \frac{1}{248}$$

$$= \frac{1}{6}\big[1.743\big] = 0.29$$

تكامل

Exp// Integrate $f(x) = x^3$ between $x=1$ and $x=2$

(6)

$$\int_1^2 x^3\,dx = \frac{1}{4}x^4\Big]_1^2 = \frac{1}{4}(2^4 - 1^4) = 3.75$$

حل تكامل اعتيادي

وما بالتحليل العددي نستخدم ① طريقة المستطيلات

let $n = 4$

$$\Delta x = \frac{2-1}{4} = 0.25$$

نصف نقطة طريقة المستطيلات في midpoint

the midpoint is $\frac{0.25}{2} = 0.125$

$$\int_a^b f(x)\,dx = \Delta x\left[f(x_1^*) + f(x_2^*) + f(x_3^*) + f(x_4^*)\right]$$

$x_1^* = 1 + 0.125 = 1.125$ نضيف اول نقطة ثابتة نصف المسافة midpoint

$x_2^* = 1.125 + 0.25 = 1.375$

$x_3^* = 1.375 + 0.25 = 1.625$

$x_4^* = 1.625 + 0.25 = 1.875$

$$\therefore I = 0.25\left[f(1.125) + f(1.375) + f(1.625) + f(1.875)\right]$$

$$= 0.25\left[1.42 + 2.60 + 4.29 + 6.59\right]$$

$$= 3.725$$

$n=4$

$$\int_1^b f(x)\,dx = \Delta x\left[\frac{1}{2}f(a)+f(x_1)+f(x_2)+f(x_3)+\frac{1}{2}f(b)\right]$$

$$\Delta x = h = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4} = 0.25$$

$x_0 = a = 1$

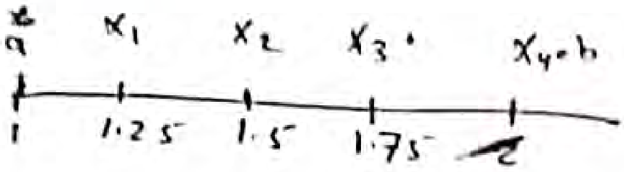
$x_1 = x_0 + h$

$= 1 + 0.25 = 1.25$

$x_2 = x_1 + h = 1.25 + 0.25 = 1.5$

$x_3 = 1.5 + 0.25 = 1.75$

$b = 2$




$$I = \int f(x)\,dx$$

$$= \Delta x\left[\frac{1}{2}f(x_0)+f(x_1)+f(x_2)+f(x_3)+\frac{1}{2}f(x_4)\right]$$

$$= 0.25\left[\frac{1}{2}f(1)+f(1.25)+f(1.5)+f(1.75)+\frac{1}{2}f\right.$$

$$= 0.25\left[\frac{1}{2}\times 1 + 1.25 + 3.38 + 5.36 + \frac{1}{2}\times 8\right.$$

$$= 0.25\,(15.19)$$

$$= 3.8$$

Simpson's rule الطريقة ③

$$\int_a^b f(x)\,dx = \frac{\Delta x}{3}\left[f(a)+4f(x_1)+2f(x_2)+4\right.$$

$$\left.+ f(x_b)\right]$$

$$= \frac{0.25}{3}\left[f(1)+4f(1.25)+2f(1.5)+4f(1.\right.$$

$$= \frac{0.25}{3}(45) = 3.75$$

Solve by T.R & S.R H.w ...

1) $\int_1^8 (3x+2)\,dx \qquad h = 1 = \Delta x = 1$

شبه المنحرف

1) T.R

$$I = \Delta x \left[ \tfrac{1}{2} f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + \tfrac{1}{2} f(x_7) \right.$$

نفس القانون or

$$I = \frac{\Delta x}{2} \left[ f_0 + f_7 + 2(f_1 + f_2 + f_3 + f_4 + f_5 + f_6 \right.$$

$$= 1 \left[ \tfrac{1}{2} f(1) + f(2) + f(3) + f(4) + f(6) + f(6) + f(7) + \tfrac{1}{2} f(8) \right)$$

$$= 1 \left[ \tfrac{1}{2} \times 5 + 8 + 11 + 14 + 18 + 20 + 23 + \tfrac{1}{2} \times 26 = \right.$$

2) S-R سمبسون

$$I = \frac{\Delta x}{3} \left[ f(x_0) + 4 f(x_1) + 2 f(x_2) + 4 f(x_3) + 2 f(x_4) + 4 f(x_5) + 2 f(x_6) + 4 f(x_7) \right]$$

$$= \frac{1}{3} \left[ f(1) + 4 f(2) + 2 f(3) + 4 f(4) + 2 f( + 4 f(6) + 2 f(7) + f(8) \right)$$

$$= \frac{1}{3} \left[ 5 + 4 \times 8 + 2 \times 11 + 4 \times 14 + 2 \times 18 + 4 + 2 \times 23 + 26 \right]$$

$=$

Numerical Analysis

Chapter five

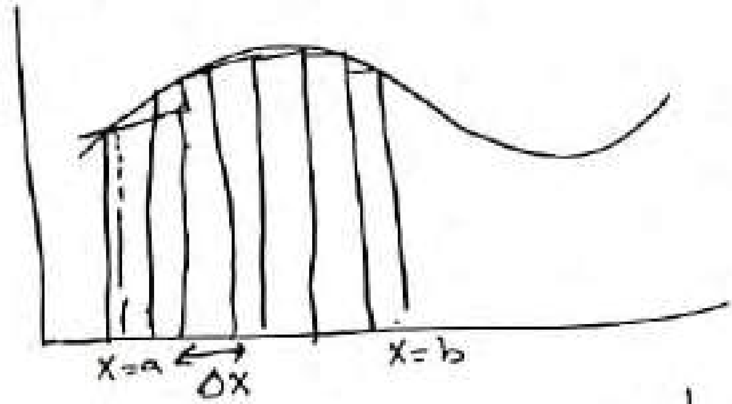
## (( Integral equation المعادلات التكاملية

### (1) Rectangular Rule قاعدة المستطيلات

Approximate the integration

$$\int_a^b f(x)\,dx$$

that is the Area under the curve by a series of rectangles as shown




The base of each of these rectangles is $h = \Delta x = \frac{b-a}{n}$

$$I \simeq \int_a^b f(x)\,dx = f(x_1^*)\,\Delta x + f(x_2^*)\,\Delta \ldots + f(x_n)\,\Delta x$$

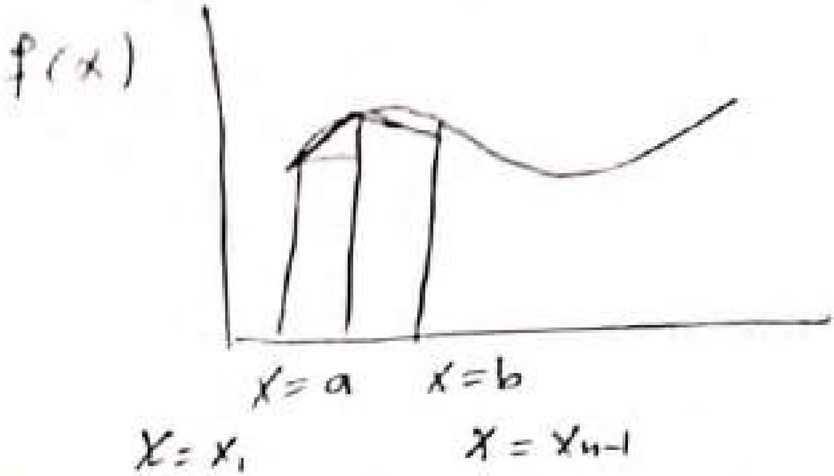
حفظ قاعدة المستطيل

$$I \text{ or } = \Delta x \left[ f(x_1^*) + f(x_2^*) + \cdots f(x \ldots \right.$$

## ② Trepezoidal Rule

قاعدة شبه المنحرف

the rectangular rule can be made more accurate by using trapezoids to replace the rectangle as shown




A Linear approximation of the function locally sometimes work much better than using the averged value like the rectangular rule does

$$\int_a^b f(x)\,dx = \frac{\Delta x}{2}\left[f(a)+f(x_1)\right]+\frac{\Delta x}{2}\left[f(x_1)+f(x_2)\right]+\cdots+\frac{\Delta x}{2}\left[f(x_{n-1})+f(b)\right]$$

شبه المنحرف

$$= \Delta x\left[\frac{1}{2}f(a)+f(x_1)+f(x_{n-1})+\frac{1}{2}f(b)\right]$$

(3)

Solve by T.R

$$\int_0^3 \frac{1}{5+x^5}\,dx \qquad n=6$$

$$y=f(x)$$

Solve

$$\Delta x=\frac{b-a}{n}=\frac{3-0}{6}=\frac{1}{2}$$

| $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ |
|---|---|---|---|---|---|---|
| $0$ | $\frac{1}{2}$ | $1$ | $\frac{3}{2}$ | $2$ | $\frac{5}{2}$ | $3$ |

$$I=\Delta x\left[\tfrac{1}{2}f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)+\tfrac{1}{2}f(x_6)\right]$$

$y_0 \quad y_1$

$$=\frac{1}{2}\left[\tfrac{1}{2}\times f(0)+f(\tfrac{1}{2})+f(1)+f(\tfrac{3}{2})+f(2)+f(\tfrac{5}{2})+\tfrac{1}{2}f(3)\right]$$

$$=\frac{1}{2}\left[\tfrac{1}{2}\times\tfrac{1}{5}+\frac{32}{161}+\frac{1}{6}+\frac{32}{403}+\frac{1}{37}+\frac{32}{3285}+\tfrac{1}{2}\times\frac{1}{248}\right]$$

$$=0.292$$

of numerical analysis (1)

## 4) Simpson's Rule

The simpson's rule and the formula is given as

$$\int_a^b f(x)\,dx = \frac{\Delta x}{3}\Big[f(a)+4f(x_1)+2f(x_2)+4f(x_3) + \cdots + 2f(x_{n-2})+4f(x_{n-1})+f(b)\Big]$$

$X_n = b$ for trapezoidal rule

$$\int_{x_0=a}^{x_n=b} f(x)\,dx = \Big[y_0+2y_1+2y_2+2y_3+\cdots+2y_{n-1}+y_n\Big]$$

For simpson's rule

$$\int_{x_0=a}^{x_n=b} = \frac{h}{3}\Big[y_0+4y_1+2y_2+4y_3+2y_4+4y_5+2y_6+\cdots+2y_{n-2}+4y_{n-1}$$

ملاحظة ⇐ الفردي نضربه × ٤

الزوجي نضربه × ٢

$$= \frac{h}{3}\Big[y_0+y_{n-1}+2(y_2+y_4+y_6)+4(y_1+y_3+y_5)$$